

# Computer algebra independent integration tests

Summer 2022 edition

4-Trig-functions/4.1-Sine/75-4.1.2.3-g-sin<sup>p</sup>-a+b-sin<sup>m</sup>-c+d-sin<sup>n</sup>

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# Chapter 1

## Introduction

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This report gives the result of running the computer algebra independent integration test. The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [ 51 ]. This is test number [ 75 ].

## 1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

## 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 ( 51 )	0.00 ( 0 )
Mathematica	100.00 ( 51 )	0.00 ( 0 )
Maple	98.04 ( 50 )	1.96 ( 1 )
Fricas	60.78 ( 31 )	39.22 ( 20 )
Giac	41.18 ( 21 )	58.82 ( 30 )
Maxima	31.37 ( 16 )	68.63 ( 35 )
Mupad	25.49 ( 13 )	74.51 ( 38 )
Sympy	7.84 ( 4 )	92.16 ( 47 )

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> <li>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.</li> <li>2. antiderivative contains a special function and the optimal antiderivative does not.</li> <li>3. antiderivative contains the imaginary unit and the optimal antiderivative does not.</li> </ol>
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

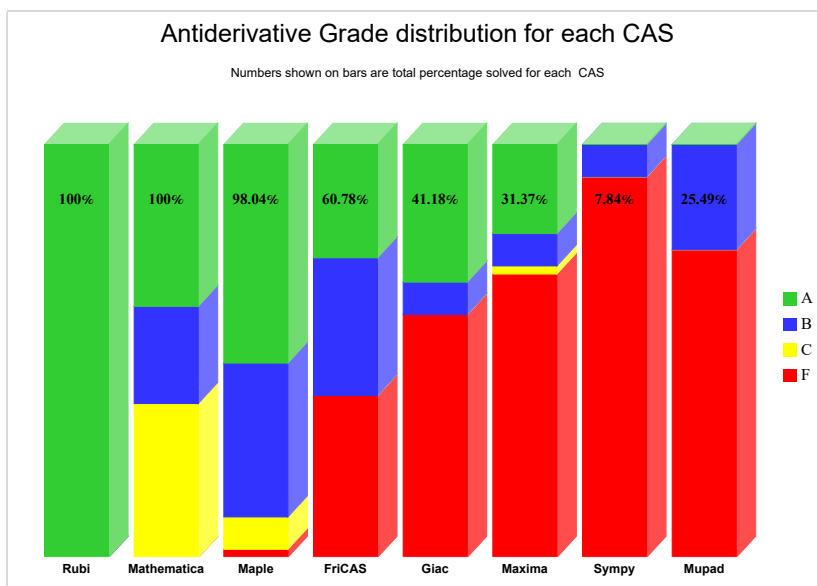
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

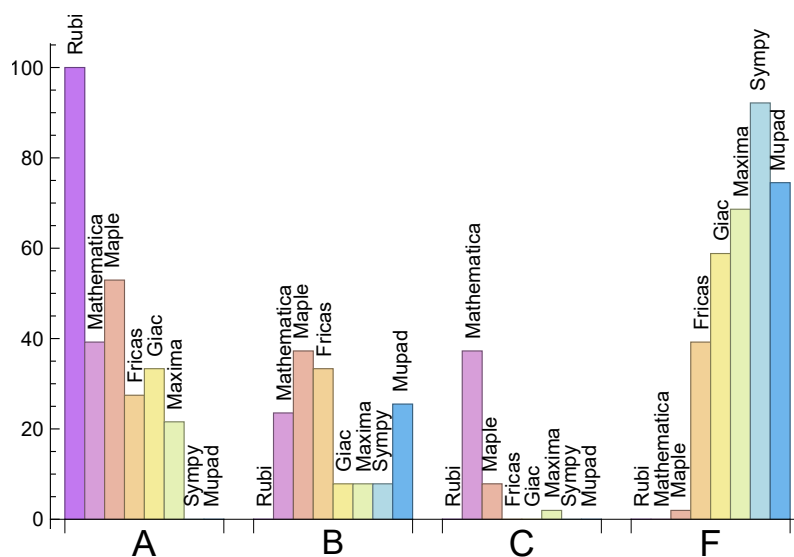
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Maple	52.94	37.25	7.84	1.96
Mathematica	39.22	23.53	37.25	0.00
Giac	33.33	7.84	0.00	58.82
Fricas	27.45	33.33	0.00	39.22
Maxima	21.57	7.84	1.96	68.63
Mupad	N/A	25.49	0.00	74.51
Sympy	0.00	7.84	0.00	92.16

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This is the typical normal failure **F**.

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	1	100.00 %	0.00 %	0.00 %
Fricas	20	50.00 %	45.00 %	5.00 %
Giac	30	70.00 %	30.00 %	0.00 %
Maxima	35	97.14 %	0.00 %	2.86 %
Sympy	47	89.36 %	6.38 %	4.26 %
Mupad	38	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS



## 1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

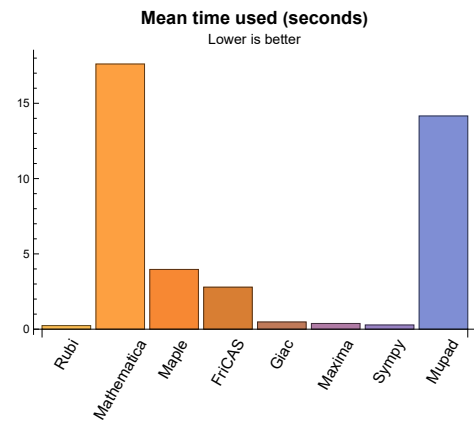
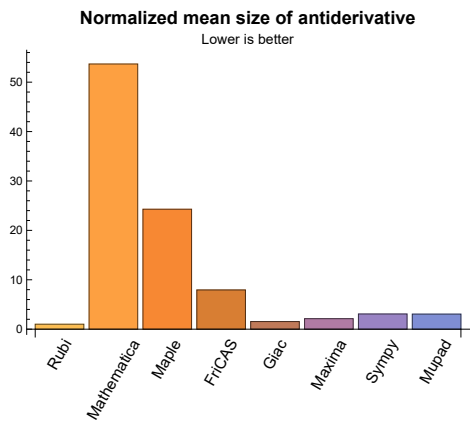
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.24	146.39	1.01	121.00	1.00
Mathematica	17.61	8158.49	53.68	204.00	1.90
Maple	3.97	7635.68	24.28	241.00	2.06
Maxima	0.38	153.50	2.13	131.00	1.68
Fricas	2.79	993.26	7.94	257.00	3.58
Sympy	0.28	273.50	3.08	273.00	3.16
Giac	0.48	133.24	1.54	123.00	1.41
Mupad	14.16	354.23	3.05	163.00	2.21

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.



## 1.4 list of integrals that has no closed form antiderivative

{

## 1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

## 1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

**Rubi** {}

**Mathematica** {26, 27, 28, 31, 32, 33, 35, 36, 37, 38, 42, 43, 44, 45, 46, 47}

**Maple** Verification phase not implemented yet.

**Maxima** Verification phase not implemented yet.

**Fricas** Verification phase not implemented yet.

**Sympy** Verification phase not implemented yet.

**Giac** Verification phase not implemented yet.

**Mupad** Verification phase not implemented yet.

## 1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

## 1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

## 1.9 Important notes about some of the results

### 1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima `integrate` was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

### 1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

### 1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at [https://ask.sagemath.org/question/57123/could-we-have-a-leaf\\_count-function-in-base-sagemath/](https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/)

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount =1
```

### 1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

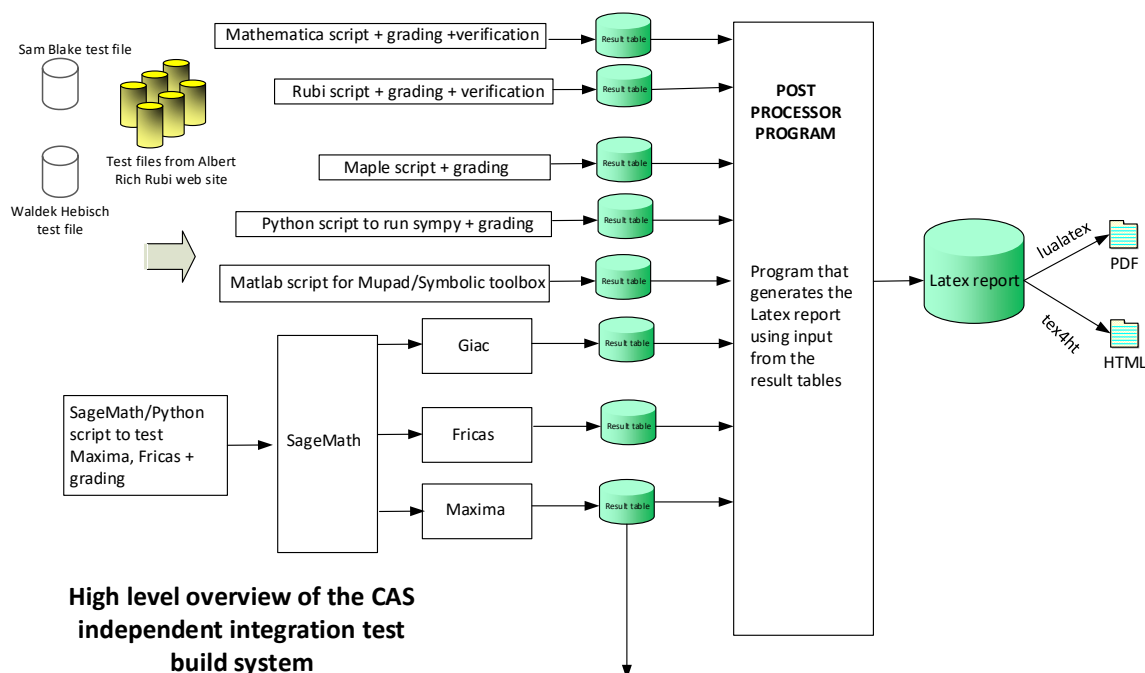
```
integrand = evalin(symengine, 'cos(x)*sin(x)')
the_variable = evalin(symengine, 'x')
anti = int(integrand,the_variable)
```

Which gives  $\sin(x)^2/2$



## 1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified.

*The following fields are present only in Rubi Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form "{n,n,..}" which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntax



# Chapter 2

## detailed summary tables of results

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## 2.1 List of integrals sorted by grade for each CAS

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### 2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51 }

B grade: { }

C grade: { }

F grade: { }

### 2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 10, 11, 12, 16, 17, 18, 19, 22, 39, 48, 49, 50, 51 }

B grade: { 8, 9, 13, 27, 28, 32, 33, 37, 43, 44, 45, 47 }

C grade: { 14, 15, 20, 21, 23, 24, 25, 26, 29, 30, 31, 34, 35, 36, 38, 40, 41, 42, 46 }

F grade: { }

### 2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 21, 23, 24, 29, 30, 39, 40, 41 }

B grade: { 15, 22, 25, 26, 27, 28, 32, 33, 34, 35, 36, 37, 38, 43, 44, 45, 47, 49, 50 }

C grade: { 31, 42, 46, 48 }

F grade: { 51 }

### 2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 10, 11, 20, 21 }

B grade: { 8, 9, 16, 17 }

C grade: { 22 }

F grade: { 12, 13, 14, 15, 18, 19, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51 }

### 2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 12, 15, 16, 17, 18, 19, 22, 27 }

B grade: { 7, 8, 9, 10, 11, 13, 14, 23, 24, 25, 26, 28, 35, 36, 37, 38, 39 }

C grade: { }

F grade: { 20, 21, 29, 30, 31, 32, 33, 34, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51 }

### 2.1.6 Sympy

A grade: { }

B grade: { 1, 2, 3, 4 }

C grade: { }

F grade: { 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51 }

### 2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 7, 9, 10, 12, 13, 14, 19, 20, 21, 23, 24, 39 }

B grade: { 6, 8, 11, 22 }

C grade: { }

F grade: { 15, 16, 17, 18, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51 }

### 2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 16, 39 }

C grade: { }

F grade: { 12, 13, 14, 15, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51 }

## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column N.S. in the table below, which stands for **normalized size** is defined as  $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$ . To help make the table fit, **Mathematica** was abbrevi-

	Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
viated to MMA.	grade	A	A	A	A	A	A	B	A	B
	verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
	size	121	121	77	147	159	97	415	119	256
	N.S.	1	1.00	0.64	1.21	1.31	0.80	3.43	0.98	2.12
	time (sec)	N/A	0.126	0.092	0.197	0.294	0.369	0.464	0.445	14.238

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	57	126	133	82	301	81	212
N.S.	1	1.00	0.59	1.31	1.39	0.85	3.14	0.84	2.21
time (sec)	N/A	0.102	0.062	0.145	0.293	0.367	0.316	0.453	14.183

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	47	106	108	67	245	62	250
N.S.	1	1.00	0.61	1.38	1.40	0.87	3.18	0.81	3.25
time (sec)	N/A	0.072	0.079	0.110	0.286	0.360	0.205	0.418	14.123

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	43	78	83	49	133	62	125
N.S.	1	1.00	0.83	1.50	1.60	0.94	2.56	1.19	2.40
time (sec)	N/A	0.042	0.242	0.090	0.285	0.383	0.131	0.453	14.301

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	61	76	79	80	0	111	88
N.S.	1	1.00	0.97	1.21	1.25	1.27	0.00	1.76	1.40
time (sec)	N/A	0.063	0.060	0.156	0.372	0.397	0.000	0.435	12.892

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	97	61	74	107	0	137	110
N.S.	1	1.00	1.83	1.15	1.40	2.02	0.00	2.58	2.08
time (sec)	N/A	0.086	0.033	0.122	0.301	0.391	0.000	0.483	12.394

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	95	89	113	147	0	123	163
N.S.	1	1.00	1.48	1.39	1.77	2.30	0.00	1.92	2.55
time (sec)	N/A	0.077	0.474	0.230	0.295	0.391	0.000	0.430	12.262

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	172	100	129	148	0	147	132
N.S.	1	1.00	2.82	1.64	2.11	2.43	0.00	2.41	2.16
time (sec)	N/A	0.111	0.058	0.203	0.298	0.371	0.000	0.449	12.223



Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	179	129	178	178	0	148	133
N.S.	1	1.00	2.08	1.50	2.07	2.07	0.00	1.72	1.55
time (sec)	N/A	0.110	0.054	0.243	0.295	0.361	0.000	0.489	12.184

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	204	152	202	217	0	184	244
N.S.	1	1.00	1.94	1.45	1.92	2.07	0.00	1.75	2.32
time (sec)	N/A	0.119	0.045	0.259	0.293	0.379	0.000	0.511	12.369

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	204	174	251	257	0	256	340
N.S.	1	1.00	1.57	1.34	1.93	1.98	0.00	1.97	2.62
time (sec)	N/A	0.137	0.051	0.248	0.299	0.376	0.000	0.487	12.542

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	165	101	78	0	155	0	99	-1
N.S.	1	1.29	0.79	0.61	0.00	1.21	0.00	0.77	-0.01
time (sec)	N/A	0.231	0.534	5.647	0.000	0.369	0.000	0.475	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	157	78	0	221	0	115	-1
N.S.	1	1.00	2.28	1.13	0.00	3.20	0.00	1.67	-0.01
time (sec)	N/A	0.204	0.235	6.502	0.000	0.468	0.000	0.559	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	234	124	0	359	0	147	-1
N.S.	1	1.00	1.95	1.03	0.00	2.99	0.00	1.22	-0.01
time (sec)	N/A	0.296	0.299	7.604	0.000	0.416	0.000	0.519	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	194	914	0	480	0	0	-1
N.S.	1	1.00	1.88	8.87	0.00	4.66	0.00	0.00	-0.01
time (sec)	N/A	0.310	0.631	28.704	0.000	0.535	0.000	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	40	45	331	44	0	0	52
N.S.	1	1.00	0.93	1.05	7.70	1.02	0.00	0.00	1.21
time (sec)	N/A	0.134	0.154	0.293	0.538	0.359	0.000	0.000	12.939

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	133	187	288	416	0	0	-1
N.S.	1	1.00	1.17	1.64	2.53	3.65	0.00	0.00	-0.01
time (sec)	N/A	0.322	0.228	0.325	0.524	0.483	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	132	117	0	422	0	0	-1
N.S.	1	1.00	1.12	0.99	0.00	3.58	0.00	0.00	-0.01
time (sec)	N/A	0.324	0.194	0.318	0.000	0.512	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	62	74	0	216	0	60	-1
N.S.	1	1.00	1.35	1.61	0.00	4.70	0.00	1.30	-0.02
time (sec)	N/A	0.111	0.072	2.620	0.000	0.575	0.000	0.462	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	145	111	63	0	0	1	-1
N.S.	1	1.00	1.42	1.09	0.62	0.00	0.00	0.01	-0.01
time (sec)	N/A	0.294	0.980	0.259	0.511	0.000	0.000	0.442	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	148	113	63	0	0	117	-1
N.S.	1	1.00	1.48	1.13	0.63	0.00	0.00	1.17	-0.01
time (sec)	N/A	0.300	0.997	0.248	0.544	0.000	0.000	0.470	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	63	111	202	205	0	187	-1
N.S.	1	1.00	1.37	2.41	4.39	4.46	0.00	4.07	-0.02
time (sec)	N/A	0.124	0.126	0.246	0.642	0.475	0.000	0.520	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	746	120	0	833	0	148	-1
N.S.	1	1.00	7.10	1.14	0.00	7.93	0.00	1.41	-0.01
time (sec)	N/A	0.201	23.957	9.704	0.000	0.764	0.000	0.495	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	331	208	0	1118	0	195	-1
N.S.	1	1.00	2.01	1.26	0.00	6.78	0.00	1.18	-0.01
time (sec)	N/A	0.314	1.583	10.938	0.000	1.735	0.000	0.501	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	661	1025	0	3409	0	0	-1
N.S.	1	1.00	4.44	6.88	0.00	22.88	0.00	0.00	-0.01
time (sec)	N/A	0.339	50.498	0.513	0.000	1.704	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	436	526	0	1341	0	0	-1
N.S.	1	1.00	5.25	6.34	0.00	16.16	0.00	0.00	-0.01
time (sec)	N/A	0.142	47.758	0.306	0.000	1.053	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	A	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	2568	613	0	3170	0	0	-1
N.S.	1	1.00	15.47	3.69	0.00	19.10	0.00	0.00	-0.01
time (sec)	N/A	0.350	24.660	0.366	0.000	1.812	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	3062	621	0	3297	0	0	-1
N.S.	1	1.00	18.23	3.70	0.00	19.62	0.00	0.00	-0.01
time (sec)	N/A	0.355	24.890	0.265	0.000	1.884	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	452	593	0	0	0	0	-1
N.S.	1	1.00	1.90	2.49	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.341	16.357	33.063	0.000	0.000	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	462	587	0	0	0	0	-1
N.S.	1	1.00	1.88	2.39	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.331	14.627	30.576	0.000	0.000	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	13199	22961	0	0	0	0	-1
N.S.	1	1.00	49.43	86.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.341	87.344	0.961	0.000	0.000	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	3415	6804	0	0	0	0	-1
N.S.	1	1.00	29.44	58.66	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.133	84.594	0.333	0.000	0.000	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	5708	6815	0	0	0	0	-1
N.S.	1	1.00	22.65	27.04	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.332	84.565	0.435	0.000	0.000	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	1661	9043	0	0	0	0	-1
N.S.	1	1.00	6.49	35.32	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.351	33.514	0.368	0.000	0.000	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	567	4546	0	3671	0	0	-1
N.S.	1	1.00	4.61	36.96	0.00	29.85	0.00	0.00	-0.01
time (sec)	N/A	0.307	2.956	0.343	0.000	1.136	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	367	238	0	1080	0	0	-1
N.S.	1	1.00	6.02	3.90	0.00	17.70	0.00	0.00	-0.02
time (sec)	N/A	0.128	2.943	20.056	0.000	0.899	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	2506	347	0	2911	0	0	-1
N.S.	1	1.00	17.90	2.48	0.00	20.79	0.00	0.00	-0.01
time (sec)	N/A	0.339	27.178	0.283	0.000	0.748	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	309693	364	0	3125	0	0	-1
N.S.	1	1.00	2212.09	2.60	0.00	22.32	0.00	0.00	-0.01
time (sec)	N/A	0.320	42.856	0.241	0.000	0.825	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	178	244	0	2889	0	299	2500
N.S.	1	1.00	0.98	1.35	0.00	15.96	0.00	1.65	13.81
time (sec)	N/A	0.355	0.778	1.969	0.000	65.596	0.000	0.559	27.407

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	179	190	0	0	0	0	-1
N.S.	1	1.00	1.16	1.23	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.339	23.283	11.436	0.000	0.000	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	203	254	0	0	0	0	-1
N.S.	1	1.00	1.39	1.74	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.320	23.395	10.957	0.000	0.000	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	23019	6211	0	0	0	0	-1
N.S.	1	1.00	90.63	24.45	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.335	39.210	2.960	0.000	0.000	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	8202	3298	0	0	0	0	-1
N.S.	1	1.00	32.81	13.19	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.344	48.673	0.289	0.000	0.000	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	3427	2933	0	0	0	0	-1
N.S.	1	1.00	30.06	25.73	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.138	47.734	0.349	0.000	0.000	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	4935	3698	0	0	0	0	-1
N.S.	1	1.00	20.06	15.03	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.366	49.277	0.293	0.000	0.000	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	23019	6200	0	0	0	0	-1
N.S.	1	1.00	90.63	24.41	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.342	39.501	0.719	0.000	0.000	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	3429	2925	0	0	0	0	-1
N.S.	1	1.00	30.08	25.66	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.139	47.477	0.389	0.000	0.000	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	391	274	268466	0	0	0	0	-1
N.S.	1	1.00	0.70	686.61	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.373	0.190	4.437	0.000	0.000	0.000	0.000	0.000



Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	197	1924	0	0	0	0	-1
N.S.	1	1.00	0.99	9.72	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.132	0.113	0.591	0.000	0.000	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	398	398	374	26840	0	0	0	0	-1
N.S.	1	1.00	0.94	67.44	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.389	1.645	0.424	0.000	0.000	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	168	0	0	0	0	0	-1
N.S.	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.204	0.799	0.204	0.000	0.000	0.000	0.000	0.000

## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio  $\frac{\text{number of rules}}{\text{integrand size}}$  is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [15] had the largest ratio of [40]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	13	4	1.00	32	0.125
2	A	11	4	1.00	32	0.125
3	A	10	5	1.00	30	0.167
4	A	4	4	1.00	24	0.167
5	A	6	5	1.00	30	0.167
6	A	8	7	1.00	32	0.219
7	A	7	5	1.00	32	0.156
8	A	6	6	1.00	32	0.188
9	A	11	5	1.00	32	0.156
10	A	11	4	1.00	32	0.125
11	A	13	4	1.00	32	0.125
12	A	5	5	1.29	34	0.147
13	A	5	5	1.00	34	0.147
14	A	8	7	1.00	34	0.206
15	A	6	6	1.00	40	0.150
16	A	3	3	1.00	40	0.075
17	A	6	6	1.00	40	0.150
18	A	6	6	1.00	40	0.150
19	A	2	2	1.00	36	0.056
20	A	6	6	1.00	36	0.167
21	A	6	6	1.00	36	0.167
22	A	3	3	1.00	36	0.083
23	A	5	4	1.00	33	0.121
24	A	8	6	1.00	33	0.182
25	A	5	4	1.00	39	0.103

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	2	2	1.00	39	0.051
27	A	5	4	1.00	39	0.103
28	A	5	4	1.00	39	0.103
29	A	9	9	1.00	33	0.273
30	A	9	9	1.00	33	0.273
31	A	3	3	1.00	39	0.077
32	A	1	1	1.00	39	0.026
33	A	3	3	1.00	39	0.077
34	A	3	3	1.00	39	0.077
35	A	5	5	1.00	35	0.143
36	A	2	2	1.00	35	0.057
37	A	5	5	1.00	35	0.143
38	A	5	5	1.00	35	0.143
39	A	8	5	1.00	33	0.152
40	A	5	3	1.00	33	0.091
41	A	5	3	1.00	33	0.091
42	A	3	3	1.00	39	0.077
43	A	3	3	1.00	39	0.077
44	A	1	1	1.00	39	0.026
45	A	3	3	1.00	39	0.077
46	A	3	3	1.00	39	0.077
47	A	1	1	1.00	39	0.026
48	A	3	3	1.00	35	0.086
49	A	1	1	1.00	35	0.029
50	A	3	3	1.00	35	0.086
51	A	4	4	1.00	38	0.105



# Chapter 3

## Listing of integrals

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### 3.1 $\int \sin^3(e + fx)(a + a \sin(e + fx))^2(c - c \sin(e + fx)) dx$

**Optimal.** Leaf size=121

$$\frac{1}{16}a^2cx - \frac{a^2c \cos^3(e + fx)}{3f} + \frac{a^2c \cos^5(e + fx)}{5f} - \frac{a^2c \cos(e + fx) \sin(e + fx)}{16f} - \frac{a^2c \cos(e + fx) \sin^3(e + fx)}{24f} + \frac{a^2c \cos^3(e + fx) \sin(e + fx)}{24f}$$

[Out]  $1/16*a^2*c*x - 1/3*a^2*c*\cos(f*x+e)^3/f + 1/5*a^2*c*\cos(f*x+e)^5/f - 1/16*a^2*c*\cos(f*x+e)*\sin(f*x+e)/f - 1/24*a^2*c*\cos(f*x+e)*\sin(f*x+e)^3/f + 1/6*a^2*c*\cos(f*x+e)*\sin(f*x+e)^5/f$

**Rubi [A]**

time = 0.13, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 4, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {3045, 2713, 2715, 8}

$$\frac{a^2c \cos^5(e + fx)}{5f} - \frac{a^2c \cos^3(e + fx)}{3f} + \frac{a^2c \sin^5(e + fx) \cos(e + fx)}{6f} - \frac{a^2c \sin^3(e + fx) \cos(e + fx)}{24f} - \frac{a^2c \sin(e + fx) \cos(e + fx)}{16f} + \frac{1}{16}a^2cx$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sin}[e + f*x]^3*(a + a*\text{Sin}[e + f*x])^2*(c - c*\text{Sin}[e + f*x]), x]$

[Out]  $(a^2*c*x)/16 - (a^2*c*\text{Cos}[e + f*x]^3)/(3*f) + (a^2*c*\text{Cos}[e + f*x]^5)/(5*f) - (a^2*c*\text{Cos}[e + f*x]*\text{Sin}[e + f*x])/(16*f) - (a^2*c*\text{Cos}[e + f*x]*\text{Sin}[e + f*x]^3)/(24*f) + (a^2*c*\text{Cos}[e + f*x]*\text{Sin}[e + f*x]^5)/(6*f)$

**Rule 8**

$\text{Int}[a_, x\_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

**Rule 2713**

$\text{Int}[\text{sin}[(c_.) + (d_.)*(x_)]^{(n_)}, x\_Symbol] \rightarrow \text{Dist}[-d^{(-1)}, \text{Subst}[\text{Int}[\text{Expand}[(1 - x^2)^{((n - 1)/2)}, x], x], x, \text{Cos}[c + d*x]], x] /; \text{FreeQ}[\{c, d\}, x] \&\& \text{IGtQ}[(n - 1)/2, 0]$

**Rule 2715**

$\text{Int}[(b_.)*\text{sin}[(c_.) + (d_.)*(x_)]^{(n_)}, x\_Symbol] \rightarrow \text{Simp}[(-b)*\text{Cos}[c + d*x]*((b*\text{Sin}[c + d*x])^{(n - 1)})/(d*n), x] + \text{Dist}[b^2*((n - 1)/n), \text{Int}[(b*\text{Sin}[c + d*x])^{(n - 2)}, x], x] /; \text{FreeQ}[\{b, c, d\}, x] \&\& \text{GtQ}[n, 1] \&\& \text{IntegerQ}[2*n]$

**Rule 3045**

$\text{Int}[\text{sin}[(e_.) + (f_.)*(x_)]^{(n_.)}*((a_.) + (b_.)*\text{sin}[(e_.) + (f_.)*(x_)]^{(m_.)}*((A_.) + (B_.)*\text{sin}[(e_.) + (f_.)*(x_)]), x\_Symbol] \rightarrow \text{Int}[\text{ExpandTrig}[\text{si}$



`n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && IntegerQ[n]`

Rubi steps

$$\begin{aligned}
 \int \sin^3(e + fx)(a + a \sin(e + fx))^2(c - c \sin(e + fx)) dx &= \int (a^2 c \sin^3(e + fx) + a^2 c \sin^4(e + fx) - a^2 c \sin^5(e + fx)) dx \\
 &= (a^2 c) \int \sin^3(e + fx) dx + (a^2 c) \int \sin^4(e + fx) dx - (a^2 c) \int \sin^5(e + fx) dx \\
 &= -\frac{a^2 c \cos(e + fx) \sin^3(e + fx)}{4f} + \frac{a^2 c \cos(e + fx) \sin^4(e + fx)}{6f} - \frac{a^2 c \cos(e + fx) \sin^5(e + fx)}{6f} \\
 &= -\frac{a^2 c \cos^3(e + fx)}{3f} + \frac{a^2 c \cos^5(e + fx)}{5f} - \frac{3a^2 c \cos^6(e + fx)}{6f} \\
 &= \frac{3}{8} a^2 c x - \frac{a^2 c \cos^3(e + fx)}{3f} + \frac{a^2 c \cos^5(e + fx)}{5f} - \frac{3a^2 c \cos^6(e + fx)}{6f} \\
 &= \frac{1}{16} a^2 c x - \frac{a^2 c \cos^3(e + fx)}{3f} + \frac{a^2 c \cos^5(e + fx)}{5f}
 \end{aligned}$$

**Mathematica [A]**

time = 0.09, size = 77, normalized size = 0.64

$$\frac{a^2 c (60e + 60fx - 120 \cos(e + fx) - 20 \cos(3(e + fx)) + 12 \cos(5(e + fx)) - 15 \sin(2(e + fx)) - 15 \sin(4(e + fx)) + 5 \sin(6(e + fx)))}{960f}$$

Antiderivative was successfully verified.

[In] `Integrate[Sin[e + f*x]^3*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x]),x]`

[Out] `(a^2*c*(60*e + 60*f*x - 120*Cos[e + f*x] - 20*Cos[3*(e + f*x)] + 12*Cos[5*(e + f*x)] - 15*Sin[2*(e + f*x)] - 15*Sin[4*(e + f*x)] + 5*Sin[6*(e + f*x)])/(960*f)`

**Maple [A]**

time = 0.20, size = 147, normalized size = 1.21

method	result
risch	$\frac{a^2 c x}{16} - \frac{a^2 c \cos(fx+e)}{8f} + \frac{a^2 c \sin(6fx+6e)}{192f} + \frac{a^2 c \cos(5fx+5e)}{80f} - \frac{a^2 c \sin(4fx+4e)}{64f} - \frac{a^2 c \cos(3fx+3e)}{48f} - \frac{a^2 c \sin(2fx+2e)}{32f} + \frac{a^2 c \cos(fx+e)}{24f}$
derivativedivides	$-a^2 c \left( -\frac{\left( \sin^5(fx+e) + \frac{5 \sin^3(fx+e)}{4} + \frac{15 \sin(fx+e)}{8} \right) \cos(fx+e)}{6} + \frac{5fx + 5e}{16} \right) + \frac{a^2 c \left( \frac{8}{3} + \sin^4(fx+e) + \frac{4 \sin^2(fx+e)}{3} \right) \cos(fx+e)}{5}$

default	$-a^2c \left( -\frac{\left( \sin^5(fx+e) + \frac{5(\sin^3(fx+e))}{4} + \frac{15\sin(fx+e)}{8} \right) \cos(fx+e)}{6} + \frac{5fx}{16} + \frac{5e}{16} \right) + \frac{a^2c \left( \frac{8}{3} + \sin^4(fx+e) + \frac{4(\sin^2(fx+e))}{3} \right) \cos(fx+e)}{5}$
norman	$\frac{-\frac{4a^2c}{15f} + \frac{a^2cx}{16} - \frac{8a^2c(\tan^6(\frac{fx}{2} + \frac{e}{2}))}{3f} - \frac{8a^2c(\tan^2(\frac{fx}{2} + \frac{e}{2}))}{5f} - \frac{4a^2c(\tan^8(\frac{fx}{2} + \frac{e}{2}))}{f} - \frac{a^2c \tan(\frac{fx}{2} + \frac{e}{2})}{8f} - \frac{17a^2c(\tan^3(\frac{fx}{2} + \frac{e}{2}))}{24f}}{f}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(f*x+e)^3*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x,method=_RETURNVERBOS E)`

[Out]  $1/f * (-a^2c * (-1/6 * (\sin(f*x+e))^5 + 5/4 * \sin(f*x+e)^3 + 15/8 * \sin(f*x+e)) * \cos(f*x+e) + 5/16 * f*x + 5/16 * e) + 1/5 * a^2c * (8/3 + \sin(f*x+e)^4 + 4/3 * \sin(f*x+e)^2) * \cos(f*x+e) + a^2c * (-1/4 * (\sin(f*x+e))^3 + 3/2 * \sin(f*x+e)) * \cos(f*x+e) + 3/8 * f*x + 3/8 * e) - 1/3 * a^2c * (2 + \sin(f*x+e)^2) * \cos(f*x+e)$

**Maxima** [A]

time = 0.29, size = 159, normalized size = 1.31

$$\frac{64(3 \cos(fx+e)^5 - 10 \cos(fx+e)^3 + 15 \cos(fx+e))a^2c + 320(\cos(fx+e)^3 - 3 \cos(fx+e))a^2c - 5(4 \sin(2fx+2e)^3 + 60fx + 60e + 9 \sin(4fx+4e) - 48 \sin(2fx+2e))a^2c + 30(12fx + 12e + \sin(4fx+4e) - 8 \sin(2fx+2e))a^2c}{960f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(f*x+e)^3*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="maxima")`

[Out]  $1/960 * (64 * (3 * \cos(f*x + e)^5 - 10 * \cos(f*x + e)^3 + 15 * \cos(f*x + e)) * a^2 * c + 320 * (\cos(f*x + e)^3 - 3 * \cos(f*x + e)) * a^2 * c - 5 * (4 * \sin(2 * f * x + 2 * e)^3 + 60 * f * x + 60 * e + 9 * \sin(4 * f * x + 4 * e) - 48 * \sin(2 * f * x + 2 * e)) * a^2 * c + 30 * (12 * f * x + 12 * e + \sin(4 * f * x + 4 * e) - 8 * \sin(2 * f * x + 2 * e)) * a^2 * c) / f$

**Fricas** [A]

time = 0.37, size = 97, normalized size = 0.80

$$\frac{48a^2c \cos(fx+e)^5 - 80a^2c \cos(fx+e)^3 + 15a^2cfx + 5(8a^2c \cos(fx+e)^5 - 14a^2c \cos(fx+e)^3 + 3a^2c \cos(fx+e)) \sin(fx+e)}{240f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(f*x+e)^3*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="fricas")`

[Out]  $1/240 * (48 * a^2 * c * \cos(f*x + e)^5 - 80 * a^2 * c * \cos(f*x + e)^3 + 15 * a^2 * c * f * x + 5 * (8 * a^2 * c * \cos(f*x + e)^5 - 14 * a^2 * c * \cos(f*x + e)^3 + 3 * a^2 * c * \cos(f*x + e)) * \sin(f*x + e)) / f$



$$\begin{aligned} &^9 + 30*\tan(e/2 + (f*x)/2)^{11} + 15*f*x + 90*\tan(e/2 + (f*x)/2)^2*(e + f*x) \\ &+ 225*\tan(e/2 + (f*x)/2)^4*(e + f*x) + 300*\tan(e/2 + (f*x)/2)^6*(e + f*x) + \\ &225*\tan(e/2 + (f*x)/2)^8*(e + f*x) + 90*\tan(e/2 + (f*x)/2)^{10}*(e + f*x) + \\ &15*\tan(e/2 + (f*x)/2)^{12}*(e + f*x) - 64)) / (240*f*(\tan(e/2 + (f*x)/2)^2 + 1) \\ &^6) \end{aligned}$$

### 3.2 $\int \sin^2(e + fx)(a + a \sin(e + fx))^2(c - c \sin(e + fx)) dx$

**Optimal.** Leaf size=96

$$\frac{1}{8}a^2cx - \frac{a^2c \cos^3(e + fx)}{3f} + \frac{a^2c \cos^5(e + fx)}{5f} - \frac{a^2c \cos(e + fx) \sin(e + fx)}{8f} + \frac{a^2c \cos(e + fx) \sin^3(e + fx)}{4f}$$

[Out]  $1/8*a^2*c*x-1/3*a^2*c*\cos(f*x+e)^3/f+1/5*a^2*c*\cos(f*x+e)^5/f-1/8*a^2*c*\cos(f*x+e)*\sin(f*x+e)/f+1/4*a^2*c*\cos(f*x+e)*\sin(f*x+e)^3/f$

**Rubi [A]**

time = 0.10, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {3045, 2715, 8, 2713}

$$\frac{a^2c \cos^5(e + fx)}{5f} - \frac{a^2c \cos^3(e + fx)}{3f} + \frac{a^2c \sin^3(e + fx) \cos(e + fx)}{4f} - \frac{a^2c \sin(e + fx) \cos(e + fx)}{8f} + \frac{1}{8}a^2cx$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sin}[e + f*x]^2*(a + a*\text{Sin}[e + f*x])^2*(c - c*\text{Sin}[e + f*x]), x]$

[Out]  $(a^2*c*x)/8 - (a^2*c*\text{Cos}[e + f*x]^3)/(3*f) + (a^2*c*\text{Cos}[e + f*x]^5)/(5*f) - (a^2*c*\text{Cos}[e + f*x]*\text{Sin}[e + f*x])/(8*f) + (a^2*c*\text{Cos}[e + f*x]*\text{Sin}[e + f*x]^3)/(4*f)$

**Rule 8**

$\text{Int}[a_, x\_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

**Rule 2713**

$\text{Int}[\text{sin}[(c_.) + (d_.)*(x_.)]^{(n_.)}, x\_Symbol] \rightarrow \text{Dist}[-d^{(-1)}, \text{Subst}[\text{Int}[\text{Expand}[(1 - x^2)^{((n - 1)/2)}, x], x], x, \text{Cos}[c + d*x]], x] /; \text{FreeQ}\{c, d\}, x \&\& \text{IGtQ}[(n - 1)/2, 0]$

**Rule 2715**

$\text{Int}[(b_.)*\text{sin}[(c_.) + (d_.)*(x_.)]^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[(-b)*\text{Cos}[c + d*x]*(b*\text{Sin}[c + d*x])^{(n - 1)}/(d*n), x] + \text{Dist}[b^2*((n - 1)/n), \text{Int}[(b*\text{Sin}[c + d*x])^{(n - 2)}, x], x] /; \text{FreeQ}\{b, c, d\}, x \&\& \text{GtQ}[n, 1] \&\& \text{IntegerQ}[2*n]$

**Rule 3045**

$\text{Int}[\text{sin}[(e_.) + (f_.)*(x_.)]^{(n_.)*((a_.) + (b_.)*\text{sin}[(e_.) + (f_.)*(x_.)])^{(m_.)*((A_.) + (B_.)*\text{sin}[(e_.) + (f_.)*(x_.)])}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandTrig}[\text{si}$

```
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned} \int \sin^2(e + fx)(a + a \sin(e + fx))^2(c - c \sin(e + fx)) dx &= \int (a^2c \sin^2(e + fx) + a^2c \sin^3(e + fx) - a^2c \sin^4(e + fx)) dx \\ &= (a^2c) \int \sin^2(e + fx) dx + (a^2c) \int \sin^3(e + fx) dx - (a^2c) \int \sin^4(e + fx) dx \\ &= -\frac{a^2c \cos(e + fx) \sin(e + fx)}{2f} + \frac{a^2c \cos(e + fx)}{4f} - \frac{a^2c \sin^3(e + fx)}{3f} \\ &= \frac{1}{2}a^2cx - \frac{a^2c \cos^3(e + fx)}{3f} + \frac{a^2c \cos^5(e + fx)}{5f} - \frac{a^2c \sin^4(e + fx)}{4f} \\ &= \frac{1}{8}a^2cx - \frac{a^2c \cos^3(e + fx)}{3f} + \frac{a^2c \cos^5(e + fx)}{5f} - \frac{a^2c \sin^4(e + fx)}{4f} \end{aligned}$$

**Mathematica [A]**

time = 0.06, size = 57, normalized size = 0.59

$$\frac{a^2c(60e + 60fx - 60 \cos(e + fx) - 10 \cos(3(e + fx)) + 6 \cos(5(e + fx)) - 15 \sin(4(e + fx)))}{480f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x]),x]
```

```
[Out] (a^2*c*(60*e + 60*f*x - 60*Cos[e + f*x] - 10*Cos[3*(e + f*x)] + 6*Cos[5*(e + f*x)] - 15*Sin[4*(e + f*x)])/(480*f)
```

**Maple [A]**

time = 0.14, size = 126, normalized size = 1.31

method	result
risch	$\frac{a^2cx}{8} - \frac{a^2c \cos(fx+e)}{8f} + \frac{a^2c \cos(5fx+5e)}{80f} - \frac{a^2c \sin(4fx+4e)}{32f} - \frac{a^2c \cos(3fx+3e)}{48f}$
derivativedivides	$\frac{a^2c \left( \frac{8}{3} + \sin^4(fx+e) + \frac{4(\sin^2(fx+e))}{3} \right) \cos(fx+e)}{5} - a^2c \left( -\frac{(\sin^3(fx+e) + \frac{3 \sin(fx+e)}{2}) \cos(fx+e)}{4} + \frac{3fx + 3e}{8} \right) - \frac{a^2c(2 + \sin^2(fx+e))}{3}$
default	$\frac{a^2c \left( \frac{8}{3} + \sin^4(fx+e) + \frac{4(\sin^2(fx+e))}{3} \right) \cos(fx+e)}{5} - a^2c \left( -\frac{(\sin^3(fx+e) + \frac{3 \sin(fx+e)}{2}) \cos(fx+e)}{4} + \frac{3fx + 3e}{8} \right) - \frac{a^2c(2 + \sin^2(fx+e))}{3}$

norman	$\frac{-\frac{4a^2c}{15f} + \frac{a^2cx}{8} - \frac{4a^2c(\tan^6(\frac{fx}{2} + \frac{e}{2}))}{f} + \frac{4a^2c(\tan^4(\frac{fx}{2} + \frac{e}{2}))}{3f} - \frac{4a^2c(\tan^2(\frac{fx}{2} + \frac{e}{2}))}{3f} - \frac{a^2c \tan(\frac{fx}{2} + \frac{e}{2})}{4f} + \frac{3a^2c(\tan^3(\frac{fx}{2} + \frac{e}{2}))}{2f}}$
--------	---

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(f*x+e)^2*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x,method=_RETURNVERBOS E)`

[Out]  $1/f*(1/5*a^2*c*(8/3+\sin(f*x+e)^4+4/3*\sin(f*x+e)^2)*\cos(f*x+e)-a^2*c*(-1/4*(\sin(f*x+e)^3+3/2*\sin(f*x+e))*\cos(f*x+e)+3/8*f*x+3/8*e)-1/3*a^2*c*(2+\sin(f*x+e)^2)*\cos(f*x+e)+a^2*c*(-1/2*\cos(f*x+e)*\sin(f*x+e)+1/2*f*x+1/2*e))$

**Maxima** [A]

time = 0.29, size = 133, normalized size = 1.39

$$\frac{32(3\cos(fx+e)^5 - 10\cos(fx+e)^3 + 15\cos(fx+e))a^2c + 160(\cos(fx+e)^3 - 3\cos(fx+e))a^2c - 15(12fx + 12e + \sin(4fx + 4e) - 8\sin(2fx + 2e))a^2c + 120(2fx + 2e - \sin(2fx + 2e))a^2c}{480f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(f*x+e)^2*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="maxima")`

[Out]  $1/480*(32*(3*\cos(f*x + e)^5 - 10*\cos(f*x + e)^3 + 15*\cos(f*x + e))*a^2*c + 160*(\cos(f*x + e)^3 - 3*\cos(f*x + e))*a^2*c - 15*(12*f*x + 12*e + \sin(4*f*x + 4*e) - 8*\sin(2*f*x + 2*e))*a^2*c + 120*(2*f*x + 2*e - \sin(2*f*x + 2*e))*a^2*c)/f$

**Fricas** [A]

time = 0.37, size = 82, normalized size = 0.85

$$\frac{24a^2c\cos(fx+e)^5 - 40a^2c\cos(fx+e)^3 + 15a^2cfx - 15(2a^2c\cos(fx+e)^3 - a^2c\cos(fx+e))\sin(fx+e)}{120f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(f*x+e)^2*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="fricas")`

[Out]  $1/120*(24*a^2*c*\cos(f*x + e)^5 - 40*a^2*c*\cos(f*x + e)^3 + 15*a^2*c*f*x - 15*(2*a^2*c*\cos(f*x + e)^3 - a^2*c*\cos(f*x + e))*\sin(f*x + e))/f$

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 301 vs.  $2(87) = 174$ .

time = 0.32, size = 301, normalized size = 3.14

$$\begin{cases} \frac{3a^2cx\sin^4(e+fx) - 3a^2cx\sin^2(e+fx)\cos^2(e+fx) + a^2cx\sin^2(e+fx) - 3a^2cx\cos^4(e+fx) + a^2cx\cos^2(e+fx) + a^2c\sin^4(e+fx)\cos(e+fx) + 3a^2c\sin^2(e+fx)\cos(e+fx) + 4a^2c\sin^2(e+fx)\cos^2(e+fx) - a^2c\sin^2(e+fx)\cos(e+fx) + 3a^2c\sin(e+fx)\cos^2(e+fx) - a^2c\sin(e+fx)\cos(e+fx) + \frac{3a^2c\cos^4(e+fx)}{15f} - \frac{3a^2c\cos^2(e+fx)}{3f}}{x(a\sin(e)+a)^2(-c\sin(e)+c)\sin^2(e)} & \text{for } f \neq 0 \\ \text{otherwise} & \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(f\*x+e)\*\*2\*(a+a\*sin(f\*x+e))\*\*2\*(c-c\*sin(f\*x+e)),x)

[Out] Piecewise((-3\*a\*\*2\*c\*x\*sin(e + f\*x)\*\*4/8 - 3\*a\*\*2\*c\*x\*sin(e + f\*x)\*\*2\*cos(e + f\*x)\*\*2/4 + a\*\*2\*c\*x\*sin(e + f\*x)\*\*2/2 - 3\*a\*\*2\*c\*x\*cos(e + f\*x)\*\*4/8 + a\*\*2\*c\*x\*cos(e + f\*x)\*\*2/2 + a\*\*2\*c\*sin(e + f\*x)\*\*4\*cos(e + f\*x)/f + 5\*a\*\*2\*c\*sin(e + f\*x)\*\*3\*cos(e + f\*x)/(8\*f) + 4\*a\*\*2\*c\*sin(e + f\*x)\*\*2\*cos(e + f\*x)\*\*3/(3\*f) - a\*\*2\*c\*sin(e + f\*x)\*\*2\*cos(e + f\*x)/f + 3\*a\*\*2\*c\*sin(e + f\*x)\*cos(e + f\*x)\*\*3/(8\*f) - a\*\*2\*c\*sin(e + f\*x)\*cos(e + f\*x)/(2\*f) + 8\*a\*\*2\*c\*cos(e + f\*x)\*\*5/(15\*f) - 2\*a\*\*2\*c\*cos(e + f\*x)\*\*3/(3\*f), Ne(f, 0)), (x\*(a\*sin(e) + a)\*\*2\*(-c\*sin(e) + c)\*sin(e)\*\*2, True))

**Giac [A]**

time = 0.45, size = 81, normalized size = 0.84

$$\frac{1}{8} a^2 c x + \frac{a^2 c \cos(5 f x + 5 e)}{80 f} - \frac{a^2 c \cos(3 f x + 3 e)}{48 f} - \frac{a^2 c \cos(f x + e)}{8 f} - \frac{a^2 c \sin(4 f x + 4 e)}{32 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(f\*x+e)^2\*(a+a\*sin(f\*x+e))^2\*(c-c\*sin(f\*x+e)),x, algorithm="giac")

[Out] 1/8\*a^2\*c\*x + 1/80\*a^2\*c\*cos(5\*f\*x + 5\*e)/f - 1/48\*a^2\*c\*cos(3\*f\*x + 3\*e)/f - 1/8\*a^2\*c\*cos(f\*x + e)/f - 1/32\*a^2\*c\*sin(4\*f\*x + 4\*e)/f

**Mupad [B]**

time = 14.18, size = 212, normalized size = 2.21

$$\frac{a^2 c (15 e - 30 \tan(\frac{e}{2} + \frac{f x}{2}) - 160 \tan(\frac{e}{2} + \frac{f x}{2})^2 + 180 \tan(\frac{e}{2} + \frac{f x}{2})^3 + 160 \tan(\frac{e}{2} + \frac{f x}{2})^4 - 480 \tan(\frac{e}{2} + \frac{f x}{2})^5 - 180 \tan(\frac{e}{2} + \frac{f x}{2})^6 + 30 \tan(\frac{e}{2} + \frac{f x}{2})^7 + 15 f x + 75 \tan(\frac{e}{2} + \frac{f x}{2})^2 (e + f x) + 150 \tan(\frac{e}{2} + \frac{f x}{2})^3 (e + f x) + 150 \tan(\frac{e}{2} + \frac{f x}{2})^4 (e + f x) + 75 \tan(\frac{e}{2} + \frac{f x}{2})^5 (e + f x) + 15 \tan(\frac{e}{2} + \frac{f x}{2})^6 (e + f x) - 32)}{120 f (\tan(\frac{e}{2} + \frac{f x}{2})^2 + 1)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(e + f\*x)^2\*(a + a\*sin(e + f\*x))^2\*(c - c\*sin(e + f\*x)),x)

[Out] (a^2\*c\*(15\*e - 30\*tan(e/2 + (f\*x)/2) - 160\*tan(e/2 + (f\*x)/2)^2 + 180\*tan(e/2 + (f\*x)/2)^3 + 160\*tan(e/2 + (f\*x)/2)^4 - 480\*tan(e/2 + (f\*x)/2)^5 - 180\*tan(e/2 + (f\*x)/2)^6 + 30\*tan(e/2 + (f\*x)/2)^7 + 15\*f\*x + 75\*tan(e/2 + (f\*x)/2)^2\*(e + f\*x) + 150\*tan(e/2 + (f\*x)/2)^3\*(e + f\*x) + 150\*tan(e/2 + (f\*x)/2)^4\*(e + f\*x) + 75\*tan(e/2 + (f\*x)/2)^5\*(e + f\*x) + 15\*tan(e/2 + (f\*x)/2)^6\*(e + f\*x) - 32)/(120\*f\*(tan(e/2 + (f\*x)/2)^2 + 1)^5)



### 3.3 $\int \sin(e + fx)(a + a \sin(e + fx))^2(c - c \sin(e + fx)) dx$

**Optimal.** Leaf size=77

$$\frac{1}{8}a^2cx - \frac{a^2c \cos^3(e + fx)}{3f} - \frac{a^2c \cos(e + fx) \sin(e + fx)}{8f} + \frac{a^2c \cos(e + fx) \sin^3(e + fx)}{4f}$$

[Out]  $1/8*a^2*c*x-1/3*a^2*c*\cos(f*x+e)^3/f-1/8*a^2*c*\cos(f*x+e)*\sin(f*x+e)/f+1/4*a^2*c*\cos(f*x+e)*\sin(f*x+e)^3/f$

**Rubi [A]**

time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ ,

Rules used = {3045, 2718, 2715, 8, 2713}

$$-\frac{a^2c \cos^3(e + fx)}{3f} + \frac{a^2c \sin^3(e + fx) \cos(e + fx)}{4f} - \frac{a^2c \sin(e + fx) \cos(e + fx)}{8f} + \frac{1}{8}a^2cx$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sin}[e + f*x]*(a + a*\text{Sin}[e + f*x])^2*(c - c*\text{Sin}[e + f*x]),x]$

[Out]  $(a^2*c*x)/8 - (a^2*c*\text{Cos}[e + f*x]^3)/(3*f) - (a^2*c*\text{Cos}[e + f*x]*\text{Sin}[e + f*x])/ (8*f) + (a^2*c*\text{Cos}[e + f*x]*\text{Sin}[e + f*x]^3)/(4*f)$

**Rule 8**

$\text{Int}[a_, x\_Symbol] \text{ :> Simp}[a*x, x] \text{ /; FreeQ}[a, x]$

**Rule 2713**

$\text{Int}[\text{sin}[(c_.) + (d_.)*(x_.)]^{(n_.)}, x\_Symbol] \text{ :> Dist}[-d^{(-1)}, \text{Subst}[\text{Int}[\text{Expand}[(1 - x^2)^{((n - 1)/2)}, x], x], x, \text{Cos}[c + d*x]], x] \text{ /; FreeQ}\{c, d\}, x] \&\& \text{IGtQ}[(n - 1)/2, 0]$

**Rule 2715**

$\text{Int}[(b_.)*\text{sin}[(c_.) + (d_.)*(x_.)]^{(n_.)}, x\_Symbol] \text{ :> Simp}[(b)*\text{Cos}[c + d*x]*((b*\text{Sin}[c + d*x])^{(n - 1)})/(d*n), x] + \text{Dist}[b^2*((n - 1)/n), \text{Int}[(b*\text{Sin}[c + d*x])^{(n - 2)}, x], x] \text{ /; FreeQ}\{b, c, d\}, x] \&\& \text{GtQ}[n, 1] \&\& \text{IntegerQ}[2*n]$

**Rule 2718**

$\text{Int}[\text{sin}[(c_.) + (d_.)*(x_.)], x\_Symbol] \text{ :> Simp}[-\text{Cos}[c + d*x]/d, x] \text{ /; FreeQ}\{c, d\}, x]$

## Rule 3045

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Int[ExpandTrig[sin[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && IntegerQ[n]
```

## Rubi steps

$$\begin{aligned} \int \sin(e + fx)(a + a \sin(e + fx))^2(c - c \sin(e + fx)) dx &= \int (a^2c \sin(e + fx) + a^2c \sin^2(e + fx) - a^2c \sin^3(e + fx)) dx \\ &= (a^2c) \int \sin(e + fx) dx + (a^2c) \int \sin^2(e + fx) dx \\ &= -\frac{a^2c \cos(e + fx)}{f} - \frac{a^2c \cos(e + fx) \sin(e + fx)}{2f} \\ &= \frac{1}{2}a^2cx - \frac{a^2c \cos^3(e + fx)}{3f} - \frac{a^2c \cos(e + fx) \sin(e + fx)}{8f} \\ &= \frac{1}{8}a^2cx - \frac{a^2c \cos^3(e + fx)}{3f} - \frac{a^2c \cos(e + fx) \sin(e + fx)}{8f} \end{aligned}$$

**Mathematica [A]**

time = 0.08, size = 47, normalized size = 0.61

$$\frac{a^2c(12e + 12fx - 24 \cos(e + fx) - 8 \cos(3(e + fx)) - 3 \sin(4(e + fx)))}{96f}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[e + f\*x]\*(a + a\*Sin[e + f\*x])^2\*(c - c\*Sin[e + f\*x]),x]

[Out] (a^2\*c\*(12\*e + 12\*f\*x - 24\*Cos[e + f\*x] - 8\*Cos[3\*(e + f\*x)] - 3\*Sin[4\*(e + f\*x)]))/(96\*f)

**Maple [A]**

time = 0.11, size = 106, normalized size = 1.38

method	result
risch	$\frac{a^2cx}{8} - \frac{a^2c \cos(fx+e)}{4f} - \frac{a^2c \sin(4fx+4e)}{32f} - \frac{a^2c \cos(3fx+3e)}{12f}$
derivativedivides	$-a^2c \left( -\frac{(\sin^3(fx+e) + \frac{3 \sin(fx+e)}{2}) \cos(fx+e)}{4} + \frac{3fx + 3e}{8} \right) + \frac{a^2c(2 + \sin^2(fx+e)) \cos(fx+e)}{3} + a^2c \left( -\frac{\cos(fx+e) \sin(fx+e)}{2} + \frac{fx}{2} \right)$

default	$-a^2c \left( -\frac{\left( \sin^3(fx+e) + \frac{3\sin(\frac{fx+e}{2})}{2} \right) \cos(fx+e)}{4} + \frac{3fx}{8} + \frac{3e}{8} \right) + \frac{a^2c(2+\sin^2(fx+e)) \cos(fx+e)}{3} + a^2c \left( -\frac{\cos(fx+e)\sin(fx+e)}{2} + \frac{fx}{2} \right)$
norman	$\frac{-\frac{2a^2c}{3f} + \frac{a^2cx}{8} - \frac{2a^2c(\tan^2(\frac{fx}{2} + \frac{e}{2}))}{3f} - \frac{2a^2c(\tan^4(\frac{fx}{2} + \frac{e}{2}))}{f} - \frac{2a^2c(\tan^6(\frac{fx}{2} + \frac{e}{2}))}{f} - \frac{a^2c \tan(\frac{fx}{2} + \frac{e}{2})}{4f} + \frac{7a^2c(\tan^3(\frac{fx}{2} + \frac{e}{2}))}{4f}}{f} + \frac{1}{1+t}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(f*x+e)*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x,method=_RETURNVERBOSE)`

[Out]  $1/f * (-a^2 * c * (-1/4 * (\sin(f*x+e)^3 + 3/2 * \sin(f*x+e)) * \cos(f*x+e) + 3/8 * f*x + 3/8 * e) + 1/3 * a^2 * c * (2 + \sin(f*x+e)^2) * \cos(f*x+e) + a^2 * c * (-1/2 * \cos(f*x+e) * \sin(f*x+e) + 1/2 * f*x + 1/2 * e) - a^2 * c * \cos(f*x+e))$

**Maxima [A]**

time = 0.29, size = 108, normalized size = 1.40

$$\frac{32(\cos(fx+e)^3 - 3\cos(fx+e))a^2c + 3(12fx + 12e + \sin(4fx+4e) - 8\sin(2fx+2e))a^2c - 24(2fx+2e - \sin(2fx+2e))a^2c + 96a^2c\cos(fx+e)}{96f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(f*x+e)*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="maxima")`

[Out]  $-1/96 * (32 * (\cos(f*x + e)^3 - 3 * \cos(f*x + e)) * a^2 * c + 3 * (12 * f * x + 12 * e + \sin(4 * f * x + 4 * e) - 8 * \sin(2 * f * x + 2 * e)) * a^2 * c - 24 * (2 * f * x + 2 * e - \sin(2 * f * x + 2 * e)) * a^2 * c + 96 * a^2 * c * \cos(f*x + e)) / f$

**Fricas [A]**

time = 0.36, size = 67, normalized size = 0.87

$$\frac{8a^2c\cos(fx+e)^3 - 3a^2cfx + 3(2a^2c\cos(fx+e)^3 - a^2c\cos(fx+e))\sin(fx+e)}{24f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(f*x+e)*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="fricas")`

[Out]  $-1/24 * (8 * a^2 * c * \cos(f*x + e)^3 - 3 * a^2 * c * f * x + 3 * (2 * a^2 * c * \cos(f*x + e)^3 - a^2 * c * \cos(f*x + e)) * \sin(f*x + e)) / f$

**Sympy [B]** Leaf count of result is larger than twice the leaf count of optimal. 245 vs. 2(70) = 140.

time = 0.21, size = 245, normalized size = 3.18

$$\begin{cases} \frac{-\frac{3a^2cx\sin^4(\frac{c+x}{2})}{8} - \frac{3a^2cx\sin^2(\frac{c+x}{2})\cos^2(\frac{c+x}{2})}{4} + \frac{a^2cx\sin^2(\frac{c+x}{2})}{2} - \frac{3a^2cx\cos^4(\frac{c+x}{2})}{8} + \frac{a^2cx\cos^2(\frac{c+x}{2})}{2} + \frac{3a^2c\sin^3(\frac{c+x}{2})\cos(\frac{c+x}{2})}{8f} + \frac{a^2c\sin^2(\frac{c+x}{2})\cos(\frac{c+x}{2})}{f} + \frac{3a^2c\sin(\frac{c+x}{2})\cos^3(\frac{c+x}{2})}{8f} - \frac{a^2c\sin(\frac{c+x}{2})\cos(\frac{c+x}{2})}{2f} + \frac{2a^2c\cos^3(\frac{c+x}{2})}{3f} - \frac{a^2c\cos(\frac{c+x}{2})}{f} & \text{for } f \neq 0 \\ x(a\sin(e) + a)^2(-c\sin(e) + c)\sin(e) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(f\*x+e)\*(a+a\*sin(f\*x+e))^2\*(c-c\*sin(f\*x+e)),x)

[Out] Piecewise((-3\*a\*\*2\*c\*x\*sin(e + f\*x)\*\*4/8 - 3\*a\*\*2\*c\*x\*sin(e + f\*x)\*\*2\*cos(e + f\*x)\*\*2/4 + a\*\*2\*c\*x\*sin(e + f\*x)\*\*2/2 - 3\*a\*\*2\*c\*x\*cos(e + f\*x)\*\*4/8 + a\*\*2\*c\*x\*cos(e + f\*x)\*\*2/2 + 5\*a\*\*2\*c\*sin(e + f\*x)\*\*3\*cos(e + f\*x)/(8\*f) + a\*\*2\*c\*sin(e + f\*x)\*\*2\*cos(e + f\*x)/f + 3\*a\*\*2\*c\*sin(e + f\*x)\*cos(e + f\*x)\*3/(8\*f) - a\*\*2\*c\*sin(e + f\*x)\*cos(e + f\*x)/(2\*f) + 2\*a\*\*2\*c\*cos(e + f\*x)\*\*3/(3\*f) - a\*\*2\*c\*cos(e + f\*x)/f, Ne(f, 0)), (x\*(a\*sin(e) + a)\*\*2\*(-c\*sin(e) + c)\*sin(e), True))

**Giac [A]**

time = 0.42, size = 62, normalized size = 0.81

$$\frac{1}{8}a^2cx - \frac{a^2c \cos(3fx + 3e)}{12f} - \frac{a^2c \cos(fx + e)}{4f} - \frac{a^2c \sin(4fx + 4e)}{32f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(f\*x+e)\*(a+a\*sin(f\*x+e))^2\*(c-c\*sin(f\*x+e)),x, algorithm="giac")

[Out] 1/8\*a^2\*c\*x - 1/12\*a^2\*c\*cos(3\*f\*x + 3\*e)/f - 1/4\*a^2\*c\*cos(f\*x + e)/f - 1/32\*a^2\*c\*sin(4\*f\*x + 4\*e)/f

**Mupad [B]**

time = 14.12, size = 250, normalized size = 3.25

$$\frac{a^2cx}{8} - \frac{\tan\left(\frac{e}{2} + \frac{f*x}{2}\right)^2 \left(\frac{a^2c(3e+3f*x)}{6} - \frac{a^2c(12e+12f*x-16)}{24}\right) + \tan\left(\frac{e}{2} + \frac{f*x}{2}\right)^6 \left(\frac{a^2c(3e+3f*x)}{6} - \frac{a^2c(12e+12f*x-48)}{24}\right) + \tan\left(\frac{e}{2} + \frac{f*x}{2}\right)^4 \left(\frac{a^2c(3e+3f*x)}{4} - \frac{a^2c(18e+18f*x-48)}{24}\right) + \frac{a^2c \tan\left(\frac{e}{2} + \frac{f*x}{2}\right) - 7a^2c \tan\left(\frac{e}{2} + \frac{f*x}{2}\right)^3 + 7a^2c \tan\left(\frac{e}{2} + \frac{f*x}{2}\right)^5 - a^2c \tan\left(\frac{e}{2} + \frac{f*x}{2}\right)^7}{f \left(\tan\left(\frac{e}{2} + \frac{f*x}{2}\right) + 1\right)^4} + \frac{a^2c(3e+3f*x)}{24} - \frac{a^2c(3e+3f*x-16)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(e + f\*x)\*(a + a\*sin(e + f\*x))^2\*(c - c\*sin(e + f\*x)),x)

[Out] (a^2\*c\*x)/8 - (tan(e/2 + (f\*x)/2)^2\*((a^2\*c\*(3\*e + 3\*f\*x))/6 - (a^2\*c\*(12\*e + 12\*f\*x - 16))/24) + tan(e/2 + (f\*x)/2)^6\*((a^2\*c\*(3\*e + 3\*f\*x))/6 - (a^2\*c\*(12\*e + 12\*f\*x - 48))/24) + tan(e/2 + (f\*x)/2)^4\*((a^2\*c\*(3\*e + 3\*f\*x))/4 - (a^2\*c\*(18\*e + 18\*f\*x - 48))/24) + (a^2\*c\*tan(e/2 + (f\*x)/2))/4 - (7\*a^2\*c\*tan(e/2 + (f\*x)/2)^3)/4 + (7\*a^2\*c\*tan(e/2 + (f\*x)/2)^5)/4 - (a^2\*c\*tan(e/2 + (f\*x)/2)^7)/4 + (a^2\*c\*(3\*e + 3\*f\*x))/24 - (a^2\*c\*(3\*e + 3\*f\*x - 16))/24)/(f\*(tan(e/2 + (f\*x)/2)^2 + 1)^4)

### 3.4 $\int (a + a \sin(e + fx))^2 (c - c \sin(e + fx)) dx$

Optimal. Leaf size=52

$$\frac{1}{2}a^2cx - \frac{a^2c \cos^3(e + fx)}{3f} + \frac{a^2c \cos(e + fx) \sin(e + fx)}{2f}$$

[Out]  $1/2*a^2*c*x-1/3*a^2*c*\cos(f*x+e)^3/f+1/2*a^2*c*\cos(f*x+e)*\sin(f*x+e)/f$

**Rubi [A]**

time = 0.04, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {2815, 2748, 2715, 8}

$$-\frac{a^2c \cos^3(e + fx)}{3f} + \frac{a^2c \sin(e + fx) \cos(e + fx)}{2f} + \frac{1}{2}a^2cx$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + a*\text{Sin}[e + f*x])^2*(c - c*\text{Sin}[e + f*x]),x]$

[Out]  $(a^2*c*x)/2 - (a^2*c*\text{Cos}[e + f*x]^3)/(3*f) + (a^2*c*\text{Cos}[e + f*x]*\text{Sin}[e + f*x])/(2*f)$

Rule 8

$\text{Int}[a_, x\_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rule 2715

$\text{Int}[(b_.*\text{sin}[(c_.) + (d_.)*(x_)])^{(n_)}, x\_Symbol] \rightarrow \text{Simp}[(-b)*\text{Cos}[c + d*x]*((b*\text{Sin}[c + d*x])^{(n-1)})/(d*n), x] + \text{Dist}[b^2*((n-1)/n), \text{Int}[(b*\text{Sin}[c + d*x])^{(n-2)}, x], x] /; \text{FreeQ}\{b, c, d\}, x \ \&\& \text{GtQ}[n, 1] \ \&\& \text{IntegerQ}[2*n]$

Rule 2748

$\text{Int}[(\text{cos}[(e_.) + (f_.)*(x_)]*(g_.)^{(p_)}*((a_.) + (b_.)*\text{sin}[(e_.) + (f_.)*(x_)])), x\_Symbol] \rightarrow \text{Simp}[(-b)*((g*\text{Cos}[e + f*x])^{(p+1)})/(f*g*(p+1)), x] + \text{Dist}[a, \text{Int}[(g*\text{Cos}[e + f*x])^p, x], x] /; \text{FreeQ}\{a, b, e, f, g, p\}, x \ \&\& (\text{IntegerQ}[2*p] \ \|\ \text{NeQ}[a^2 - b^2, 0])$

Rule 2815

$\text{Int}[(a_.) + (b_.)*\text{sin}[(e_.) + (f_.)*(x_)]^{(m_)}*((c_.) + (d_.)*\text{sin}[(e_.) + (f_.)*(x_)]^{(n_)}), x\_Symbol] \rightarrow \text{Dist}[a^m*c^m, \text{Int}[\text{Cos}[e + f*x]^{(2*m)}*(c + d*\text{Sin}[e + f*x])^{(n-m)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, n\}, x \ \&\& \text{EqQ}[b*c + a*d, 0] \ \&\& \text{EqQ}[a^2 - b^2, 0] \ \&\& \text{IntegerQ}[m] \ \&\& !(\text{IntegerQ}[n] \ \&\& ((\text{LtQ}$

[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))

### Rubi steps

$$\begin{aligned}
 \int (a + a \sin(e + fx))^2 (c - c \sin(e + fx)) dx &= (ac) \int \cos^2(e + fx)(a + a \sin(e + fx)) dx \\
 &= -\frac{a^2 c \cos^3(e + fx)}{3f} + (a^2 c) \int \cos^2(e + fx) dx \\
 &= -\frac{a^2 c \cos^3(e + fx)}{3f} + \frac{a^2 c \cos(e + fx) \sin(e + fx)}{2f} + \frac{1}{2}(a^2 c) \int \\
 &= \frac{1}{2} a^2 c x - \frac{a^2 c \cos^3(e + fx)}{3f} + \frac{a^2 c \cos(e + fx) \sin(e + fx)}{2f}
 \end{aligned}$$

### Mathematica [A]

time = 0.24, size = 43, normalized size = 0.83

$$-\frac{a^2 c (3 \cos(e + fx) + \cos(3(e + fx)) - 3(2fx + \sin(2(e + fx))))}{12f}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a\*Sin[e + f\*x])^2\*(c - c\*Sin[e + f\*x]),x]

[Out] -1/12\*(a^2\*c\*(3\*Cos[e + f\*x] + Cos[3\*(e + f\*x)] - 3\*(2\*f\*x + Sin[2\*(e + f\*x)])))/f

### Maple [A]

time = 0.09, size = 78, normalized size = 1.50

method	result
risch	$\frac{a^2 c x}{2} - \frac{a^2 c \cos(fx+e)}{4f} - \frac{a^2 c \cos(3fx+3e)}{12f} + \frac{a^2 c \sin(2fx+2e)}{4f}$
derivativdivides	$\frac{\frac{a^2 c (2 + \sin^2(fx+e)) \cos(fx+e)}{3} - a^2 c \left( -\frac{\cos(fx+e) \sin(fx+e)}{2} + \frac{fx}{2} + \frac{e}{2} \right) - a^2 c \cos(fx+e) + a^2 c (fx+e)}{f}$
default	$\frac{\frac{a^2 c (2 + \sin^2(fx+e)) \cos(fx+e)}{3} - a^2 c \left( -\frac{\cos(fx+e) \sin(fx+e)}{2} + \frac{fx}{2} + \frac{e}{2} \right) - a^2 c \cos(fx+e) + a^2 c (fx+e)}{f}$
norman	$\frac{\frac{a^2 c \tan\left(\frac{fx}{2} + \frac{e}{2}\right)}{f} - \frac{2a^2 c}{3f} + \frac{a^2 c x}{2} - \frac{2a^2 c \left(\tan^4\left(\frac{fx}{2} + \frac{e}{2}\right)\right)}{f} - \frac{a^2 c \left(\tan^5\left(\frac{fx}{2} + \frac{e}{2}\right)\right)}{f} + \frac{3a^2 c x \left(\tan^2\left(\frac{fx}{2} + \frac{e}{2}\right)\right)}{2} + \frac{3a^2 c x \left(\tan^4\left(\frac{fx}{2} + \frac{e}{2}\right)\right)}{2} + \frac{a^2 c x}{2}}{\left(1 + \tan^2\left(\frac{fx}{2} + \frac{e}{2}\right)\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a\*sin(f\*x+e))^2\*(c-c\*sin(f\*x+e)),x,method=\_RETURNVERBOSE)

[Out]  $1/f*(1/3*a^2*c*(2+\sin(f*x+e))^2*\cos(f*x+e)-a^2*c*(-1/2*\cos(f*x+e)*\sin(f*x+e)+1/2*f*x+1/2*e)-a^2*c*\cos(f*x+e)+a^2*c*(f*x+e))$

**Maxima** [A]

time = 0.28, size = 83, normalized size = 1.60

$$\frac{4(\cos(fx+e)^3 - 3\cos(fx+e))a^2c + 3(2fx+2e - \sin(2fx+2e))a^2c - 12(fx+e)a^2c + 12a^2c\cos(fx+e)}{12f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="maxima")`

[Out]  $-1/12*(4*(\cos(f*x+e))^3 - 3*\cos(f*x+e))*a^2*c + 3*(2*f*x + 2*e - \sin(2*f*x + 2*e))*a^2*c - 12*(f*x + e)*a^2*c + 12*a^2*c*\cos(f*x + e))/f$

**Fricas** [A]

time = 0.38, size = 49, normalized size = 0.94

$$\frac{2a^2c\cos(fx+e)^3 - 3a^2cfx - 3a^2c\cos(fx+e)\sin(fx+e)}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="fricas")`

[Out]  $-1/6*(2*a^2*c*\cos(f*x+e)^3 - 3*a^2*c*f*x - 3*a^2*c*\cos(f*x+e)*\sin(f*x+e))/f$

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 133 vs.  $2(46) = 92$ .

time = 0.13, size = 133, normalized size = 2.56

$$\begin{cases} -\frac{a^2cx\sin^2(e+fx)}{2} - \frac{a^2cx\cos^2(e+fx)}{2} + a^2cx + \frac{a^2c\sin^2(e+fx)\cos(e+fx)}{f} + \frac{a^2c\sin(e+fx)\cos(e+fx)}{2f} + \frac{2a^2c\cos^3(e+fx)}{3f} - \frac{a^2c\cos(e+fx)}{f} & \text{for } f \neq 0 \\ x(a\sin(e)+a)^2(-c\sin(e)+c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x)`

[Out] `Piecewise((-a**2*c*x*sin(e+f*x)**2/2 - a**2*c*x*cos(e+f*x)**2/2 + a**2*c*x + a**2*c*sin(e+f*x)**2*cos(e+f*x)/f + a**2*c*sin(e+f*x)*cos(e+f*x)/(2*f) + 2*a**2*c*cos(e+f*x)**3/(3*f) - a**2*c*cos(e+f*x)/f, Ne(f, 0)), (x*(a*sin(e)+a)**2*(-c*sin(e)+c), True))`

**Giac** [A]

time = 0.45, size = 62, normalized size = 1.19

$$\frac{1}{2}a^2cx - \frac{a^2c\cos(3fx+3e)}{12f} - \frac{a^2c\cos(fx+e)}{4f} + \frac{a^2c\sin(2fx+2e)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*sin(f\*x+e))^2\*(c-c\*sin(f\*x+e)),x, algorithm="giac")

[Out]  $\frac{1}{2}a^2c*x - \frac{1}{12}a^2c*\cos(3*f*x + 3*e)/f - \frac{1}{4}a^2c*\cos(f*x + e)/f + \frac{1}{4}a^2c*\sin(2*f*x + 2*e)/f$

**Mupad [B]**

time = 14.30, size = 125, normalized size = 2.40

$$\frac{a^2 c x}{2} - \frac{\tan\left(\frac{e}{2} + \frac{f x}{2}\right)^4 \left(\frac{3 a^2 c (e + f x)}{2} - \frac{a^2 c (9 e + 9 f x - 12)}{6}\right) - a^2 c \tan\left(\frac{e}{2} + \frac{f x}{2}\right) + \frac{a^2 c (e + f x)}{2} + a^2 c \tan\left(\frac{e}{2} + \frac{f x}{2}\right)^5 - \frac{a^2 c (3 e + 3 f x - 4)}{6}}{f \left(\tan\left(\frac{e}{2} + \frac{f x}{2}\right)^2 + 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a\*sin(e + f\*x))^2\*(c - c\*sin(e + f\*x)),x)

[Out]  $(a^2*c*x)/2 - (\tan(e/2 + (f*x)/2))^4*((3*a^2*c*(e + f*x))/2 - (a^2*c*(9*e + 9*f*x - 12))/6) - a^2*c*\tan(e/2 + (f*x)/2) + (a^2*c*(e + f*x))/2 + a^2*c*\tan(e/2 + (f*x)/2)^5 - (a^2*c*(3*e + 3*f*x - 4))/6)/(f*(\tan(e/2 + (f*x)/2)^2 + 1)^3)$



### 3.5 $\int \csc(e + fx)(a + a \sin(e + fx))^2(c - c \sin(e + fx)) dx$

Optimal. Leaf size=63

$$\frac{1}{2}a^2cx - \frac{a^2c \tanh^{-1}(\cos(e + fx))}{f} + \frac{a^2c \cos(e + fx)}{f} + \frac{a^2c \cos(e + fx) \sin(e + fx)}{2f}$$

[Out]  $1/2*a^2*c*x - a^2*c*\operatorname{arctanh}(\cos(f*x+e))/f + a^2*c*\cos(f*x+e)/f + 1/2*a^2*c*\cos(f*x+e)*\sin(f*x+e)/f$

Rubi [A]

time = 0.06, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {3045, 3855, 2718, 2715, 8}

$$\frac{a^2c \cos(e + fx)}{f} + \frac{a^2c \sin(e + fx) \cos(e + fx)}{2f} - \frac{a^2c \tanh^{-1}(\cos(e + fx))}{f} + \frac{1}{2}a^2cx$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Csc}[e + f*x]*(a + a*\operatorname{Sin}[e + f*x])^2*(c - c*\operatorname{Sin}[e + f*x]), x]$

[Out]  $(a^2*c*x)/2 - (a^2*c*\operatorname{ArcTanh}[\operatorname{Cos}[e + f*x]])/f + (a^2*c*\operatorname{Cos}[e + f*x])/f + (a^2*c*\operatorname{Cos}[e + f*x]*\operatorname{Sin}[e + f*x])/(2*f)$

Rule 8

$\operatorname{Int}[a_, x\_Symbol] \rightarrow \operatorname{Simp}[a*x, x] /; \operatorname{FreeQ}[a, x]$

Rule 2715

$\operatorname{Int}[(b_*)*\operatorname{sin}[(c_*) + (d_*)(x_)]^{(n_)}, x\_Symbol] \rightarrow \operatorname{Simp}[(-b)*\operatorname{Cos}[c + d*x]*((b*\operatorname{Sin}[c + d*x])^{(n-1)})/(d*n), x] + \operatorname{Dist}[b^2*((n-1)/n), \operatorname{Int}[(b*\operatorname{Sin}[c + d*x])^{(n-2)}, x], x] /; \operatorname{FreeQ}\{b, c, d, x\} \&\& \operatorname{GtQ}[n, 1] \&\& \operatorname{IntegerQ}[2*n]$

Rule 2718

$\operatorname{Int}[\operatorname{sin}[(c_*) + (d_*)(x_)], x\_Symbol] \rightarrow \operatorname{Simp}[-\operatorname{Cos}[c + d*x]/d, x] /; \operatorname{FreeQ}\{c, d, x\}$

Rule 3045

$\operatorname{Int}[\operatorname{sin}[(e_*) + (f_*)(x_)]^{(n_)*}((a_*) + (b_*)*\operatorname{sin}[(e_*) + (f_*)(x_)])^{(m_)*}((A_*) + (B_*)*\operatorname{sin}[(e_*) + (f_*)(x_)]), x\_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandTrig}[\operatorname{sin}[e + f*x]^n*(a + b*\operatorname{sin}[e + f*x])^m*(A + B*\operatorname{sin}[e + f*x]), x], x] /; \operatorname{FreeQ}\{$

a, b, e, f, A, B}, x] && EqQ[A\*b + a\*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && IntegerQ[n]

### Rule 3855

Int[csc[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[-ArcTanh[Cos[c + d\*x]]/d, x] /; FreeQ[{c, d}, x]

### Rubi steps

$$\begin{aligned} \int \csc(e + fx)(a + a \sin(e + fx))^2(c - c \sin(e + fx)) dx &= \int (a^2c + a^2c \csc(e + fx) - a^2c \sin(e + fx) - a^2c \sin^2(e + fx)) dx \\ &= a^2cx + (a^2c) \int \csc(e + fx) dx - (a^2c) \int \sin(e + fx) dx - \frac{a^2c}{2} \int \sin(2(e + fx)) dx \\ &= a^2cx - \frac{a^2c \tanh^{-1}(\cos(e + fx))}{f} + \frac{a^2c \cos(e + fx)}{f} - \frac{a^2c \sin(2(e + fx))}{2f} \\ &= \frac{1}{2}a^2cx - \frac{a^2c \tanh^{-1}(\cos(e + fx))}{f} + \frac{a^2c \cos(e + fx)}{f} - \frac{a^2c \sin(2(e + fx))}{2f} \end{aligned}$$

### Mathematica [A]

time = 0.06, size = 61, normalized size = 0.97

$$\frac{a^2c(-2e + 2fx + 4 \cos(e + fx) - 4 \log(\cos(\frac{1}{2}(e + fx))) + 4 \log(\sin(\frac{1}{2}(e + fx))) + \sin(2(e + fx)))}{4f}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[e + f\*x]\*(a + a\*Sin[e + f\*x])^2\*(c - c\*Sin[e + f\*x]),x]

[Out] (a^2\*c\*(-2\*e + 2\*f\*x + 4\*Cos[e + f\*x] - 4\*Log[Cos[(e + f\*x)/2]] + 4\*Log[Sin[(e + f\*x)/2]] + Sin[2\*(e + f\*x)]))/(4\*f)

### Maple [A]

time = 0.16, size = 76, normalized size = 1.21

method	result
derivativedivides	$\frac{a^2c(fx+e)+a^2c \ln(\csc(fx+e)-\cot(fx+e))+a^2c \cos(fx+e)-a^2c \left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}$
default	$\frac{a^2c(fx+e)+a^2c \ln(\csc(fx+e)-\cot(fx+e))+a^2c \cos(fx+e)-a^2c \left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}$
risch	$\frac{a^2cx}{2} + \frac{a^2ce^{i(fx+e)}}{2f} + \frac{a^2ce^{-i(fx+e)}}{2f} + \frac{a^2c \ln(e^{i(fx+e)}-1)}{f} - \frac{a^2c \ln(e^{i(fx+e)}+1)}{f} + \frac{a^2c \sin(2fx+2e)}{4f}$

norman	$\frac{\frac{a^2 c \tan\left(\frac{fx}{2} + \frac{e}{2}\right)}{f} + \frac{a^2 c x}{2} - \frac{2a^2 c \left(\tan^2\left(\frac{fx}{2} + \frac{e}{2}\right)\right)}{f} - \frac{2a^2 c \left(\tan^6\left(\frac{fx}{2} + \frac{e}{2}\right)\right)}{f} - \frac{4a^2 c \left(\tan^4\left(\frac{fx}{2} + \frac{e}{2}\right)\right)}{f} - \frac{a^2 c \left(\tan^5\left(\frac{fx}{2} + \frac{e}{2}\right)\right)}{f} + \frac{3a^2 c x \left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right)\right)}{f}}{\left(1 + \tan^2\left(\frac{fx}{2} + \frac{e}{2}\right)\right)^3}$
--------	---

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csc(f*x+e)*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x,method=_RETURNVERBOSE)`

[Out] `1/f*(a^2*c*(f*x+e)+a^2*c*ln(csc(f*x+e)-cot(f*x+e))+a^2*c*cos(f*x+e)-a^2*c*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e))`

**Maxima [A]**

time = 0.37, size = 79, normalized size = 1.25

$$\frac{(2fx + 2e - \sin(2fx + 2e))a^2c - 4(fx + e)a^2c - 4a^2c \cos(fx + e) + 4a^2c \log(\cot(fx + e) + \csc(fx + e))}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(f*x+e)*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="maxima")`

[Out] `-1/4*((2*f*x + 2*e - sin(2*f*x + 2*e))*a^2*c - 4*(f*x + e)*a^2*c - 4*a^2*c*cos(f*x + e) + 4*a^2*c*log(cot(f*x + e) + csc(f*x + e)))/f`

**Fricas [A]**

time = 0.40, size = 80, normalized size = 1.27

$$\frac{a^2cfx + a^2c \cos(fx + e) \sin(fx + e) + 2a^2c \cos(fx + e) - a^2c \log\left(\frac{1}{2} \cos(fx + e) + \frac{1}{2}\right) + a^2c \log\left(-\frac{1}{2} \cos(fx + e) + \frac{1}{2}\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(f*x+e)*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="fricas")`

[Out] `1/2*(a^2*c*f*x + a^2*c*cos(f*x + e)*sin(f*x + e) + 2*a^2*c*cos(f*x + e) - a^2*c*log(1/2*cos(f*x + e) + 1/2) + a^2*c*log(-1/2*cos(f*x + e) + 1/2))/f`

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$-a^2c \left( \int (-\sin(e + fx) \csc(e + fx)) dx + \int \sin^2(e + fx) \csc(e + fx) dx + \int \sin^3(e + fx) \csc(e + fx) dx + \int (-\csc(e + fx)) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(f*x+e)*(a+a*sin(f*x+e))**2*(c-c*sin(f*x+e)),x)`

[Out] `-a**2*c*(Integral(-sin(e + f*x)*csc(e + f*x), x) + Integral(sin(e + f*x)**2*csc(e + f*x), x) + Integral(sin(e + f*x)**3*csc(e + f*x), x) + Integral(-csc(e + f*x), x))`

**Giac [A]**

time = 0.44, size = 111, normalized size = 1.76

$$\frac{(fx + e)a^2c + 2a^2c \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)\right|\right) - \frac{2\left(a^2c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 - 2a^2c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - a^2c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - 2a^2c\right)}{\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 1\right)^2}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*((f*x + e)*a^2*c + 2*a^2*c*log(abs(tan(1/2*f*x + 1/2*e))) - 2*(a^2*c*tan(1/2*f*x + 1/2*e)^3 - 2*a^2*c*tan(1/2*f*x + 1/2*e)^2 - a^2*c*tan(1/2*f*x + 1/2*e) - 2*a^2*c)/(tan(1/2*f*x + 1/2*e)^2 + 1)^2)/f
```

**Mupad [B]**

time = 12.89, size = 88, normalized size = 1.40

$$\frac{a^2c \left( \cos(e + fx) + \ln\left(\frac{\sin\left(\frac{e}{2} + \frac{fx}{2}\right)}{\cos\left(\frac{e}{2} + \frac{fx}{2}\right)}\right) + \frac{\sin(2e + 2fx)}{4} + \operatorname{atan}\left(\frac{\sqrt{5} \left(\cos\left(\frac{e}{2} + \frac{fx}{2}\right) + 2 \sin\left(\frac{e}{2} + \frac{fx}{2}\right)\right)}{5 \cos\left(\frac{e}{2} + \operatorname{atan}\left(\frac{1}{2}\right) + \frac{fx}{2}\right)}\right) \right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x)))/sin(e + f*x),x)
```

```
[Out] (a^2*c*(cos(e + f*x) + log(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)) + sin(2*e + 2*f*x)/4 + atan((5^(1/2)*(cos(e/2 + (f*x)/2) + 2*sin(e/2 + (f*x)/2))))/(5*cos(e/2 + atan(1/2) + (f*x)/2))))/f
```

### 3.6 $\int \csc^2(e + fx)(a + a \sin(e + fx))^2(c - c \sin(e + fx)) dx$

Optimal. Leaf size=53

$$-a^2cx - \frac{a^2c \tanh^{-1}(\cos(e + fx))}{f} + \frac{a^2c \cos(e + fx)}{f} - \frac{a^2c \cot(e + fx)}{f}$$

[Out]  $-a^2c*x - a^2c*\operatorname{arctanh}(\cos(f*x+e))/f + a^2c*\cos(f*x+e)/f - a^2c*\cot(f*x+e)/f$

Rubi [A]

time = 0.09, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$ , Rules used = {3029, 2789, 2672, 327, 212, 3554, 8}

$$\frac{a^2c \cos(e + fx)}{f} - \frac{a^2c \cot(e + fx)}{f} - \frac{a^2c \tanh^{-1}(\cos(e + fx))}{f} + a^2(-c)x$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Csc}[e + f*x]^2*(a + a*\operatorname{Sin}[e + f*x])^2*(c - c*\operatorname{Sin}[e + f*x]), x]$

[Out]  $-(a^2c*x) - (a^2c*\operatorname{ArcTanh}[\operatorname{Cos}[e + f*x]])/f + (a^2c*\operatorname{Cos}[e + f*x])/f - (a^2c*\operatorname{Cot}[e + f*x])/f$

Rule 8

$\operatorname{Int}[a_, x\_Symbol] \rightarrow \operatorname{Simp}[a*x, x] \text{ ; FreeQ}[a, x]$

Rule 212

$\operatorname{Int}[(a_ + (b_)*(x_)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]))*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[a, 2])], x] \text{ ; FreeQ}\{[a, b], x\} \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$

Rule 327

$\operatorname{Int}[(c_)*(x_)^m*((a_ + (b_)*(x_)^n))^p, x\_Symbol] \rightarrow \operatorname{Simp}[c^{(n-1)}*(c*x)^{(m-n+1)}*((a + b*x^n)^{(p+1)}/(b*(m+n*p+1))), x] - \operatorname{Dist}[a*c^n*((m-n+1)/(b*(m+n*p+1))), \operatorname{Int}[(c*x)^{(m-n)}*(a + b*x^n)^p, x], x] \text{ ; FreeQ}\{[a, b, c, p], x\} \ \&\& \operatorname{IGtQ}[n, 0] \ \&\& \operatorname{GtQ}[m, n-1] \ \&\& \operatorname{NeQ}[m+n*p+1, 0] \ \&\& \operatorname{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 2672

$\operatorname{Int}[(a_)*\sin[(e_ + (f_)*(x_))]^{m_}*\tan[(e_ + (f_)*(x_))]^{n_}, x\_Symbol] \rightarrow \operatorname{With}\{ff = \operatorname{FreeFactors}[\operatorname{Sin}[e + f*x], x]\}, \operatorname{Dist}[ff/f, \operatorname{Subst}[\operatorname{Int}[($

```
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff), x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

### Rule 2789

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((g_)*tan[(e_) + (f_)*(x_)])^(p_), x_Symbol] := Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

### Rule 3029

```
Int[sin[(e_) + (f_)*(x_)]^(p_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist[a^n*c^n, Int[Tan[e + f*x]^p*(a + b*Sin[e + f*x])^(m - n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[p + 2*n, 0] && IntegerQ[n]
```

### Rule 3554

```
Int[((b_)*tan[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[b*((b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

### Rubi steps

$$\begin{aligned}
\int \csc^2(e + fx)(a + a \sin(e + fx))^2(c - c \sin(e + fx)) dx &= (ac) \int \cot^2(e + fx)(a + a \sin(e + fx)) dx \\
&= (ac) \int (a \cos(e + fx) \cot(e + fx) + a \cot^2(e + fx)) dx \\
&= (a^2c) \int \cos(e + fx) \cot(e + fx) dx + (a^2c) \int \cot^2(e + fx) dx \\
&= -\frac{a^2c \cot(e + fx)}{f} - (a^2c) \int 1 dx - \frac{(a^2c) \operatorname{Subst}(u, \sin(e + fx), u)}{f} \\
&= -a^2cx + \frac{a^2c \cos(e + fx)}{f} - \frac{a^2c \cot(e + fx)}{f} - \frac{(a^2c) \operatorname{Subst}(u, \sin(e + fx), u)}{f} \\
&= -a^2cx - \frac{a^2c \tanh^{-1}(\cos(e + fx))}{f} + \frac{a^2c \cos(e + fx)}{f}
\end{aligned}$$

### Mathematica [A]

time = 0.03, size = 97, normalized size = 1.83

$$-a^2cx + \frac{a^2c \cos(e) \cos(fx)}{f} - \frac{a^2c \cot(e + fx)}{f} - \frac{a^2c \log\left(\cos\left(\frac{e}{2} + \frac{fx}{2}\right)\right)}{f} + \frac{a^2c \log\left(\sin\left(\frac{e}{2} + \frac{fx}{2}\right)\right)}{f} - \frac{a^2c \sin(e) \sin(fx)}{f}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[e + f\*x]^2\*(a + a\*Sin[e + f\*x])^2\*(c - c\*Sin[e + f\*x]),x]

[Out]  $-(a^2*c*x) + (a^2*c*\cos[e]*\cos[f*x])/f - (a^2*c*\cot[e + f*x])/f - (a^2*c*\log[\cos[e/2 + (f*x)/2]])/f + (a^2*c*\log[\sin[e/2 + (f*x)/2]])/f - (a^2*c*\sin[e]*\sin[f*x])/f$

**Maple [A]**

time = 0.12, size = 61, normalized size = 1.15

method	result
derivativdivides	$\frac{a^2 c \ln(\csc(fx+e) - \cot(fx+e)) - a^2 c \cot(fx+e) - a^2 c(fx+e) + a^2 c \cos(fx+e)}{f}$
default	$\frac{a^2 c \ln(\csc(fx+e) - \cot(fx+e)) - a^2 c \cot(fx+e) - a^2 c(fx+e) + a^2 c \cos(fx+e)}{f}$
risch	$-a^2 c x + \frac{a^2 c e^{i(fx+e)}}{2f} + \frac{a^2 c e^{-i(fx+e)}}{2f} - \frac{2ia^2 c}{f(e^{2i(fx+e)} - 1)} - \frac{a^2 c \ln(e^{i(fx+e)} + 1)}{f} + \frac{a^2 c \ln(e^{i(fx+e)} - 1)}{f}$
norman	$\frac{\frac{a^2 c (\tan^6(\frac{fx}{2} + \frac{e}{2}))}{f} - \frac{a^2 c}{2f} - \frac{2a^2 c (\tan^3(\frac{fx}{2} + \frac{e}{2}))}{f} - \frac{2a^2 c (\tan^7(\frac{fx}{2} + \frac{e}{2}))}{f} - \frac{4a^2 c (\tan^5(\frac{fx}{2} + \frac{e}{2}))}{f} - \frac{a^2 c (\tan^2(\frac{fx}{2} + \frac{e}{2}))}{f} + \frac{a^2 c (\tan^4(\frac{fx}{2} + \frac{e}{2}))}{f}}{\tan(\frac{fx}{2} + \frac{e}{2})(1 + \tan^2(\frac{fx}{2} + \frac{e}{2}))}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(f\*x+e)^2\*(a+a\*sin(f\*x+e))^2\*(c-c\*sin(f\*x+e)),x,method=\_RETURNVERBOS E)

[Out]  $1/f*(a^2*c*\ln(\csc(f*x+e) - \cot(f*x+e)) - a^2*c*\cot(f*x+e) - a^2*c*(f*x+e) + a^2*c*\cos(f*x+e))$

**Maxima [A]**

time = 0.30, size = 74, normalized size = 1.40

$$\frac{2(fx+e)a^2c + a^2c(\log(\cos(fx+e)+1) - \log(\cos(fx+e)-1)) - 2a^2c\cos(fx+e) + \frac{2a^2c}{\tan(fx+e)}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f\*x+e)^2\*(a+a\*sin(f\*x+e))^2\*(c-c\*sin(f\*x+e)),x, algorithm="maxima")

[Out]  $-1/2*(2*(f*x + e)*a^2*c + a^2*c*(\log(\cos(f*x + e) + 1) - \log(\cos(f*x + e) - 1)) - 2*a^2*c*\cos(f*x + e) + 2*a^2*c/\tan(f*x + e))/f$

**Fricas [A]**

time = 0.39, size = 107, normalized size = 2.02

$$\frac{a^2 c \log(\frac{1}{2} \cos(fx+e) + \frac{1}{2}) \sin(fx+e) - a^2 c \log(-\frac{1}{2} \cos(fx+e) + \frac{1}{2}) \sin(fx+e) + 2a^2 c \cos(fx+e) + 2(a^2 c f x - a^2 c \cos(fx+e)) \sin(fx+e)}{2f \sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f\*x+e)^2\*(a+a\*sin(f\*x+e))^2\*(c-c\*sin(f\*x+e)),x, algorithm="fricas")

[Out]  $-1/2*(a^2*c*\log(1/2*\cos(f*x + e) + 1/2)*\sin(f*x + e) - a^2*c*\log(-1/2*\cos(f*x + e) + 1/2)*\sin(f*x + e) + 2*a^2*c*\cos(f*x + e) + 2*(a^2*c*f*x - a^2*c*\cos(f*x + e))*\sin(f*x + e))/(f*\sin(f*x + e))$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$-a^2c \left( \int (-\sin(e+fx)) \csc^2(e+fx) dx + \int \sin^2(e+fx) \csc^2(e+fx) dx + \int \sin^3(e+fx) \csc^2(e+fx) dx + \int (-\csc^2(e+fx)) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f\*x+e)\*\*2\*(a+a\*sin(f\*x+e))\*\*2\*(c-c\*sin(f\*x+e)),x)

[Out]  $-a^{**2}*c*(Integral(-\sin(e + f*x)*\csc(e + f*x)**2, x) + Integral(\sin(e + f*x)**2*\csc(e + f*x)**2, x) + Integral(\sin(e + f*x)**3*\csc(e + f*x)**2, x) + Integral(-\csc(e + f*x)**2, x))$

**Giac [B]** Leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(56) = 112.

time = 0.48, size = 137, normalized size = 2.58

$$\frac{6(fx+e)a^2c - 6a^2c \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)\right|\right) - 3a^2c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + \frac{2a^2c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + 3a^2c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - 10a^2c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 3a^2c}{\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)} + \frac{3a^2c}{6f}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f\*x+e)^2\*(a+a\*sin(f\*x+e))^2\*(c-c\*sin(f\*x+e)),x, algorithm="giac")

[Out]  $-1/6*(6*(f*x + e)*a^2*c - 6*a^2*c*\log(\text{abs}(\tan(1/2*f*x + 1/2*e)))) - 3*a^2*c*\tan(1/2*f*x + 1/2*e) + (2*a^2*c*\tan(1/2*f*x + 1/2*e)^3 + 3*a^2*c*\tan(1/2*f*x + 1/2*e)^2 - 10*a^2*c*\tan(1/2*f*x + 1/2*e) + 3*a^2*c)/(\tan(1/2*f*x + 1/2*e)^3 + \tan(1/2*f*x + 1/2*e))/f$

**Mupad [B]**

time = 12.39, size = 110, normalized size = 2.08

$$\frac{a^2c \left( 2 \operatorname{atan} \left( \frac{\sqrt{2} \left( \cos\left(\frac{e}{2} + \frac{fx}{2}\right) - \sin\left(\frac{e}{2} + \frac{fx}{2}\right) \right)}{2 \cos\left(\frac{e}{2} - \frac{\pi}{4} + \frac{fx}{2}\right)} \right) + \ln \left( \frac{\sin\left(\frac{e}{2} + \frac{fx}{2}\right)}{\cos\left(\frac{e}{2} + \frac{fx}{2}\right)} \right) \right)}{f} - \frac{a^2c \left( \cos(e + fx) - \frac{\sin(2e + 2fx)}{2} \right)}{f \sin(e + fx)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + a\*sin(e + f\*x))^2\*(c - c\*sin(e + f\*x)))/sin(e + f\*x)^2,x)

[Out]  $(a^2*c*(2*\operatorname{atan}((2^{(1/2)}*(\cos(e/2 + (f*x)/2) - \sin(e/2 + (f*x)/2))))/(2*\cos(e/2 - \pi/4 + (f*x)/2))) + \log(\sin(e/2 + (f*x)/2)/\cos(e/2 + (f*x)/2)))/f - (a^2*c*(\cos(e + f*x) - \sin(2*e + 2*f*x)/2))/(f*\sin(e + f*x))$



### 3.7 $\int \csc^3(e + fx)(a + a \sin(e + fx))^2(c - c \sin(e + fx)) dx$

Optimal. Leaf size=64

$$-a^2cx + \frac{a^2c \tanh^{-1}(\cos(e + fx))}{2f} - \frac{a^2c \cot(e + fx)}{f} - \frac{a^2c \cot(e + fx) \csc(e + fx)}{2f}$$

[Out]  $-a^2cx + 1/2a^2c \operatorname{arctanh}(\cos(fx+e))/f - a^2c \cot(fx+e)/f - 1/2a^2c \cot(fx+e) \csc(fx+e)/f$

Rubi [A]

time = 0.08, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$ , Rules used = {3045, 3855, 3852, 8, 3853}

$$-\frac{a^2c \cot(e + fx)}{f} + \frac{a^2c \tanh^{-1}(\cos(e + fx))}{2f} - \frac{a^2c \cot(e + fx) \csc(e + fx)}{2f} + a^2(-c)x$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Csc}[e + fx]^3(a + a \sin[e + fx])^2(c - c \sin[e + fx]), x]$

[Out]  $-(a^2cx) + (a^2c \operatorname{ArcTanh}[\text{Cos}[e + fx]])/(2f) - (a^2c \cot[e + fx])/f - (a^2c \cot[e + fx] \csc[e + fx])/(2f)$

Rule 8

$\text{Int}[a_, x\_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rule 3045

$\text{Int}[\sin[(e_) + (f_)*(x_)]^{(n_)}*((a_) + (b_)*\sin[(e_) + (f_)*(x_)])^{(m_)}*((A_) + (B_)*\sin[(e_) + (f_)*(x_)]), x\_Symbol] \rightarrow \text{Int}[\text{ExpandTrig}[\sin[e + fx]^n(a + b \sin[e + fx])^m(A + B \sin[e + fx]), x], x] /; \text{FreeQ}[\{a, b, e, f, A, B\}, x] \ \&\& \ \text{EqQ}[A*b + a*B, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{IntegerQ}[m] \ \&\& \ \text{IntegerQ}[n]$

Rule 3852

$\text{Int}[\csc[(c_) + (d_)*(x_)]^{(n_)}, x\_Symbol] \rightarrow \text{Dist}[-d^{(-1)}, \text{Subst}[\text{Int}[\text{ExpandIntegrand}[(1 + x^2)^{(n/2 - 1)}, x], x], x, \text{Cot}[c + d*x]], x] /; \text{FreeQ}[\{c, d\}, x] \ \&\& \ \text{IGtQ}[n/2, 0]$

Rule 3853

$\text{Int}[(\csc[(c_) + (d_)*(x_)]*(b_))^{(n_)}, x\_Symbol] \rightarrow \text{Simp}[(-b)*\text{Cos}[c + d*x]*((b*\csc[c + d*x])^{(n-1)}(d*(n-1))), x] + \text{Dist}[b^2*((n-2)/(n-1)),$

Int[(b\*Csc[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &  
& IntegerQ[2\*n]

### Rule 3855

Int[csc[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[-ArcTanh[Cos[c + d\*x]]/d, x]  
/; FreeQ[{c, d}, x]

### Rubi steps

$$\begin{aligned} \int \csc^3(e + fx)(a + a \sin(e + fx))^2(c - c \sin(e + fx)) dx &= \int (-a^2c - a^2c \csc(e + fx) + a^2c \csc^2(e + fx) + \\ &= -a^2cx - (a^2c) \int \csc(e + fx) dx + (a^2c) \int \csc^2(e + fx) dx \\ &= -a^2cx + \frac{a^2c \tanh^{-1}(\cos(e + fx))}{f} - \frac{a^2c \cot(e + fx)}{f} \\ &= -a^2cx + \frac{a^2c \tanh^{-1}(\cos(e + fx))}{2f} - \frac{a^2c \cot(e + fx)}{f} \end{aligned}$$

### Mathematica [A]

time = 0.47, size = 95, normalized size = 1.48

$$\frac{-a^2c(8e + 8fx + 4 \cot(\frac{1}{2}(e + fx)) + \csc^2(\frac{1}{2}(e + fx)) - 4 \log(\cos(\frac{1}{2}(e + fx))) + 4 \log(\sin(\frac{1}{2}(e + fx))) - \sec^2(\frac{1}{2}(e + fx)) - 4 \tan(\frac{1}{2}(e + fx)))}{8f}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[e + f\*x]^3\*(a + a\*Sin[e + f\*x])^2\*(c - c\*Sin[e + f\*x]),x]

[Out] -1/8\*(a^2\*c\*(8\*e + 8\*f\*x + 4\*Cot[(e + f\*x)/2] + Csc[(e + f\*x)/2]^2 - 4\*Log[Cos[(e + f\*x)/2]] + 4\*Log[Sin[(e + f\*x)/2]] - Sec[(e + f\*x)/2]^2 - 4\*Tan[(e + f\*x)/2]))/f

### Maple [A]

time = 0.23, size = 89, normalized size = 1.39

method	result
derivativedivides	$\frac{-a^2c \cot(fx+e) + a^2c \left( -\frac{\csc(fx+e) \cot(fx+e)}{2} + \frac{\ln(\csc(fx+e) - \cot(fx+e))}{2} \right) - a^2c \ln(\csc(fx+e) - \cot(fx+e)) - a^2c(fx+e)}{f}$
default	$\frac{-a^2c \cot(fx+e) + a^2c \left( -\frac{\csc(fx+e) \cot(fx+e)}{2} + \frac{\ln(\csc(fx+e) - \cot(fx+e))}{2} \right) - a^2c \ln(\csc(fx+e) - \cot(fx+e)) - a^2c(fx+e)}{f}$
risch	$-a^2cx + \frac{a^2c(e^{3i(fx+e)} + e^{i(fx+e)} - 2ie^{2i(fx+e)} + 2i)}{f(e^{2i(fx+e)} - 1)^2} + \frac{a^2c \ln(e^{i(fx+e)} + 1)}{2f} - \frac{a^2c \ln(e^{i(fx+e)} - 1)}{2f}$

norman	$\frac{\frac{a^2 c \left( \tan^7 \left( \frac{fx}{2} + \frac{e}{2} \right) \right)}{f} - \frac{a^2 c}{8f} - \frac{3a^2 c \left( \tan^2 \left( \frac{fx}{2} + \frac{e}{2} \right) \right)}{4f} - \frac{7a^2 c \left( \tan^6 \left( \frac{fx}{2} + \frac{e}{2} \right) \right)}{8f} - \frac{11a^2 c \left( \tan^4 \left( \frac{fx}{2} + \frac{e}{2} \right) \right)}{8f} - \frac{a^2 c \tan \left( \frac{fx}{2} + \frac{e}{2} \right)}{2f} - \frac{a^2 c \left( \tan^3 \right)}{2f}}{\tan}$
--------	---

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csc(f*x+e)^3*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x,method=_RETURNVERBOSE)`

[Out]  $1/f*(-a^2*c*\cot(f*x+e)+a^2*c*(-1/2*csc(f*x+e)*\cot(f*x+e)+1/2*\ln(csc(f*x+e)-\cot(f*x+e)))-a^2*c*\ln(csc(f*x+e)-\cot(f*x+e))-a^2*c*(f*x+e)$

**Maxima [A]**

time = 0.29, size = 113, normalized size = 1.77

$$\frac{4(fx+e)a^2c - a^2c \left( \frac{2 \cos(fx+e)}{\cos(fx+e)^2 - 1} - \log(\cos(fx+e) + 1) + \log(\cos(fx+e) - 1) \right) - 2a^2c(\log(\cos(fx+e) + 1) - \log(\cos(fx+e) - 1)) + \frac{4a^2c}{\tan(fx+e)}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(f*x+e)^3*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="maxima")`

[Out]  $-1/4*(4*(f*x + e)*a^2*c - a^2*c*(2*\cos(f*x + e)/(\cos(f*x + e)^2 - 1) - \log(\cos(f*x + e) + 1) + \log(\cos(f*x + e) - 1)) - 2*a^2*c*(\log(\cos(f*x + e) + 1) - \log(\cos(f*x + e) - 1)) + 4*a^2*c/\tan(f*x + e))/f$

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 147 vs. 2(64) = 128.

time = 0.39, size = 147, normalized size = 2.30

$$\frac{4a^2cfx \cos(fx+e)^2 - 4a^2cfx - 4a^2c \cos(fx+e) \sin(fx+e) - 2a^2c \cos(fx+e) - (a^2c \cos(fx+e)^2 - a^2c) \log\left(\frac{1}{2} \cos(fx+e) + \frac{1}{2}\right) + (a^2c \cos(fx+e)^2 - a^2c) \log\left(-\frac{1}{2} \cos(fx+e) + \frac{1}{2}\right)}{4(f \cos(fx+e)^2 - f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(f*x+e)^3*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="fricas")`

[Out]  $-1/4*(4*a^2*c*f*x*\cos(f*x + e)^2 - 4*a^2*c*f*x - 4*a^2*c*\cos(f*x + e)*\sin(f*x + e) - 2*a^2*c*\cos(f*x + e) - (a^2*c*\cos(f*x + e)^2 - a^2*c)*\log(1/2*\cos(f*x + e) + 1/2) + (a^2*c*\cos(f*x + e)^2 - a^2*c)*\log(-1/2*\cos(f*x + e) + 1/2))/(f*\cos(f*x + e)^2 - f)$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$-a^2c \left( \int (-\sin(e+fx)) \csc^3(e+fx) dx + \int \sin^2(e+fx) \csc^3(e+fx) dx + \int \sin^3(e+fx) \csc^3(e+fx) dx + \int (-\csc^3(e+fx)) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f\*x+e)\*\*3\*(a+a\*sin(f\*x+e))\*\*2\*(c-c\*sin(f\*x+e)),x)

[Out] -a\*\*2\*c\*(Integral(-sin(e + f\*x)\*csc(e + f\*x)\*\*3, x) + Integral(sin(e + f\*x)\*\*2\*csc(e + f\*x)\*\*3, x) + Integral(sin(e + f\*x)\*\*3\*csc(e + f\*x)\*\*3, x) + Integral(-csc(e + f\*x)\*\*3, x))

**Giac [A]**

time = 0.43, size = 123, normalized size = 1.92

$$\frac{a^2 c \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^2 - 8(f x + e) a^2 c - 4 a^2 c \log\left(\left|\tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)\right|\right) + 4 a^2 c \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) + \frac{6 a^2 c \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^2 - 4 a^2 c \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) - a^2 c}{\tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^2}}{8 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f\*x+e)^3\*(a+a\*sin(f\*x+e))^2\*(c-c\*sin(f\*x+e)),x, algorithm="giac")

[Out] 1/8\*(a^2\*c\*tan(1/2\*f\*x + 1/2\*e)^2 - 8\*(f\*x + e)\*a^2\*c - 4\*a^2\*c\*log(abs(tan(1/2\*f\*x + 1/2\*e))) + 4\*a^2\*c\*tan(1/2\*f\*x + 1/2\*e) + (6\*a^2\*c\*tan(1/2\*f\*x + 1/2\*e)^2 - 4\*a^2\*c\*tan(1/2\*f\*x + 1/2\*e) - a^2\*c)/tan(1/2\*f\*x + 1/2\*e)^2)/f

**Mupad [B]**

time = 12.26, size = 163, normalized size = 2.55

$$\frac{a^2 c \tan\left(\frac{e}{2} + \frac{f x}{2}\right)}{2 f} - \frac{a^2 c \ln\left(\frac{\sin\left(\frac{e}{2} + \frac{f x}{2}\right)}{\cos\left(\frac{e}{2} + \frac{f x}{2}\right)}\right)}{2 f} - \frac{2 a^2 c \operatorname{atan}\left(\frac{2 \cos\left(\frac{e}{2} + \frac{f x}{2}\right) + \sin\left(\frac{e}{2} + \frac{f x}{2}\right)}{\cos\left(\frac{e}{2} + \frac{f x}{2}\right) - 2 \sin\left(\frac{e}{2} + \frac{f x}{2}\right)}\right)}{f} - \frac{a^2 c \cot\left(\frac{e}{2} + \frac{f x}{2}\right)}{2 f} - \frac{a^2 c \cot\left(\frac{e}{2} + \frac{f x}{2}\right)^2}{8 f} + \frac{a^2 c \tan\left(\frac{e}{2} + \frac{f x}{2}\right)^2}{8 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + a\*sin(e + f\*x))^2\*(c - c\*sin(e + f\*x)))/sin(e + f\*x)^3,x)

[Out] (a^2\*c\*tan(e/2 + (f\*x)/2))/(2\*f) - (a^2\*c\*log(sin(e/2 + (f\*x)/2)/cos(e/2 + (f\*x)/2)))/(2\*f) - (2\*a^2\*c\*atan((2\*cos(e/2 + (f\*x)/2) + sin(e/2 + (f\*x)/2))/(cos(e/2 + (f\*x)/2) - 2\*sin(e/2 + (f\*x)/2)))/f - (a^2\*c\*cot(e/2 + (f\*x)/2))/(2\*f) - (a^2\*c\*cot(e/2 + (f\*x)/2)^2)/(8\*f) + (a^2\*c\*tan(e/2 + (f\*x)/2)^2)/(8\*f)

### 3.8 $\int \csc^4(e + fx)(a + a \sin(e + fx))^2(c - c \sin(e + fx)) dx$

Optimal. Leaf size=61

$$\frac{a^2 c \tanh^{-1}(\cos(e + fx))}{2f} - \frac{a^2 c \cot^3(e + fx)}{3f} - \frac{a^2 c \cot(e + fx) \csc(e + fx)}{2f}$$

[Out] 1/2\*a^2\*c\*arctanh(cos(f\*x+e))/f-1/3\*a^2\*c\*cot(f\*x+e)^3/f-1/2\*a^2\*c\*cot(f\*x+e)\*csc(f\*x+e)/f

Rubi [A]

time = 0.11, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$ , Rules used = {3029, 2785, 2687, 30, 2691, 3855}

$$-\frac{a^2 c \cot^3(e + fx)}{3f} + \frac{a^2 c \tanh^{-1}(\cos(e + fx))}{2f} - \frac{a^2 c \cot(e + fx) \csc(e + fx)}{2f}$$

Antiderivative was successfully verified.

[In] Int[Csc[e + f\*x]^4\*(a + a\*Sin[e + f\*x])^2\*(c - c\*Sin[e + f\*x]),x]

[Out] (a^2\*c\*ArcTanh[Cos[e + f\*x]])/(2\*f) - (a^2\*c\*Cot[e + f\*x]^3)/(3\*f) - (a^2\*c\*Cot[e + f\*x]\*Csc[e + f\*x])/(2\*f)

Rule 30

Int[(x\_)^(m\_), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 2687

Int[sec[(e\_) + (f\_)\*(x\_)]^(m\_)\*((b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[1/f, Subst[Int[(b\*x)^n\*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f\*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/2] && LtQ[0, n, m - 1])

Rule 2691

Int[((a\_)\*sec[(e\_) + (f\_)\*(x\_)])^(m\_)\*((b\_)\*tan[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[b\*(a\*Sec[e + f\*x])^m\*((b\*Tan[e + f\*x])^(n - 1)/(f\*(m + n - 1))), x] - Dist[b^2\*((n - 1)/(m + n - 1)), Int[(a\*Sec[e + f\*x])^m\*(b\*Tan[e + f\*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && NeQ[m + n - 1, 0] && IntegerQ[2\*m, 2\*n]

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)]^(p_.)/((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x] - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

### Rule 3029

```
Int[sin[(e_.) + (f_.)*(x_)]^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(m_.))*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] := Dist[a^n*c^n, Int[Tan[e + f*x]^p*(a + b*Sin[e + f*x])^(m - n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[p + 2*n, 0] && IntegerQ[n]
```

### Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]
```

### Rubi steps

$$\begin{aligned} \int \csc^4(e + fx)(a + a \sin(e + fx))^2(c - c \sin(e + fx)) dx &= (a^2 c^2) \int \frac{\cot^4(e + fx)}{c - c \sin(e + fx)} dx \\ &= (a^2 c) \int \cot^2(e + fx) \csc(e + fx) dx + (a^2 c) \int \csc(e + fx) dx \\ &= -\frac{a^2 c \cot(e + fx) \csc(e + fx)}{2f} - \frac{1}{2}(a^2 c) \int \csc(e + fx) dx \\ &= \frac{a^2 c \tanh^{-1}(\cos(e + fx))}{2f} - \frac{a^2 c \cot^3(e + fx)}{3f} - \frac{a^2 c \cot(e + fx) \csc(e + fx)}{2f} \end{aligned}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 172 vs. 2(61) = 122.

time = 0.06, size = 172, normalized size = 2.82

$$a^2 c \left( \frac{\cot\left(\frac{1}{2}(e + fx)\right)}{6f} - \frac{\csc^2\left(\frac{1}{2}(e + fx)\right)}{8f} - \frac{\cot\left(\frac{1}{2}(e + fx)\right) \csc^2\left(\frac{1}{2}(e + fx)\right)}{24f} + \frac{\log\left(\cos\left(\frac{1}{2}(e + fx)\right)\right)}{2f} - \frac{\log\left(\sin\left(\frac{1}{2}(e + fx)\right)\right)}{2f} + \frac{\sec^2\left(\frac{1}{2}(e + fx)\right)}{8f} - \frac{\tan\left(\frac{1}{2}(e + fx)\right)}{6f} + \frac{\sec^2\left(\frac{1}{2}(e + fx)\right) \tan\left(\frac{1}{2}(e + fx)\right)}{24f} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x]),x]
```

```
[Out] a^2*c*(Cot[(e + f*x)/2]/(6*f) - Csc[(e + f*x)/2]^2/(8*f) - (Cot[(e + f*x)/2]*Csc[(e + f*x)/2]^2)/(24*f) + Log[Cos[(e + f*x)/2]]/(2*f) - Log[Sin[(e + f*x)/2]]/(2*f) + Sec[(e + f*x)/2]^2/(8*f) - Tan[(e + f*x)/2]/(6*f) + (Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(24*f)
```

**Maple [A]**

time = 0.20, size = 100, normalized size = 1.64

method	result
derivativedivides	$a^2c\left(-\frac{\csc(fx+e)\cot(fx+e)}{2} + \frac{\ln(\csc(fx+e)-\cot(fx+e))}{2}\right) + a^2c\left(-\frac{2}{3} - \frac{\csc^2(fx+e)}{3}\right) \cot(fx+e) + a^2c\cot(fx+e) - a^2c\ln(\csc(fx+e))$
default	$a^2c\left(-\frac{\csc(fx+e)\cot(fx+e)}{2} + \frac{\ln(\csc(fx+e)-\cot(fx+e))}{2}\right) + a^2c\left(-\frac{2}{3} - \frac{\csc^2(fx+e)}{3}\right) \cot(fx+e) + a^2c\cot(fx+e) - a^2c\ln(\csc(fx+e))$
risch	$\frac{a^2c(6ie^{4i(fx+e)} + 3e^{5i(fx+e)} + 2i - 3e^{i(fx+e)})}{3f(e^{2i(fx+e)} - 1)^3} + \frac{a^2c\ln(e^{i(fx+e)} + 1)}{2f} - \frac{a^2c\ln(e^{i(fx+e)} - 1)}{2f}$
norman	$\frac{-\frac{a^2c}{24f} - \frac{3a^2c(\tan^3(\frac{fx}{2} + \frac{e}{2}))}{4f} - \frac{7a^2c(\tan^7(\frac{fx}{2} + \frac{e}{2}))}{8f} - \frac{11a^2c(\tan^5(\frac{fx}{2} + \frac{e}{2}))}{8f} - \frac{a^2c\tan(\frac{fx}{2} + \frac{e}{2})}{8f} + \frac{a^2c(\tan^4(\frac{fx}{2} + \frac{e}{2}))}{8f} - \frac{a^2c(\tan^2(\frac{fx}{2} + \frac{e}{2}))}{8f}}{\tan(\frac{fx}{2} + \frac{e}{2})^3(1 + \tan^2(\frac{fx}{2} + \frac{e}{2}))^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/f*(a^2*c*(-1/2*csc(f*x+e)*cot(f*x+e)+1/2*ln(csc(f*x+e)-cot(f*x+e)))+a^2*c
*(-2/3-1/3*csc(f*x+e)^2)*cot(f*x+e)+a^2*c*cot(f*x+e)-a^2*c*ln(csc(f*x+e)-co
t(f*x+e)))
```

**Maxima [B]** Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(59) = 118.

time = 0.30, size = 129, normalized size = 2.11

$$\frac{3a^2c\left(\frac{2\cos(fx+e)}{\cos(fx+e)^2-1} - \log(\cos(fx+e)+1) + \log(\cos(fx+e)-1)\right) + 6a^2c(\log(\cos(fx+e)+1) - \log(\cos(fx+e)-1)) + \frac{12a^2c}{\tan(fx+e)} - \frac{4(3\tan(fx+e)^2+1)a^2c}{\tan(fx+e)^3}}{12f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] 1/12*(3*a^2*c*(2*cos(f*x + e)/(cos(f*x + e)^2 - 1) - log(cos(f*x + e) + 1)
+ log(cos(f*x + e) - 1)) + 6*a^2*c*(log(cos(f*x + e) + 1) - log(cos(f*x + e)
) - 1)) + 12*a^2*c/tan(f*x + e) - 4*(3*tan(f*x + e)^2 + 1)*a^2*c/tan(f*x +
e)^3)/f
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 148 vs. 2(59) = 118.

time = 0.37, size = 148, normalized size = 2.43

$$\frac{4a^2c\cos(fx+e)^3 + 6a^2c\cos(fx+e)\sin(fx+e) + 3(a^2c\cos(fx+e)^2 - a^2c)\log\left(\frac{1}{2}\cos(fx+e) + \frac{1}{2}\right)\sin(fx+e) - 3(a^2c\cos(fx+e)^2 - a^2c)\log\left(-\frac{1}{2}\cos(fx+e) + \frac{1}{2}\right)\sin(fx+e)}{12(f\cos(fx+e)^2 - f)\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f\*x+e)^4\*(a+a\*sin(f\*x+e))^2\*(c-c\*sin(f\*x+e)),x, algorithm="fricas")

[Out]  $\frac{1}{12}*(4*a^2*c*\cos(f*x + e)^3 + 6*a^2*c*\cos(f*x + e)*\sin(f*x + e) + 3*(a^2*c*\cos(f*x + e)^2 - a^2*c)*\log(1/2*\cos(f*x + e) + 1/2)*\sin(f*x + e) - 3*(a^2*c*\cos(f*x + e)^2 - a^2*c)*\log(-1/2*\cos(f*x + e) + 1/2)*\sin(f*x + e))/((f*\cos(f*x + e)^2 - f)*\sin(f*x + e))$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$-a^2c \left( \int (-\sin(e+fx)) \csc^4(e+fx) dx + \int \sin^2(e+fx) \csc^4(e+fx) dx + \int \sin^3(e+fx) \csc^4(e+fx) dx + \int (-\csc^4(e+fx)) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f\*x+e)\*\*4\*(a+a\*sin(f\*x+e))\*\*2\*(c-c\*sin(f\*x+e)),x)

[Out]  $-a^{**2}*c*(\text{Integral}(-\sin(e + f*x)*\csc(e + f*x)**4, x) + \text{Integral}(\sin(e + f*x)**2*\csc(e + f*x)**4, x) + \text{Integral}(\sin(e + f*x)**3*\csc(e + f*x)**4, x) + \text{Integral}(-\csc(e + f*x)**4, x))$

**Giac [B]** Leaf count of result is larger than twice the leaf count of optimal. 147 vs. 2(59) = 118.

time = 0.45, size = 147, normalized size = 2.41

$$\frac{a^2c \tan(\frac{1}{2}fx + \frac{1}{2}e)^3 + 3a^2c \tan(\frac{1}{2}fx + \frac{1}{2}e)^2 - 12a^2c \log(|\tan(\frac{1}{2}fx + \frac{1}{2}e)|) - 3a^2c \tan(\frac{1}{2}fx + \frac{1}{2}e) + \frac{22a^2c \tan(\frac{1}{2}fx + \frac{1}{2}e)^3 + 3a^2c \tan(\frac{1}{2}fx + \frac{1}{2}e)^2 - 3a^2c \tan(\frac{1}{2}fx + \frac{1}{2}e) - a^2c}{\tan(\frac{1}{2}fx + \frac{1}{2}e)^3}}{24f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f\*x+e)^4\*(a+a\*sin(f\*x+e))^2\*(c-c\*sin(f\*x+e)),x, algorithm="giac")

[Out]  $\frac{1}{24}*(a^2*c*\tan(1/2*f*x + 1/2*e)^3 + 3*a^2*c*\tan(1/2*f*x + 1/2*e)^2 - 12*a^2*c*\log(\text{abs}(\tan(1/2*f*x + 1/2*e))) - 3*a^2*c*\tan(1/2*f*x + 1/2*e) + (22*a^2*c*\tan(1/2*f*x + 1/2*e)^3 + 3*a^2*c*\tan(1/2*f*x + 1/2*e)^2 - 3*a^2*c*\tan(1/2*f*x + 1/2*e) - a^2*c)/\tan(1/2*f*x + 1/2*e)^3)/f$

**Mupad [B]**

time = 12.22, size = 132, normalized size = 2.16

$$\frac{a^2c \tan(\frac{e}{2} + \frac{fx}{2})^2}{8f} - \frac{a^2c \tan(\frac{e}{2} + \frac{fx}{2})}{8f} - \frac{\cot(\frac{e}{2} + \frac{fx}{2})^3 \left( -ca^2 \tan(\frac{e}{2} + \frac{fx}{2})^2 + ca^2 \tan(\frac{e}{2} + \frac{fx}{2}) + \frac{ca^2}{3} \right)}{8f} + \frac{a^2c \tan(\frac{e}{2} + \frac{fx}{2})^3}{24f} - \frac{a^2c \ln(\tan(\frac{e}{2} + \frac{fx}{2}))}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + a\*sin(e + f\*x))^2\*(c - c\*sin(e + f\*x)))/sin(e + f\*x)^4,x)

[Out]  $(a^2*c*\tan(e/2 + (f*x)/2)^2)/(8*f) - (a^2*c*\tan(e/2 + (f*x)/2))/(8*f) - (\cot(e/2 + (f*x)/2)^3*((a^2*c)/3 + a^2*c*\tan(e/2 + (f*x)/2) - a^2*c*\tan(e/2 + (f*x)/2)^2))/(8*f) + (a^2*c*\tan(e/2 + (f*x)/2)^3)/(24*f) - (a^2*c*\log(\tan(e/2 + (f*x)/2)))/(2*f)$



### 3.9 $\int \csc^5(e + fx)(a + a \sin(e + fx))^2(c - c \sin(e + fx)) dx$

**Optimal.** Leaf size=86

$$\frac{a^2 c \tanh^{-1}(\cos(e + fx))}{8f} - \frac{a^2 c \cot^3(e + fx)}{3f} + \frac{a^2 c \cot(e + fx) \csc(e + fx)}{8f} - \frac{a^2 c \cot(e + fx) \csc^3(e + fx)}{4f}$$

[Out]  $1/8*a^2*c*\operatorname{arctanh}(\cos(f*x+e))/f-1/3*a^2*c*\cot(f*x+e)^3/f+1/8*a^2*c*\cot(f*x+e)*\csc(f*x+e)/f-1/4*a^2*c*\cot(f*x+e)*\csc(f*x+e)^3/f$

**Rubi [A]**

time = 0.11, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$ , Rules used = {3045, 3852, 8, 3853, 3855}

$$-\frac{a^2 c \cot^3(e + fx)}{3f} + \frac{a^2 c \tanh^{-1}(\cos(e + fx))}{8f} - \frac{a^2 c \cot(e + fx) \csc^3(e + fx)}{4f} + \frac{a^2 c \cot(e + fx) \csc(e + fx)}{8f}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Csc}[e + f*x]^5*(a + a*\operatorname{Sin}[e + f*x])^2*(c - c*\operatorname{Sin}[e + f*x]), x]$

[Out]  $(a^2*c*\operatorname{ArcTanh}[\operatorname{Cos}[e + f*x]])/(8*f) - (a^2*c*\operatorname{Cot}[e + f*x]^3)/(3*f) + (a^2*c*\operatorname{Cot}[e + f*x]*\operatorname{Csc}[e + f*x])/(8*f) - (a^2*c*\operatorname{Cot}[e + f*x]*\operatorname{Csc}[e + f*x]^3)/(4*f)$

Rule 8

$\operatorname{Int}[a_, x\_Symbol] \rightarrow \operatorname{Simp}[a*x, x] /; \operatorname{FreeQ}[a, x]$

Rule 3045

$\operatorname{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(n_.)*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)])}^{(m_.)*((A_.) + (B_.)*\sin[(e_.) + (f_.)*(x_.)])}, x\_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandTrig}[\sin[e + f*x]^{n*(a + b*\sin[e + f*x])}^{m*(A + B*\sin[e + f*x])}, x], x] /; \operatorname{FreeQ}[\{a, b, e, f, A, B\}, x] \ \&\& \operatorname{EqQ}[A*b + a*B, 0] \ \&\& \operatorname{EqQ}[a^2 - b^2, 0] \ \&\& \operatorname{IntegerQ}[m] \ \&\& \operatorname{IntegerQ}[n]$

Rule 3852

$\operatorname{Int}[\operatorname{csc}[(c_.) + (d_.)*(x_.)]^{(n_.)}, x\_Symbol] \rightarrow \operatorname{Dist}[-d^{(-1)}, \operatorname{Subst}[\operatorname{Int}[\operatorname{ExpandIntegrand}[(1 + x^2)^{(n/2 - 1)}, x], x], x, \operatorname{Cot}[c + d*x]], x] /; \operatorname{FreeQ}[\{c, d\}, x] \ \&\& \operatorname{IGtQ}[n/2, 0]$

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^n], x_Symbol] := Simp[(-b)*Cos[c + d*x]
*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

### Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]
```

### Rubi steps

$$\begin{aligned} \int \csc^5(e + fx)(a + a \sin(e + fx))^2(c - c \sin(e + fx)) dx &= \int (-a^2c \csc^2(e + fx) - a^2c \csc^3(e + fx) + a^2c \csc^4(e + fx)) dx \\ &= -\left( (a^2c) \int \csc^2(e + fx) dx \right) - (a^2c) \int \csc^3(e + fx) dx \\ &= \frac{a^2c \cot(e + fx) \csc(e + fx)}{2f} - \frac{a^2c \cot(e + fx) \csc^2(e + fx)}{4f} \\ &= \frac{a^2c \tanh^{-1}(\cos(e + fx))}{2f} - \frac{a^2c \cot^3(e + fx)}{3f} + \frac{a^2c \cot^2(e + fx)}{4f} \\ &= \frac{a^2c \tanh^{-1}(\cos(e + fx))}{8f} - \frac{a^2c \cot^3(e + fx)}{3f} + \frac{a^2c \cot^2(e + fx)}{4f} \end{aligned}$$

**Mathematica [B]** Leaf count is larger than twice the leaf count of optimal. 179 vs. 2(86) = 172.

time = 0.05, size = 179, normalized size = 2.08

$$\frac{a^2c \cot(e + fx)}{3f} + \frac{a^2c \csc^2(\frac{1}{2}(e + fx))}{32f} - \frac{a^2c \csc^4(\frac{1}{2}(e + fx))}{64f} - \frac{a^2c \cot(e + fx) \csc^2(e + fx)}{3f} + \frac{a^2c \log(\cos(\frac{1}{2}(e + fx)))}{8f} - \frac{a^2c \log(\sin(\frac{1}{2}(e + fx)))}{8f} - \frac{a^2c \sec^2(\frac{1}{2}(e + fx))}{32f} + \frac{a^2c \sec^4(\frac{1}{2}(e + fx))}{64f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x]),x]
```

```
[Out] (a^2*c*Cot[e + f*x])/(3*f) + (a^2*c*Csc[(e + f*x)/2]^2)/(32*f) - (a^2*c*Csc
[(e + f*x)/2]^4)/(64*f) - (a^2*c*Cot[e + f*x]*Csc[e + f*x]^2)/(3*f) + (a^2*
c*Log[Cos[(e + f*x)/2]])/(8*f) - (a^2*c*Log[Sin[(e + f*x)/2]])/(8*f) - (a^2
*c*Sec[(e + f*x)/2]^2)/(32*f) + (a^2*c*Sec[(e + f*x)/2]^4)/(64*f)
```

### Maple [A]

time = 0.24, size = 129, normalized size = 1.50

method	result
--------	--------

derivativdivides	$\frac{a^2c\left(-\frac{2}{3}-\frac{\csc^2(fx+e)}{3}\right)\cot(fx+e)+a^2c\left(\left(-\frac{\csc^3(fx+e)}{4}-\frac{3\csc(fx+e)}{8}\right)\cot(fx+e)+\frac{3\ln(\csc(fx+e)-\cot(fx+e))}{8}\right)}{f}-a^2c\left(-\frac{2}{3}-\frac{\csc^2(fx+e)}{3}\right)\cot(fx+e)+a^2c\left(\left(-\frac{\csc^3(fx+e)}{4}-\frac{3\csc(fx+e)}{8}\right)\cot(fx+e)+\frac{3\ln(\csc(fx+e)-\cot(fx+e))}{8}\right)}{f}$
default	$\frac{a^2c\left(-\frac{2}{3}-\frac{\csc^2(fx+e)}{3}\right)\cot(fx+e)+a^2c\left(\left(-\frac{\csc^3(fx+e)}{4}-\frac{3\csc(fx+e)}{8}\right)\cot(fx+e)+\frac{3\ln(\csc(fx+e)-\cot(fx+e))}{8}\right)}{f}$
risch	$-\frac{a^2c(3e^{7i(fx+e)}+21e^{5i(fx+e)}-24ie^{6i(fx+e)}+21e^{3i(fx+e)}+24ie^{4i(fx+e)}+3e^{i(fx+e)}-8ie^{2i(fx+e)}+8i)}{12f(e^{2i(fx+e)}-1)^4}-\frac{a^2c\ln(e^{i(fx+e)}-1)}{8}$
norman	$\frac{\frac{a^2c}{64f}-\frac{a^2c(\tan^8(\frac{fx}{2}+\frac{e}{2}))}{8f}-\frac{3a^2c(\tan^4(\frac{fx}{2}+\frac{e}{2}))}{32f}-\frac{5a^2c(\tan^6(\frac{fx}{2}+\frac{e}{2}))}{32f}-\frac{a^2c\tan(\frac{fx}{2}+\frac{e}{2})}{24f}-\frac{3a^2c(\tan^2(\frac{fx}{2}+\frac{e}{2}))}{64f}}{\tan(\frac{fx}{2}+\frac{e}{2})^4(1+\tan^2(\frac{fx}{2}+\frac{e}{2}))}+\frac{a^2c(\tan^5(\frac{fx}{2}+\frac{e}{2}))}{64f}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csc(f*x+e)^5*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x,method=_RETURNVERBOSE)`

[Out]  $\frac{1}{f}*(a^2*c*(-2/3-1/3*\csc(f*x+e)^2)*\cot(f*x+e)+a^2*c*((-1/4*\csc(f*x+e)^3-3/8*\csc(f*x+e))*\cot(f*x+e)+3/8*\ln(\csc(f*x+e)-\cot(f*x+e))))-a^2*c*(-1/2*\csc(f*x+e)*\cot(f*x+e)+1/2*\ln(\csc(f*x+e)-\cot(f*x+e)))+a^2*c*\cot(f*x+e)$

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(84) = 168.

time = 0.29, size = 178, normalized size = 2.07

$$\frac{3a^2c\left(\frac{2(3\cos(fx+e)^3-5\cos(fx+e))}{\cos(fx+e)^3-2\cos(fx+e)+1}-3\log(\cos(fx+e)+1)+3\log(\cos(fx+e)-1)\right)-12a^2c\left(\frac{2\cos(fx+e)}{\cos(fx+e)^2-1}-\log(\cos(fx+e)+1)+\log(\cos(fx+e)-1)\right)+\frac{48a^2c}{\tan(fx+e)}-\frac{16(3\tan(fx+e)^2+1)a^2c}{\tan(fx+e)^3}}{48f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(f*x+e)^5*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="maxima")`

[Out]  $\frac{1}{48}*(3*a^2*c*(2*(3*\cos(f*x+e)^3-5*\cos(f*x+e))/(\cos(f*x+e)^4-2*\cos(f*x+e)^2+1)-3*\log(\cos(f*x+e)+1)+3*\log(\cos(f*x+e)-1))-12*a^2*c*(2*\cos(f*x+e)/(\cos(f*x+e)^2-1)-\log(\cos(f*x+e)+1)+\log(\cos(f*x+e)-1))+48*a^2*c/\tan(f*x+e)-16*(3*\tan(f*x+e)^2+1)*a^2*c/\tan(f*x+e)^3)/f$

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(84) = 168.

time = 0.36, size = 178, normalized size = 2.07

$$\frac{16a^2c\cos(fx+e)^3\sin(fx+e)+6a^2c\cos(fx+e)^3+6a^2c\cos(fx+e)-3(a^2c\cos(fx+e)^4-2a^2c\cos(fx+e)^2+a^2c)\log(\frac{1}{2}\cos(fx+e)+\frac{1}{2})+3(a^2c\cos(fx+e)^4-2a^2c\cos(fx+e)^2+a^2c)\log(-\frac{1}{2}\cos(fx+e)+\frac{1}{2})}{48(f\cos(fx+e)^4-2f\cos(fx+e)^2+f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(f*x+e)^5*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="fricas")`

[Out]  $-1/48*(16*a^2*c*cos(f*x + e)^3*sin(f*x + e) + 6*a^2*c*cos(f*x + e)^3 + 6*a^2*c*cos(f*x + e) - 3*(a^2*c*cos(f*x + e)^4 - 2*a^2*c*cos(f*x + e)^2 + a^2*c)*log(1/2*cos(f*x + e) + 1/2) + 3*(a^2*c*cos(f*x + e)^4 - 2*a^2*c*cos(f*x + e)^2 + a^2*c)*log(-1/2*cos(f*x + e) + 1/2))/(f*cos(f*x + e)^4 - 2*f*cos(f*x + e)^2 + f)$

**Sympy [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(f*x+e)**5*(a+a*sin(f*x+e))**2*(c-c*sin(f*x+e)),x)`

[Out] Timed out

**Giac [A]**

time = 0.49, size = 148, normalized size = 1.72

$$\frac{3a^2c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 + 8a^2c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 - 24a^2c \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)\right|\right) - 24a^2c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + \frac{50a^2c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 + 24a^2c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 - 8a^2c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - 3a^2c}{\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4}}{192f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(f*x+e)^5*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="giac")`

[Out]  $1/192*(3*a^2*c*tan(1/2*f*x + 1/2*e)^4 + 8*a^2*c*tan(1/2*f*x + 1/2*e)^3 - 24*a^2*c*log(abs(tan(1/2*f*x + 1/2*e))) - 24*a^2*c*tan(1/2*f*x + 1/2*e) + (50*a^2*c*tan(1/2*f*x + 1/2*e)^4 + 24*a^2*c*tan(1/2*f*x + 1/2*e)^3 - 8*a^2*c*tan(1/2*f*x + 1/2*e) - 3*a^2*c)/tan(1/2*f*x + 1/2*e)^4)/f$

**Mupad [B]**

time = 12.18, size = 133, normalized size = 1.55

$$\frac{a^2c \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^3}{24f} - \frac{a^2c \tan\left(\frac{e}{2} + \frac{fx}{2}\right)}{8f} - \frac{\cot\left(\frac{e}{2} + \frac{fx}{2}\right)^4 \left(-2ca^2 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^3 + \frac{2ca^2 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)}{3} + \frac{ca^2}{4}\right)}{16f} + \frac{a^2c \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^4}{64f} - \frac{a^2c \ln\left(\tan\left(\frac{e}{2} + \frac{fx}{2}\right)\right)}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x)))/sin(e + f*x)^5,x)`

[Out]  $(a^2*c*tan(e/2 + (f*x)/2)^3)/(24*f) - (a^2*c*tan(e/2 + (f*x)/2))/(8*f) - (cot(e/2 + (f*x)/2)^4*((a^2*c)/4 + (2*a^2*c*tan(e/2 + (f*x)/2))/3 - 2*a^2*c*tan(e/2 + (f*x)/2)^3))/(16*f) + (a^2*c*tan(e/2 + (f*x)/2)^4)/(64*f) - (a^2*c*log(tan(e/2 + (f*x)/2)))/(8*f)$

### 3.10 $\int \csc^6(e + fx)(a + a \sin(e + fx))^2(c - c \sin(e + fx)) dx$

**Optimal.** Leaf size=105

$$\frac{a^2 c \tanh^{-1}(\cos(e + fx))}{8f} - \frac{a^2 c \cot^3(e + fx)}{3f} - \frac{a^2 c \cot^5(e + fx)}{5f} + \frac{a^2 c \cot(e + fx) \csc(e + fx)}{8f} - \frac{a^2 c \cot(e + fx) \csc^3(e + fx)}{4f}$$

[Out]  $1/8*a^2*c*\operatorname{arctanh}(\cos(f*x+e))/f-1/3*a^2*c*\cot(f*x+e)^3/f-1/5*a^2*c*\cot(f*x+e)^5/f+1/8*a^2*c*\cot(f*x+e)*\csc(f*x+e)/f-1/4*a^2*c*\cot(f*x+e)*\csc(f*x+e)^3/f$

**Rubi [A]**

time = 0.12, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ ,

Rules used = {3045, 3853, 3855, 3852}

$$-\frac{a^2 c \cot^5(e + fx)}{5f} - \frac{a^2 c \cot^3(e + fx)}{3f} + \frac{a^2 c \tanh^{-1}(\cos(e + fx))}{8f} - \frac{a^2 c \cot(e + fx) \csc^3(e + fx)}{4f} + \frac{a^2 c \cot(e + fx) \csc(e + fx)}{8f}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Csc}[e + f*x]^6*(a + a*\operatorname{Sin}[e + f*x])^2*(c - c*\operatorname{Sin}[e + f*x]), x]$

[Out]  $(a^2*c*\operatorname{ArcTanh}[\operatorname{Cos}[e + f*x]])/(8*f) - (a^2*c*\operatorname{Cot}[e + f*x]^3)/(3*f) - (a^2*c*\operatorname{Cot}[e + f*x]^5)/(5*f) + (a^2*c*\operatorname{Cot}[e + f*x]*\operatorname{Csc}[e + f*x])/(8*f) - (a^2*c*\operatorname{Cot}[e + f*x]*\operatorname{Csc}[e + f*x]^3)/(4*f)$

Rule 3045

$\operatorname{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(n_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)])^{(m_.)}*((A_.) + (B_.)*\sin[(e_.) + (f_.)*(x_.)]), x\_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandTrig}[\sin[e + f*x]^n*(a + b*\sin[e + f*x])^m*(A + B*\sin[e + f*x]), x], x] /;$  FreeQ[{a, b, e, f, A, B}, x] && EqQ[A\*b + a\*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && IntegerQ[n]

Rule 3852

$\operatorname{Int}[\csc[(c_.) + (d_.)*(x_.)]^{(n_.)}, x\_Symbol] \rightarrow \operatorname{Dist}[-d^{(-1)}, \operatorname{Subst}[\operatorname{Int}[\operatorname{ExpandIntegrand}[(1 + x^2)^{(n/2 - 1)}, x], x], x, \operatorname{Cot}[c + d*x]], x] /;$  FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 3853

$\operatorname{Int}[(\csc[(c_.) + (d_.)*(x_.)]*(b_.))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{Simp}[(-b)*\operatorname{Cos}[c + d*x]*((b*\operatorname{Csc}[c + d*x])^{(n - 1)}/(d*(n - 1))), x] + \operatorname{Dist}[b^2*((n - 2)/(n - 1)), \operatorname{Int}[(b*\operatorname{Csc}[c + d*x])^{(n - 2)}, x], x] /;$  FreeQ[{b, c, d}, x] && GtQ[n, 1] &

& IntegerQ[2\*n]

### Rule 3855

Int[csc[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] := Simp[-ArcTanh[Cos[c + d\*x]]/d, x]  
/; FreeQ[{c, d}, x]

### Rubi steps

$$\begin{aligned} \int \csc^6(e + fx)(a + a \sin(e + fx))^2(c - c \sin(e + fx)) dx &= \int (-a^2 c \csc^3(e + fx) - a^2 c \csc^4(e + fx) + a^2 c \csc^5(e + fx)) dx \\ &= -\left( (a^2 c) \int \csc^3(e + fx) dx \right) - (a^2 c) \int \csc^4(e + fx) dx - (a^2 c) \int \csc^5(e + fx) dx \\ &= \frac{a^2 c \cot(e + fx) \csc(e + fx)}{2f} - \frac{a^2 c \cot(e + fx) \csc^3(e + fx)}{4f} - \frac{a^2 c \cot(e + fx) \csc^5(e + fx)}{6f} \\ &= \frac{a^2 c \tanh^{-1}(\cos(e + fx))}{2f} - \frac{a^2 c \cot^3(e + fx)}{3f} - \frac{a^2 c \cot^5(e + fx)}{5f} \\ &= \frac{a^2 c \tanh^{-1}(\cos(e + fx))}{8f} - \frac{a^2 c \cot^3(e + fx)}{3f} - \frac{a^2 c \cot^5(e + fx)}{5f} \end{aligned}$$

### Mathematica [A]

time = 0.05, size = 204, normalized size = 1.94

$$\frac{2a^2 c \cot(e + fx)}{15f} + \frac{a^2 c \csc^2\left(\frac{1}{2}(e + fx)\right)}{32f} - \frac{a^2 c \csc^4\left(\frac{1}{2}(e + fx)\right)}{64f} + \frac{a^2 c \cot(e + fx) \csc^2(e + fx)}{15f} - \frac{a^2 c \cot(e + fx) \csc^4(e + fx)}{5f} + \frac{a^2 c \log(\cos\left(\frac{1}{2}(e + fx)\right))}{8f} - \frac{a^2 c \log(\sin\left(\frac{1}{2}(e + fx)\right))}{8f} - \frac{a^2 c \sec^2\left(\frac{1}{2}(e + fx)\right)}{32f} + \frac{a^2 c \sec^4\left(\frac{1}{2}(e + fx)\right)}{64f}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[e + f\*x]^6\*(a + a\*Sin[e + f\*x])^2\*(c - c\*Sin[e + f\*x]),x]

[Out] (2\*a^2\*c\*Cot[e + f\*x])/(15\*f) + (a^2\*c\*Csc[(e + f\*x)/2]^2)/(32\*f) - (a^2\*c\*Csc[(e + f\*x)/2]^4)/(64\*f) + (a^2\*c\*Cot[e + f\*x]\*Csc[e + f\*x]^2)/(15\*f) - (a^2\*c\*Cot[e + f\*x]\*Csc[e + f\*x]^4)/(5\*f) + (a^2\*c\*Log[Cos[(e + f\*x)/2]])/(8\*f) - (a^2\*c\*Log[Sin[(e + f\*x)/2]])/(8\*f) - (a^2\*c\*Sec[(e + f\*x)/2]^2)/(32\*f) + (a^2\*c\*Sec[(e + f\*x)/2]^4)/(64\*f)

### Maple [A]

time = 0.26, size = 152, normalized size = 1.45

method	result
risch	$-\frac{a^2 c (15 e^{9i(fx+e)} + 240 i e^{6i(fx+e)} + 90 e^{7i(fx+e)} + 80 i e^{4i(fx+e)} + 80 i e^{2i(fx+e)} - 90 e^{3i(fx+e)} - 16 i - 15 e^{i(fx+e)})}{60 f (e^{2i(fx+e)} - 1)^5} - \frac{a^2 c \ln\left(\frac{\cos\left(\frac{e+fx}{2}\right)}{\sin\left(\frac{e+fx}{2}\right)}\right)}{8 f}$

derivativedivides	$\frac{a^2 c \left( \left( -\frac{\csc^3(fx+e)}{4} - \frac{3 \csc(fx+e)}{8} \right) \cot(fx+e) + \frac{3 \ln(\csc(fx+e) - \cot(fx+e))}{8} \right) + a^2 c \left( -\frac{8}{15} - \frac{\csc^4(fx+e)}{5} - \frac{4(\csc^2(fx+e))}{15} \right)}{f}$
default	$\frac{a^2 c \left( \left( -\frac{\csc^3(fx+e)}{4} - \frac{3 \csc(fx+e)}{8} \right) \cot(fx+e) + \frac{3 \ln(\csc(fx+e) - \cot(fx+e))}{8} \right) + a^2 c \left( -\frac{8}{15} - \frac{\csc^4(fx+e)}{5} - \frac{4(\csc^2(fx+e))}{15} \right)}{f}$
norman	$\frac{-\frac{a^2 c}{160 f} - \frac{a^2 c (\tan^9(\frac{fx}{2} + \frac{e}{2}))}{8 f} - \frac{3 a^2 c (\tan^5(\frac{fx}{2} + \frac{e}{2}))}{32 f} - \frac{5 a^2 c (\tan^7(\frac{fx}{2} + \frac{e}{2}))}{32 f} - \frac{a^2 c \tan(\frac{fx}{2} + \frac{e}{2})}{64 f} - \frac{7 a^2 c (\tan^2(\frac{fx}{2} + \frac{e}{2}))}{240 f} - \frac{3 a^2 c (\tan^4(\frac{fx}{2} + \frac{e}{2}))}{240 f}}{240 f}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csc(f*x+e)^6*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x,method=_RETURNVERBOSE)`

[Out]  $\frac{1}{f} * (a^2 * c * ((-1/4 * \csc(f*x+e)^3 - 3/8 * \csc(f*x+e)) * \cot(f*x+e) + 3/8 * \ln(\csc(f*x+e) - \cot(f*x+e))) + a^2 * c * (-8/15 - 1/5 * \csc(f*x+e)^4 - 4/15 * \csc(f*x+e)^2) * \cot(f*x+e) - a^2 * c * (-2/3 - 1/3 * \csc(f*x+e)^2) * \cot(f*x+e) - a^2 * c * (-1/2 * \csc(f*x+e) * \cot(f*x+e) + 1/2 * \ln(\csc(f*x+e) - \cot(f*x+e))))$

**Maxima** [A]

time = 0.29, size = 202, normalized size = 1.92

$$\frac{15 a^2 c \left( \frac{2(3 \cos(fx+e)^3 - 5 \cos(fx+e))}{\cos(fx+e)^2 - 2 \cos(fx+e)^2 + 1} - 3 \log(\cos(fx+e) + 1) + 3 \log(\cos(fx+e) - 1) \right) - 60 a^2 c \left( \frac{2 \cos(fx+e)}{\cos(fx+e)^2 - 1} - \log(\cos(fx+e) + 1) + \log(\cos(fx+e) - 1) \right) + \frac{80(3 \tan(fx+e)^2 + 1) a^2 c}{\tan(fx+e)^3} - \frac{16(15 \tan(fx+e)^4 + 10 \tan(fx+e)^2 + 3) a^2 c}{\tan(fx+e)^5}}{240 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(f*x+e)^6*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="maxima")`

[Out]  $\frac{1}{240} * (15 * a^2 * c * (2 * (3 * \cos(f*x + e)^3 - 5 * \cos(f*x + e)) / (\cos(f*x + e)^4 - 2 * \cos(f*x + e)^2 + 1) - 3 * \log(\cos(f*x + e) + 1) + 3 * \log(\cos(f*x + e) - 1)) - 60 * a^2 * c * (2 * \cos(f*x + e) / (\cos(f*x + e)^2 - 1) - \log(\cos(f*x + e) + 1) + \log(\cos(f*x + e) - 1)) + 80 * (3 * \tan(f*x + e)^2 + 1) * a^2 * c / \tan(f*x + e)^3 - 16 * (15 * \tan(f*x + e)^4 + 10 * \tan(f*x + e)^2 + 3) * a^2 * c / \tan(f*x + e)^5) / f$

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 217 vs. 2(102) = 204.

time = 0.38, size = 217, normalized size = 2.07

$$\frac{32 a^2 c \cos(fx+e)^5 - 80 a^2 c \cos(fx+e)^3 + 15(a^2 c \cos(fx+e)^4 - 2 a^2 c \cos(fx+e)^2 + a^2 c) \log\left(\frac{1}{2} \cos(fx+e) + \frac{1}{2}\right) \sin(fx+e) - 15(a^2 c \cos(fx+e)^4 - 2 a^2 c \cos(fx+e)^2 + a^2 c) \log\left(-\frac{1}{2} \cos(fx+e) + \frac{1}{2}\right) \sin(fx+e) - 30(a^2 c \cos(fx+e)^3 + a^2 c \cos(fx+e)) \sin(fx+e)}{240(f \cos(fx+e)^4 - 2 f \cos(fx+e)^2 + f) \sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(f*x+e)^6*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="fricas")`

[Out]  $\frac{1}{240} * (32 * a^2 * c * \cos(f*x + e)^5 - 80 * a^2 * c * \cos(f*x + e)^3 + 15 * (a^2 * c * \cos(f*x + e)^4 - 2 * a^2 * c * \cos(f*x + e)^2 + a^2 * c) * \log(1/2 * \cos(f*x + e) + 1/2) * \sin(f*x + e) - 15 * (a^2 * c * \cos(f*x + e)^4 - 2 * a^2 * c * \cos(f*x + e)^2 + a^2 * c) * \log(-1/2 * \cos(f*x + e) + 1/2) * \sin(f*x + e) - 30 * (a^2 * c * \cos(f*x + e)^3 + a^2 * c * \cos(f*x + e)) * \sin(f*x + e)) / f$

```
f*x + e) - 15*(a^2*c*cos(f*x + e)^4 - 2*a^2*c*cos(f*x + e)^2 + a^2*c)*log(-
1/2*cos(f*x + e) + 1/2)*sin(f*x + e) - 30*(a^2*c*cos(f*x + e)^3 + a^2*c*cos
(f*x + e))*sin(f*x + e))/((f*cos(f*x + e)^4 - 2*f*cos(f*x + e)^2 + f)*sin(f
*x + e))
```

**Sympy [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6*(a+a*sin(f*x+e))**2*(c-c*sin(f*x+e)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3003 deep
```

**Giac [A]**

time = 0.51, size = 184, normalized size = 1.75

$$\frac{6a^2c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^5 + 15a^2c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 + 10a^2c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 - 120a^2c \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)\right|\right) - 60a^2c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + \frac{274a^2c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^5 + 60a^2c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 - 10a^2c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 - 15a^2c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - 6a^2c}{960f}}{\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] 1/960*(6*a^2*c*tan(1/2*f*x + 1/2*e)^5 + 15*a^2*c*tan(1/2*f*x + 1/2*e)^4 + 1
0*a^2*c*tan(1/2*f*x + 1/2*e)^3 - 120*a^2*c*log(abs(tan(1/2*f*x + 1/2*e))) -
60*a^2*c*tan(1/2*f*x + 1/2*e) + (274*a^2*c*tan(1/2*f*x + 1/2*e)^5 + 60*a^2
*c*tan(1/2*f*x + 1/2*e)^4 - 10*a^2*c*tan(1/2*f*x + 1/2*e)^3 - 15*a^2*c*tan
(1/2*f*x + 1/2*e) - 6*a^2*c)/tan(1/2*f*x + 1/2*e)^5)/f
```

**Mupad [B]**

time = 12.37, size = 244, normalized size = 2.32

$$\frac{a^2c \left( 6 \cos\left(\frac{e}{2} + \frac{f*x}{2}\right)^{10} - 6 \sin\left(\frac{e}{2} + \frac{f*x}{2}\right)^{10} - 15 \cos\left(\frac{e}{2} + \frac{f*x}{2}\right) \sin\left(\frac{e}{2} + \frac{f*x}{2}\right)^9 + 15 \cos\left(\frac{e}{2} + \frac{f*x}{2}\right)^9 \sin\left(\frac{e}{2} + \frac{f*x}{2}\right) - 10 \cos\left(\frac{e}{2} + \frac{f*x}{2}\right)^8 \sin\left(\frac{e}{2} + \frac{f*x}{2}\right)^2 + 60 \cos\left(\frac{e}{2} + \frac{f*x}{2}\right)^8 \sin\left(\frac{e}{2} + \frac{f*x}{2}\right) - 60 \cos\left(\frac{e}{2} + \frac{f*x}{2}\right)^6 \sin\left(\frac{e}{2} + \frac{f*x}{2}\right)^4 + 10 \cos\left(\frac{e}{2} + \frac{f*x}{2}\right)^8 \sin\left(\frac{e}{2} + \frac{f*x}{2}\right)^2 + 120 \cos\left(\frac{e}{2} + \frac{f*x}{2}\right)^5 \ln\left(\frac{\cos\left(\frac{e}{2} + \frac{f*x}{2}\right)}{\sin\left(\frac{e}{2} + \frac{f*x}{2}\right)}\right) \sin\left(\frac{e}{2} + \frac{f*x}{2}\right)^5 \right)}{960f \cos\left(\frac{e}{2} + \frac{f*x}{2}\right)^5 \sin\left(\frac{e}{2} + \frac{f*x}{2}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x)))/sin(e + f*x)^6,x)
```

```
[Out] -(a^2*c*(6*cos(e/2 + (f*x)/2)^10 - 6*sin(e/2 + (f*x)/2)^10 - 15*cos(e/2 + (
f*x)/2)*sin(e/2 + (f*x)/2)^9 + 15*cos(e/2 + (f*x)/2)^9*sin(e/2 + (f*x)/2) -
10*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2)^8 + 60*cos(e/2 + (f*x)/2)^4*sin
(e/2 + (f*x)/2)^6 - 60*cos(e/2 + (f*x)/2)^6*sin(e/2 + (f*x)/2)^4 + 10*cos(e
/2 + (f*x)/2)^8*sin(e/2 + (f*x)/2)^2 + 120*cos(e/2 + (f*x)/2)^5*log(sin(e/2
+ (f*x)/2)/cos(e/2 + (f*x)/2))*sin(e/2 + (f*x)/2)^5))/(960*f*cos(e/2 + (f*
x)/2)^5*sin(e/2 + (f*x)/2)^5)
```



### 3.11 $\int \csc^7(e + fx)(a + a \sin(e + fx))^2(c - c \sin(e + fx)) dx$

**Optimal.** Leaf size=130

$$\frac{a^2 c \tanh^{-1}(\cos(e + fx))}{16f} - \frac{a^2 c \cot^3(e + fx)}{3f} - \frac{a^2 c \cot^5(e + fx)}{5f} + \frac{a^2 c \cot(e + fx) \csc(e + fx)}{16f} + \frac{a^2 c \cot(e + fx) \csc^3(e + fx)}{24f}$$

[Out]  $1/16*a^2*c*\operatorname{arctanh}(\cos(f*x+e))/f-1/3*a^2*c*\cot(f*x+e)^3/f-1/5*a^2*c*\cot(f*x+e)^5/f+1/16*a^2*c*\cot(f*x+e)*\csc(f*x+e)/f+1/24*a^2*c*\cot(f*x+e)*\csc(f*x+e)^3/f-1/6*a^2*c*\cot(f*x+e)*\csc(f*x+e)^5/f$

**Rubi [A]**

time = 0.14, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 4, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {3045, 3852, 3853, 3855}

$$-\frac{a^2 c \cot^5(e + fx)}{5f} - \frac{a^2 c \cot^3(e + fx)}{3f} + \frac{a^2 c \tanh^{-1}(\cos(e + fx))}{16f} - \frac{a^2 c \cot(e + fx) \csc^5(e + fx)}{6f} + \frac{a^2 c \cot(e + fx) \csc^3(e + fx)}{24f} + \frac{a^2 c \cot(e + fx) \csc(e + fx)}{16f}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Csc}[e + f*x]^7*(a + a*\operatorname{Sin}[e + f*x])^2*(c - c*\operatorname{Sin}[e + f*x]), x]$

[Out]  $(a^2*c*\operatorname{ArcTanh}[\operatorname{Cos}[e + f*x]])/(16*f) - (a^2*c*\operatorname{Cot}[e + f*x]^3)/(3*f) - (a^2*c*\operatorname{Cot}[e + f*x]^5)/(5*f) + (a^2*c*\operatorname{Cot}[e + f*x]*\operatorname{Csc}[e + f*x])/(16*f) + (a^2*c*\operatorname{Cot}[e + f*x]*\operatorname{Csc}[e + f*x]^3)/(24*f) - (a^2*c*\operatorname{Cot}[e + f*x]*\operatorname{Csc}[e + f*x]^5)/(6*f)$

Rule 3045

$\operatorname{Int}[\operatorname{sin}[(e_.) + (f_.)*(x_.)]^{(n_.)}*((a_.) + (b_.)*\operatorname{sin}[(e_.) + (f_.)*(x_.)])^{(m_.)}*((A_.) + (B_.)*\operatorname{sin}[(e_.) + (f_.)*(x_.)])], x\_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandTrig}[\operatorname{sin}[e + f*x]^n*(a + b*\operatorname{sin}[e + f*x])^m*(A + B*\operatorname{sin}[e + f*x]), x], x] /;$  FreeQ[{a, b, e, f, A, B}, x] && EqQ[A\*b + a\*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && IntegerQ[n]

Rule 3852

$\operatorname{Int}[\operatorname{csc}[(c_.) + (d_.)*(x_.)]^{(n_.)}, x\_Symbol] \rightarrow \operatorname{Dist}[-d^{(-1)}, \operatorname{Subst}[\operatorname{Int}[\operatorname{ExpandIntegrand}[(1 + x^2)^{(n/2 - 1)}, x], x], x, \operatorname{Cot}[c + d*x]], x] /;$  FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 3853

$\operatorname{Int}[(\operatorname{csc}[(c_.) + (d_.)*(x_.)]*(b_.))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{Simp}[(-b)*\operatorname{Cos}[c + d*x]*((b*\operatorname{Csc}[c + d*x])^{(n - 1)}/(d*(n - 1))), x] + \operatorname{Dist}[b^2*((n - 2)/(n - 1)), \operatorname{Int}[(b*\operatorname{Csc}[c + d*x])^{(n - 2)}, x], x] /;$  FreeQ[{b, c, d}, x] && GtQ[n, 1] &

& IntegerQ[2\*n]

Rule 3855

Int[csc[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[-ArcTanh[Cos[c + d\*x]]/d, x]  
/; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \csc^7(e + fx)(a + a \sin(e + fx))^2(c - c \sin(e + fx)) dx &= \int (-a^2 c \csc^4(e + fx) - a^2 c \csc^5(e + fx) + a^2 c \csc^6(e + fx)) dx \\ &= -\left( (a^2 c) \int \csc^4(e + fx) dx \right) - (a^2 c) \int \csc^5(e + fx) dx - (a^2 c) \int \csc^6(e + fx) dx \\ &= \frac{a^2 c \cot(e + fx) \csc^3(e + fx)}{4f} - \frac{a^2 c \cot(e + fx) \csc^4(e + fx)}{6f} \\ &= -\frac{a^2 c \cot^3(e + fx)}{3f} - \frac{a^2 c \cot^5(e + fx)}{5f} + \frac{3a^2 c \cot^6(e + fx)}{6f} \\ &= \frac{3a^2 c \tanh^{-1}(\cos(e + fx))}{8f} - \frac{a^2 c \cot^3(e + fx)}{3f} \\ &= \frac{a^2 c \tanh^{-1}(\cos(e + fx))}{16f} - \frac{a^2 c \cot^3(e + fx)}{3f} - \frac{a^2 c \cot^6(e + fx)}{6f} \end{aligned}$$

Mathematica [A]

time = 0.05, size = 204, normalized size = 1.57

$$\frac{2a^2 c \cot(e + fx)}{15f} + \frac{a^2 c \csc^2\left(\frac{1}{2}(e + fx)\right)}{64f} - \frac{a^2 c \csc^6\left(\frac{1}{2}(e + fx)\right)}{384f} + \frac{a^2 c \cot(e + fx) \csc^2(e + fx)}{15f} - \frac{a^2 c \cot(e + fx) \csc^4(e + fx)}{5f} + \frac{a^2 c \log\left(\cos\left(\frac{1}{2}(e + fx)\right)\right)}{16f} - \frac{a^2 c \log\left(\sin\left(\frac{1}{2}(e + fx)\right)\right)}{16f} - \frac{a^2 c \sec^2\left(\frac{1}{2}(e + fx)\right)}{64f} + \frac{a^2 c \sec^6\left(\frac{1}{2}(e + fx)\right)}{384f}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[e + f\*x]^7\*(a + a\*Sin[e + f\*x])^2\*(c - c\*Sin[e + f\*x]),x]

[Out] (2\*a^2\*c\*Cot[e + f\*x])/(15\*f) + (a^2\*c\*Csc[(e + f\*x)/2]^2)/(64\*f) - (a^2\*c\*Csc[(e + f\*x)/2]^6)/(384\*f) + (a^2\*c\*Cot[e + f\*x]\*Csc[e + f\*x]^2)/(15\*f) - (a^2\*c\*Cot[e + f\*x]\*Csc[e + f\*x]^4)/(5\*f) + (a^2\*c\*Log[Cos[(e + f\*x)/2]])/(16\*f) - (a^2\*c\*Log[Sin[(e + f\*x)/2]])/(16\*f) - (a^2\*c\*Sec[(e + f\*x)/2]^2)/(64\*f) + (a^2\*c\*Sec[(e + f\*x)/2]^6)/(384\*f)

Maple [A]

time = 0.25, size = 174, normalized size = 1.34

method	result
--------	--------

risch	$\frac{-a^2c(15e^{11i(fx+e)} - 85e^{9i(fx+e)} - 570e^{7i(fx+e)} + 480ie^{8i(fx+e)} - 570e^{5i(fx+e)} - 320ie^{6i(fx+e)} - 85e^{3i(fx+e)} + 15e^{i(fx+e)})}{120f(e^{2i(fx+e)} - 1)^6}$
derivativedivides	$a^2c\left(-\frac{8}{15} - \frac{\csc^4(fx+e)}{5} - \frac{4(\csc^2(fx+e))}{15}\right)\cot(fx+e) + a^2c\left(\left(-\frac{\csc^5(fx+e)}{6} - \frac{5(\csc^3(fx+e))}{24} - \frac{5\csc(fx+e)}{16}\right)\cot(fx+e)\right)$
default	$a^2c\left(-\frac{8}{15} - \frac{\csc^4(fx+e)}{5} - \frac{4(\csc^2(fx+e))}{15}\right)\cot(fx+e) + a^2c\left(\left(-\frac{\csc^5(fx+e)}{6} - \frac{5(\csc^3(fx+e))}{24} - \frac{5\csc(fx+e)}{16}\right)\cot(fx+e)\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csc(f*x+e)^7*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x,method=_RETURNVERBOSE)`

[Out]  $\frac{1}{f} * (a^2 * c * (-8/15 - 1/5 * \csc(f*x+e)^4 - 4/15 * \csc(f*x+e)^2) * \cot(f*x+e) + a^2 * c * ((-1/6 * \csc(f*x+e)^5 - 5/24 * \csc(f*x+e)^3 - 5/16 * \csc(f*x+e)) * \cot(f*x+e) + 5/16 * \ln(\csc(f*x+e) - \cot(f*x+e))) - a^2 * c * ((-1/4 * \csc(f*x+e)^3 - 3/8 * \csc(f*x+e)) * \cot(f*x+e) + 3/8 * \ln(\csc(f*x+e) - \cot(f*x+e))) - a^2 * c * (-2/3 - 1/3 * \csc(f*x+e)^2) * \cot(f*x+e))$

**Maxima** [A]

time = 0.30, size = 251, normalized size = 1.93

$$5a^2c\left(\frac{2(15\cos(fx+e)^5 - 40\cos(fx+e)^3 + 33\cos(fx+e))}{\cos(fx+e)^5 - 3\cos(fx+e)^3 + 3\cos(fx+e) - 1} - 15\log(\cos(fx+e) + 1) + 15\log(\cos(fx+e) - 1)\right) - 30a^2c\left(\frac{2(3\cos(fx+e)^3 - 5\cos(fx+e))}{\cos(fx+e)^3 - 2\cos(fx+e) + 1} - 3\log(\cos(fx+e) + 1) + 3\log(\cos(fx+e) - 1)\right) + \frac{160(3\tan(fx+e)^2 + 1)a^2c}{\tan(fx+e)^3} - \frac{32(15\tan(fx+e)^4 + 10\tan(fx+e)^2 + 3)a^2c}{\tan(fx+e)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(f*x+e)^7*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="maxima")`

[Out]  $\frac{1}{480} * (5 * a^2 * c * (2 * (15 * \cos(f*x + e)^5 - 40 * \cos(f*x + e)^3 + 33 * \cos(f*x + e)) / (\cos(f*x + e)^6 - 3 * \cos(f*x + e)^4 + 3 * \cos(f*x + e)^2 - 1) - 15 * \log(\cos(f*x + e) + 1) + 15 * \log(\cos(f*x + e) - 1)) - 30 * a^2 * c * (2 * (3 * \cos(f*x + e)^3 - 5 * \cos(f*x + e)) / (\cos(f*x + e)^4 - 2 * \cos(f*x + e)^2 + 1) - 3 * \log(\cos(f*x + e) + 1) + 3 * \log(\cos(f*x + e) - 1)) + 160 * (3 * \tan(f*x + e)^2 + 1) * a^2 * c / \tan(f*x + e)^3 - 32 * (15 * \tan(f*x + e)^4 + 10 * \tan(f*x + e)^2 + 3) * a^2 * c / \tan(f*x + e)^5) / f$

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 257 vs. 2(127) = 254.

time = 0.38, size = 257, normalized size = 1.98

$$\frac{30a^2c\cos(fx+e)^5 - 80a^2c\cos(fx+e)^3 - 30a^2c\cos(fx+e) - 15(a^2c\cos(fx+e)^5 - 3a^2c\cos(fx+e)^3 + 3a^2c\cos(fx+e) - a^2c)\log\left(\frac{1}{2}\cos(fx+e) + \frac{1}{2}\right) + 15(a^2c\cos(fx+e)^5 - 3a^2c\cos(fx+e)^3 + 3a^2c\cos(fx+e) - a^2c)\log\left(-\frac{1}{2}\cos(fx+e) + \frac{1}{2}\right) + 32(2a^2c\cos(fx+e)^5 - 5a^2c\cos(fx+e)^3)\sin(fx+e)}{480(f\cos(fx+e)^6 - 3f\cos(fx+e)^4 + 3f\cos(fx+e)^2 - f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(f*x+e)^7*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="fricas")`

```
[Out] -1/480*(30*a^2*c*cos(f*x + e)^5 - 80*a^2*c*cos(f*x + e)^3 - 30*a^2*c*cos(f*x + e) - 15*(a^2*c*cos(f*x + e)^6 - 3*a^2*c*cos(f*x + e)^4 + 3*a^2*c*cos(f*x + e)^2 - a^2*c)*log(1/2*cos(f*x + e) + 1/2) + 15*(a^2*c*cos(f*x + e)^6 - 3*a^2*c*cos(f*x + e)^4 + 3*a^2*c*cos(f*x + e)^2 - a^2*c)*log(-1/2*cos(f*x + e) + 1/2) + 32*(2*a^2*c*cos(f*x + e)^5 - 5*a^2*c*cos(f*x + e)^3)*sin(f*x + e))/(f*cos(f*x + e)^6 - 3*f*cos(f*x + e)^4 + 3*f*cos(f*x + e)^2 - f)
```

**Sympy** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**7*(a+a*sin(f*x+e))**2*(c-c*sin(f*x+e)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4368 deep
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 256 vs. 2(127) = 254.

time = 0.49, size = 256, normalized size = 1.97

$$\frac{5a^6c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 12a^6c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + 15a^6c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^5 + 20a^6c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^7 - 15a^6c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^9 - 120a^6c \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)\right|\right) - 120a^6c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + \frac{294a^6c \cos\left(\frac{1}{2}fx + \frac{1}{2}e\right)^6 + 120a^6c \cos\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 + 15a^6c \cos\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - 15a^6c \cos\left(\frac{1}{2}fx + \frac{1}{2}e\right)^0}{\sin\left(\frac{1}{2}fx + \frac{1}{2}e\right)^7}}{1920f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^7*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/1920*(5*a^2*c*tan(1/2*f*x + 1/2*e)^6 + 12*a^2*c*tan(1/2*f*x + 1/2*e)^5 + 15*a^2*c*tan(1/2*f*x + 1/2*e)^4 + 20*a^2*c*tan(1/2*f*x + 1/2*e)^3 - 15*a^2*c*tan(1/2*f*x + 1/2*e)^2 - 120*a^2*c*log(abs(tan(1/2*f*x + 1/2*e))) - 120*a^2*c*tan(1/2*f*x + 1/2*e) + (294*a^2*c*tan(1/2*f*x + 1/2*e)^6 + 120*a^2*c*tan(1/2*f*x + 1/2*e)^5 + 15*a^2*c*tan(1/2*f*x + 1/2*e)^4 - 20*a^2*c*tan(1/2*f*x + 1/2*e)^3 - 15*a^2*c*tan(1/2*f*x + 1/2*e)^2 - 12*a^2*c*tan(1/2*f*x + 1/2*e) - 5*a^2*c)/tan(1/2*f*x + 1/2*e)^6)/f
```

**Mupad** [B]

time = 12.54, size = 340, normalized size = 2.62

$$\frac{a^6c \left( 5 \cos\left(\frac{1}{2}fx + \frac{1}{2}e\right)^6 - 5 \sin\left(\frac{1}{2}fx + \frac{1}{2}e\right)^6 - 12 \cos\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 + 12 \sin\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 - 15 \cos\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 15 \sin\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - 20 \cos\left(\frac{1}{2}fx + \frac{1}{2}e\right)^0 + 20 \sin\left(\frac{1}{2}fx + \frac{1}{2}e\right)^0 + 15 \cos\left(\frac{1}{2}fx + \frac{1}{2}e\right)^8 + 120 \cos\left(\frac{1}{2}fx + \frac{1}{2}e\right)^6 + 120 \sin\left(\frac{1}{2}fx + \frac{1}{2}e\right)^6 - 120 \cos\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 + 15 \cos\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 20 \sin\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 15 \cos\left(\frac{1}{2}fx + \frac{1}{2}e\right)^0 + 120 \cos\left(\frac{1}{2}fx + \frac{1}{2}e\right)^8 + 120 \sin\left(\frac{1}{2}fx + \frac{1}{2}e\right)^8 \ln\left(\frac{\cos\left(\frac{1}{2}fx + \frac{1}{2}e\right)}{\sin\left(\frac{1}{2}fx + \frac{1}{2}e\right)}\right) \right)}{1920f \cos\left(\frac{1}{2}fx + \frac{1}{2}e\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x)))/sin(e + f*x)^7,x)
```

```
[Out] -(a^2*c*(5*cos(e/2 + (f*x)/2)^12 - 5*sin(e/2 + (f*x)/2)^12 - 12*cos(e/2 + (f*x)/2)*sin(e/2 + (f*x)/2)^11 + 12*cos(e/2 + (f*x)/2)^11*sin(e/2 + (f*x)/2) - 15*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2)^10 - 20*cos(e/2 + (f*x)/2)^3*
```

$$\begin{aligned} & \sin(e/2 + (f*x)/2)^9 + 15*\cos(e/2 + (f*x)/2)^4*\sin(e/2 + (f*x)/2)^8 + 120*c \\ & \cos(e/2 + (f*x)/2)^5*\sin(e/2 + (f*x)/2)^7 - 120*\cos(e/2 + (f*x)/2)^7*\sin(e/2 \\ & + (f*x)/2)^5 - 15*\cos(e/2 + (f*x)/2)^8*\sin(e/2 + (f*x)/2)^4 + 20*\cos(e/2 + \\ & (f*x)/2)^9*\sin(e/2 + (f*x)/2)^3 + 15*\cos(e/2 + (f*x)/2)^{10}*\sin(e/2 + (f*x) \\ & /2)^2 + 120*\cos(e/2 + (f*x)/2)^6*\log(\sin(e/2 + (f*x)/2)/\cos(e/2 + (f*x)/2)) \\ & * \sin(e/2 + (f*x)/2)^6)/(1920*f*\cos(e/2 + (f*x)/2)^6*\sin(e/2 + (f*x)/2)^6) \end{aligned}$$

### 3.12 $\int \sin^2(c+dx)(a+a\sin(c+dx))^{3/2}(c-c\sin(c+dx)) dx$

**Optimal.** Leaf size=128

$$\frac{8a^3c\cos^3(c+dx)}{63d(a+a\sin(c+dx))^{3/2}} - \frac{2a^2c\cos^3(c+dx)}{21d\sqrt{a+a\sin(c+dx)}} + \frac{4ac\cos^3(c+dx)\sqrt{a+a\sin(c+dx)}}{21d} - \frac{2c\cos^3(c+dx)}{21d}$$

[Out]  $-8/63*a^3*c*\cos(d*x+c)^3/d/(a+a*\sin(d*x+c))^(3/2)-2/9*c*\cos(d*x+c)^3*(a+a*\sin(d*x+c))^(3/2)/d-2/21*a^2*c*\cos(d*x+c)^3/d/(a+a*\sin(d*x+c))^(1/2)+4/21*a*c*\cos(d*x+c)^3*(a+a*\sin(d*x+c))^(1/2)/d$

**Rubi [A]**

time = 0.23, antiderivative size = 165, normalized size of antiderivative = 1.29, number of steps used = 5, number of rules used = 5, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$ , Rules used = {3055, 3060, 2838, 2830, 2725}

$$\frac{2a^2c\sin^3(c+dx)\cos(c+dx)}{63d\sqrt{a\sin(c+dx)+a}} - \frac{2a^2c\cos(c+dx)}{9d\sqrt{a\sin(c+dx)+a}} + \frac{2ac\sin^3(c+dx)\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{9d} - \frac{2c\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{21d} + \frac{4ac\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{63d}$$

Antiderivative was successfully verified.

[In] Int[Sin[c + d\*x]^2\*(a + a\*Sin[c + d\*x])^(3/2)\*(c - c\*Sin[c + d\*x]),x]

[Out]  $(-2*a^2*c*\cos[c + d*x])/(9*d*\sqrt{a + a*\sin[c + d*x]}) + (2*a^2*c*\cos[c + d*x]*\sin[c + d*x]^3)/(63*d*\sqrt{a + a*\sin[c + d*x]}) + (4*a*c*\cos[c + d*x]*\sqrt{a + a*\sin[c + d*x]})/(63*d) + (2*a*c*\cos[c + d*x]*\sin[c + d*x]^3*\sqrt{a + a*\sin[c + d*x]})/(9*d) - (2*c*\cos[c + d*x]*(a + a*\sin[c + d*x])^(3/2))/(21*d)$

Rule 2725

Int[Sqrt[(a\_) + (b\_)\*sin[(c\_) + (d\_)\*(x\_)]], x\_Symbol] := Simp[-2\*b\*(Cos[c + d\*x]/(d\*Sqrt[a + b\*Sin[c + d\*x]])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2830

Int[((a\_) + (b\_)\*sin[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*sin[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Simp[(-d)\*Cos[e + f\*x]\*((a + b\*Sin[e + f\*x])^m/(f\*(m + 1))), x] + Dist[(a\*d\*m + b\*c\*(m + 1))/(b\*(m + 1)), Int[(a + b\*Sin[e + f\*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]

Rule 2838

Int[sin[(e\_) + (f\_)\*(x\_)]^2\*((a\_) + (b\_)\*sin[(e\_) + (f\_)\*(x\_)])^(m\_), x\_Symbol] := Simp[(-Cos[e + f\*x])\*((a + b\*Sin[e + f\*x])^(m + 1))/(b\*f\*(m + 2)

```

)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && !L
tQ[m, -2^(-1)]

```

### Rule 3055

```

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
+ 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
+ b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1]
&& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

```

### Rule 3060

```

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])*((A_) + (B_)*sin[(e_) + (
f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]))], x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && !LtQ[n, -1]

```

### Rubi steps

$$\begin{aligned}
\int \sin^2(c + dx)(a + a \sin(c + dx))^{3/2}(c - c \sin(c + dx)) dx &= \frac{2ac \cos(c + dx) \sin^3(c + dx) \sqrt{a + a \sin(c + dx)}}{9d} \\
&= \frac{2a^2c \cos(c + dx) \sin^3(c + dx)}{63d \sqrt{a + a \sin(c + dx)}} + \frac{2ac \cos(c + dx)}{63d \sqrt{a + a \sin(c + dx)}} \\
&= \frac{2a^2c \cos(c + dx) \sin^3(c + dx)}{63d \sqrt{a + a \sin(c + dx)}} + \frac{2ac \cos(c + dx)}{63d \sqrt{a + a \sin(c + dx)}} \\
&= \frac{2a^2c \cos(c + dx) \sin^3(c + dx)}{63d \sqrt{a + a \sin(c + dx)}} + \frac{4ac \cos(c + dx)}{63d \sqrt{a + a \sin(c + dx)}} \\
&= -\frac{2a^2c \cos(c + dx)}{9d \sqrt{a + a \sin(c + dx)}} + \frac{2a^2c \cos(c + dx) \sin^3(c + dx)}{63d \sqrt{a + a \sin(c + dx)}}
\end{aligned}$$

**Mathematica [A]**

time = 0.53, size = 101, normalized size = 0.79

$$\frac{ac(\cos(\frac{1}{2}(c+dx)) - \sin(\frac{1}{2}(c+dx)))^3 \sqrt{a(1+\sin(c+dx))} (-62 + 30\cos(2(c+dx)) - 69\sin(c+dx) + 7\sin(3(c+dx)))}{126d(\cos(\frac{1}{2}(c+dx)) + \sin(\frac{1}{2}(c+dx)))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2)*(c - c*Sin[c + d*x]),x]
[Out] (a*c*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*Sqrt[a*(1 + Sin[c + d*x])]*(-62 + 30*Cos[2*(c + d*x)] - 69*Sin[c + d*x] + 7*Sin[3*(c + d*x)])/(126*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

**Maple [A]**

time = 5.65, size = 78, normalized size = 0.61

method	result	size
default	$-\frac{2(1+\sin(dx+c))a^2(\sin(dx+c)-1)^2c(7(\sin^3(dx+c))+15(\sin^2(dx+c))+12\sin(dx+c)+8)}{63\cos(dx+c)\sqrt{a+a\sin(dx+c)}d}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2*(a+a*sin(d*x+c))^(3/2)*(c-c*sin(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/63*(1+sin(d*x+c))*a^2*(sin(d*x+c)-1)^2*c*(7*sin(d*x+c)^3+15*sin(d*x+c)^2+12*sin(d*x+c)+8)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^(3/2)*(c-c*sin(d*x+c)),x, algorithm
="maxima")
```

```
[Out] -integrate((a*sin(d*x + c) + a)^(3/2)*(c*sin(d*x + c) - c)*sin(d*x + c)^2,
x)
```

**Fricas [A]**

time = 0.37, size = 155, normalized size = 1.21

$$\frac{2(7a\cos(dx+c)^5 - a\cos(dx+c)^4 - 11a\cos(dx+c)^3 + a\cos(dx+c)^2 - 4a\cos(dx+c) - 8ac - (7a\cos(dx+c)^4 + 8a\cos(dx+c)^3 - 3a\cos(dx+c)^2 - 4a\cos(dx+c) - 8ac)\sin(dx+c)\sqrt{a\sin(dx+c)+a}}{63(d\cos(dx+c) + d\sin(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^(3/2)*(c-c*sin(d*x+c)),x, algorithm
="fricas")
```



[Out]  $2/63*(7*a*c*\cos(d*x + c)^5 - a*c*\cos(d*x + c)^4 - 11*a*c*\cos(d*x + c)^3 + a*c*\cos(d*x + c)^2 - 4*a*c*\cos(d*x + c) - 8*a*c - (7*a*c*\cos(d*x + c)^4 + 8*a*c*\cos(d*x + c)^3 - 3*a*c*\cos(d*x + c)^2 - 4*a*c*\cos(d*x + c) - 8*a*c)*\sin(d*x + c)*\sqrt{a*\sin(d*x + c) + a}/(d*\cos(d*x + c) + d*\sin(d*x + c) + d)$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$-c \left( \int \left( -a \sqrt{a \sin(c + dx) + a} \sin^2(c + dx) \right) dx + \int a \sqrt{a \sin(c + dx) + a} \sin^4(c + dx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(d*x+c)**2*(a+a*sin(d*x+c))**(3/2)*(c-c*sin(d*x+c)),x)`

[Out] `-c*(Integral(-a*sqrt(a*sin(c + d*x) + a)*sin(c + d*x)**2, x) + Integral(a*sqrt(a*sin(c + d*x) + a)*sin(c + d*x)**4, x))`

**Giac** [A]

time = 0.48, size = 99, normalized size = 0.77

$$\frac{\sqrt{2} (126 \operatorname{acsgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c)) \sin(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c) - 9 \operatorname{acsgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c)) \sin(-\frac{7}{4}\pi + \frac{7}{2}dx + \frac{7}{2}c) - 7 \operatorname{acsgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c)) \sin(-\frac{9}{4}\pi + \frac{9}{2}dx + \frac{9}{2}c)) \sqrt{a}}{504d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^(3/2)*(c-c*sin(d*x+c)),x, algorithm="giac")`

[Out]  $1/504*\sqrt{2}*(126*a*c*\operatorname{sgn}(\cos(-1/4*\pi + 1/2*d*x + 1/2*c))*\sin(-1/4*\pi + 1/2*d*x + 1/2*c) - 9*a*c*\operatorname{sgn}(\cos(-1/4*\pi + 1/2*d*x + 1/2*c))*\sin(-7/4*\pi + 7/2*d*x + 7/2*c) - 7*a*c*\operatorname{sgn}(\cos(-1/4*\pi + 1/2*d*x + 1/2*c))*\sin(-9/4*\pi + 9/2*d*x + 9/2*c))*\sqrt{a}/d$

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \sin(c + dx)^2 (a + a \sin(c + dx))^{3/2} (c - c \sin(c + dx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(c + d*x)^2*(a + a*sin(c + d*x))^(3/2)*(c - c*sin(c + d*x)),x)`

[Out] `int(sin(c + d*x)^2*(a + a*sin(c + d*x))^(3/2)*(c - c*sin(c + d*x)), x)`

$$3.13 \quad \int \frac{\csc(e+fx) \sqrt{a + a \sin(e + fx)}}{c - c \sin(e+fx)} dx$$

Optimal. Leaf size=69

$$-\frac{2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{a + a \sin(e + fx)}}\right)}{cf} + \frac{2 \sec(e + fx) \sqrt{a + a \sin(e + fx)}}{cf}$$

[Out]  $-2*\operatorname{arctanh}(\cos(f*x+e)*a^{(1/2)}/(a+a*\sin(f*x+e))^{(1/2)})*a^{(1/2)}/c/f+2*\sec(f*x+e)*(a+a*\sin(f*x+e))^{(1/2)}/c/f$

Rubi [A]

time = 0.20, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$ , Rules used = {3013, 2852, 212, 2815, 2752}

$$\frac{2 \sec(e + fx) \sqrt{a \sin(e + fx) + a}}{cf} - \frac{2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{a \sin(e + fx) + a}}\right)}{cf}$$

Antiderivative was successfully verified.

[In] `Int[(Csc[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c - c*Sin[e + f*x]),x]`

[Out] `(-2*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]])/(c*f) + (2*Sec[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c*f)`

Rule 212

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

Rule 2752

`Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] := Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]`

Rule 2815

`Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && (LtQ`

[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))

### Rule 2852

Int[Sqrt[(a\_) + (b\_)\*sin[(e\_) + (f\_)\*(x\_)]]/((c\_) + (d\_)\*sin[(e\_) + (f\_)\*(x\_)]), x\_Symbol] :> Dist[-2\*(b/f), Subst[Int[1/(b\*c + a\*d - d\*x^2), x], x, b\*(Cos[e + f\*x]/Sqrt[a + b\*Sin[e + f\*x])]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

### Rule 3013

Int[Sqrt[(a\_) + (b\_)\*sin[(e\_) + (f\_)\*(x\_)]]/(sin[(e\_) + (f\_)\*(x\_)]\*((c\_) + (d\_)\*sin[(e\_) + (f\_)\*(x\_)])), x\_Symbol] :> Dist[1/c, Int[Sqrt[a + b\*Sin[e + f\*x]]/Sin[e + f\*x], x], x] - Dist[d/c, Int[Sqrt[a + b\*Sin[e + f\*x]]/(c + d\*Sin[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 - b^2, 0]

### Rubi steps

$$\begin{aligned} \int \frac{\csc(e + fx) \sqrt{a + a \sin(e + fx)}}{c - c \sin(e + fx)} dx &= \frac{\int \csc(e + fx) \sqrt{a + a \sin(e + fx)} dx}{c} + \int \frac{\sqrt{a + a \sin(e + fx)}}{c - c \sin(e + fx)} dx \\ &= \frac{\int \sec^2(e + fx) (a + a \sin(e + fx))^{3/2} dx}{ac} - \frac{(2a) \text{Subst}\left(\int \frac{1}{a-x^2} dx, \frac{\sqrt{a + a \sin(e + fx)}}{c}\right)}{cf} \\ &= -\frac{2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a} \cos(e + fx)}{\sqrt{a + a \sin(e + fx)}}\right)}{cf} + \frac{2 \sec(e + fx) \sqrt{a + a \sin(e + fx)}}{cf} \end{aligned}$$

**Mathematica [B]** Leaf count is larger than twice the leaf count of optimal. 157 vs. 2(69) = 138.

time = 0.23, size = 157, normalized size = 2.28

$\frac{\sec(e + fx) (2 + \cos(\frac{1}{2}(e + fx)) (-\log(1 + \cos(\frac{1}{2}(e + fx)) - \sin(\frac{1}{2}(e + fx))) + \log(1 - \cos(\frac{1}{2}(e + fx)) + \sin(\frac{1}{2}(e + fx)))) + (\log(1 + \cos(\frac{1}{2}(e + fx)) - \sin(\frac{1}{2}(e + fx))) - \log(1 - \cos(\frac{1}{2}(e + fx)) + \sin(\frac{1}{2}(e + fx)))) \sin(\frac{1}{2}(e + fx))}{cf} \sqrt{a(1 + \sin(e + fx))}$

Antiderivative was successfully verified.

[In] Integrate[(Csc[e + f\*x]\*Sqrt[a + a\*Sin[e + f\*x]])/(c - c\*Sin[e + f\*x]),x]  
 [Out] (Sec[e + f\*x]\*(2 + Cos[(e + f\*x)/2]\*(-Log[1 + Cos[(e + f\*x)/2] - Sin[(e + f\*x)/2]] + Log[1 - Cos[(e + f\*x)/2] + Sin[(e + f\*x)/2]])) + (Log[1 + Cos[(e + f\*x)/2] - Sin[(e + f\*x)/2]] - Log[1 - Cos[(e + f\*x)/2] + Sin[(e + f\*x)/2]])\*Sin[(e + f\*x)/2]\*Sqrt[a\*(1 + Sin[e + f\*x])]/(c\*f)

**Maple [A]**

time = 6.50, size = 78, normalized size = 1.13

method	result	size
default	$\frac{2(1+\sin(fx+e)) \left( a^{\frac{3}{2}} - \operatorname{arctanh} \left( \frac{\sqrt{a - a \sin(fx+e)}}{\sqrt{a}} \right) a \sqrt{a - a \sin(fx+e)} \right)}{\sqrt{a} c \cos(fx+e) \sqrt{a + a \sin(fx+e)} f}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)/sin(f*x+e)/(c-c*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(1+sin(f*x+e))*(a^(3/2)-arctanh((a-a*sin(f*x+e))^(1/2)/a^(1/2))*a*(a-a*sin(f*x+e))^(1/2))/a^(1/2)/c/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/sin(f*x+e)/(c-c*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(a*sin(f*x + e) + a)/((c*sin(f*x + e) - c)*sin(f*x + e)), x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 221 vs. 2(65) = 130.

time = 0.47, size = 221, normalized size = 3.20

$$\frac{\sqrt{a} \cos(fx+e) \log \left( \frac{a \cos(fx+e)^3 - 7a \cos(fx+e)^2 - 4(\cos(fx+e)^2 + (\cos(fx+e)+3)\sin(fx+e) - 2\cos(fx+e) - 3) \sqrt{a \sin(fx+e) + a} \sqrt{a} - 9a \cos(fx+e) + (a \cos(fx+e)^2 + 8a \cos(fx+e) - a) \sin(fx+e) - a}{\cos(fx+e)^3 + \cos(fx+e)^2 + (\cos(fx+e)^2 - 1) \sin(fx+e) - \cos(fx+e) - 1} \right) + 4 \sqrt{a \sin(fx+e) + a}}{2cf \cos(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/sin(f*x+e)/(c-c*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(a)*cos(f*x + e)*log((a*cos(f*x + e))^3 - 7*a*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 + (cos(f*x + e) + 3)*sin(f*x + e) - 2*cos(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a)*sqrt(a) - 9*a*cos(f*x + e) + (a*cos(f*x + e)^2 + 8*a*cos(f*x + e) - a)*sin(f*x + e) - a)/(cos(f*x + e)^3 + cos(f*x + e)^2 + (cos(f*x + e)^2 - 1)*sin(f*x + e) - cos(f*x + e) - 1)) + 4*sqrt(a*sin(f*x + e) + a))/(c*f*cos(f*x + e))
```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a \sin(e + fx) + a}}{\sin^2(e + fx) - \sin(e + fx)} dx$$


---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*sin(f\*x+e))\*\*(1/2)/sin(f\*x+e)/(c-c\*sin(f\*x+e)),x)

[Out] -Integral(sqrt(a\*sin(e + f\*x) + a)/(sin(e + f\*x)\*\*2 - sin(e + f\*x)), x)/c

**Giac [A]**

time = 0.56, size = 115, normalized size = 1.67

$$\frac{\sqrt{2} \left( \frac{\sqrt{2} \log \left( \frac{|-2\sqrt{2} + 4 \sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)|}{2\sqrt{2} + 4 \sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)} \right) \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))}{c} + \frac{2 \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))}{c \sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)} \right) \sqrt{a}}{2f}$$


---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*sin(f\*x+e))^(1/2)/sin(f\*x+e)/(c-c\*sin(f\*x+e)),x, algorithm="giac")

[Out] -1/2\*sqrt(2)\*(sqrt(2)\*log(abs(-2\*sqrt(2) + 4\*sin(-1/4\*pi + 1/2\*f\*x + 1/2\*e))/abs(2\*sqrt(2) + 4\*sin(-1/4\*pi + 1/2\*f\*x + 1/2\*e))))\*sgn(cos(-1/4\*pi + 1/2\*f\*x + 1/2\*e))/c + 2\*sgn(cos(-1/4\*pi + 1/2\*f\*x + 1/2\*e))/(c\*sin(-1/4\*pi + 1/2\*f\*x + 1/2\*e))\*sqrt(a)/f

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a + a \sin(e + fx)}}{\sin(e + fx) (c - c \sin(e + fx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a\*sin(e + f\*x))^(1/2)/(sin(e + f\*x)\*(c - c\*sin(e + f\*x))),x)

[Out] int((a + a\*sin(e + f\*x))^(1/2)/(sin(e + f\*x)\*(c - c\*sin(e + f\*x))), x)

$$3.14 \quad \int \frac{\csc(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))} dx$$

Optimal. Leaf size=120

$$-\frac{2 \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a} cf} + \frac{\tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{2} \sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{2} \sqrt{a} cf} + \frac{\sec(e+fx) \sqrt{a+a\sin(e+fx)}}{acf}$$

[Out]  $-2*\operatorname{arctanh}(\cos(f*x+e)*a^{(1/2)}/(a+a*\sin(f*x+e))^{(1/2)})/c/f/a^{(1/2)}+1/2*\operatorname{arctanh}(1/2*\cos(f*x+e)*a^{(1/2)*2^{(1/2)}}/(a+a*\sin(f*x+e))^{(1/2)})/c/f*2^{(1/2)}/a^{(1/2)}+\sec(f*x+e)*(a+a*\sin(f*x+e))^{(1/2)}/a/c/f$

Rubi [A]

time = 0.30, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.206$ , Rules used = {3019, 2815, 2752, 3064, 2728, 212, 2852}

$$\frac{\sec(e+fx) \sqrt{a\sin(e+fx)+a}}{acf} - \frac{2 \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a} cf} + \frac{\tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{2} \sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{2} \sqrt{a} cf}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Csc}[e+f*x]/(\operatorname{Sqrt}[a+a*\operatorname{Sin}[e+f*x]]*(c-c*\operatorname{Sin}[e+f*x])),x]$

[Out]  $(-2*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Cos}[e+f*x])/(\operatorname{Sqrt}[a+a*\operatorname{Sin}[e+f*x]])]/(\operatorname{Sqrt}[a]*c*f) + \operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Cos}[e+f*x])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a+a*\operatorname{Sin}[e+f*x]])]/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a]*c*f) + (\operatorname{Sec}[e+f*x]*\operatorname{Sqrt}[a+a*\operatorname{Sin}[e+f*x]])/(a*c*f)$

Rule 212

$\operatorname{Int}[(a_+ + (b_+)*(x_+)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]))*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{Gt} Q[a, 0] \ || \operatorname{Lt} Q[b, 0])$

Rule 2728

$\operatorname{Int}[1/\operatorname{Sqrt}[(a_+ + (b_+)*\sin[(c_+ + (d_+)*(x_+)])], x\_Symbol] \rightarrow \operatorname{Dist}[-2/d, \operatorname{Subst}[\operatorname{Int}[1/(2*a - x^2), x], x, b*(\operatorname{Cos}[c + d*x]/\operatorname{Sqrt}[a + b*\operatorname{Sin}[c + d*x])]], x] /; \operatorname{FreeQ}\{a, b, c, d\}, x \ \&\& \operatorname{Eq} Q[a^2 - b^2, 0]$

Rule 2752

$\operatorname{Int}[(\cos[(e_+ + (f_+)*(x_+)]*(g_+))^{(p_+)})*((a_+ + (b_+)*\sin[(e_+ + (f_+)*(x_+)]))^{(m_+)}, x\_Symbol] \rightarrow \operatorname{Simp}[b*(g*\operatorname{Cos}[e + f*x])^{(p+1)}*((a + b*\operatorname{Sin}[e + f*x])^{(m-1)})/(f*g*(m-1)), x] /; \operatorname{FreeQ}\{a, b, e, f, g, m, p\}, x \ \&\& \operatorname{Eq} Q[a^2$

- b^2, 0] && EqQ[2\*m + p - 1, 0] && NeQ[m, 1]

### Rule 2815

Int[((a\_) + (b\_)\*sin[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*sin[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[a^m\*c^m, Int[Cos[e + f\*x]^(2\*m)\*(c + d\*Sin[e + f\*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))

### Rule 2852

Int[Sqrt[(a\_) + (b\_)\*sin[(e\_) + (f\_)\*(x\_)]]/((c\_) + (d\_)\*sin[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Dist[-2\*(b/f), Subst[Int[1/(b\*c + a\*d - d\*x^2), x], x, b\*(Cos[e + f\*x]/Sqrt[a + b\*Sin[e + f\*x])]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

### Rule 3019

Int[1/(sin[(e\_) + (f\_)\*(x\_)]\*Sqrt[(a\_) + (b\_)\*sin[(e\_) + (f\_)\*(x\_)])\*(c\_) + (d\_)\*sin[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Dist[d^2/(c\*(b\*c - a\*d)), Int[Sqrt[a + b\*Sin[e + f\*x]]/(c + d\*Sin[e + f\*x]), x], x] + Dist[1/(c\*(b\*c - a\*d)), Int[(b\*c - a\*d - b\*d\*Sin[e + f\*x])/(Sin[e + f\*x]\*Sqrt[a + b\*Sin[e + f\*x])], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 - b^2, 0]

### Rule 3064

Int[((A\_) + (B\_)\*sin[(e\_) + (f\_)\*(x\_)])/(Sqrt[(a\_) + (b\_)\*sin[(e\_) + (f\_)\*(x\_)])\*(c\_) + (d\_)\*sin[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Dist[(A\*b - a\*B)/(b\*c - a\*d), Int[1/Sqrt[a + b\*Sin[e + f\*x]], x], x] + Dist[(B\*c - A\*d)/(b\*c - a\*d), Int[Sqrt[a + b\*Sin[e + f\*x]]/(c + d\*Sin[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

### Rubi steps

$$\begin{aligned}
\int \frac{\csc(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))} dx &= \frac{\int \frac{\sqrt{a+a\sin(e+fx)}}{c-c\sin(e+fx)} dx}{2a} + \frac{\int \frac{\csc(e+fx)(2ac+ac\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}} dx}{2ac^2} \\
&= -\frac{\int \frac{1}{\sqrt{a+a\sin(e+fx)}} dx}{2c} + \frac{\int \sec^2(e+fx)(a+a\sin(e+fx)) dx}{2a^2c} \\
&= \frac{\sec(e+fx)\sqrt{a+a\sin(e+fx)}}{acf} + \frac{\text{Subst}\left(\int \frac{1}{2a-x^2} dx, x, \frac{\sqrt{a+a\sin(e+fx)}}{c}\right)}{cf} \\
&= -\frac{2 \tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}cf} + \frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{2}\sqrt{a}cf}
\end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 0.30, size = 234, normalized size = 1.95

$$\frac{\cos(e+fx)(-1+\cos(\frac{1}{2}(e+fx)))(\log(1+\cos(\frac{1}{2}(e+fx)))-\sin(\frac{1}{2}(e+fx)))-\log(1-\cos(\frac{1}{2}(e+fx))+\sin(\frac{1}{2}(e+fx))) + (1+I)(-1)^{3/4}\tanh^{-1}\left(\frac{1+I}{\sqrt{2}}(-1+\tan(\frac{1}{4}(e+fx)))\right)(\cos(\frac{1}{2}(e+fx))-\sin(\frac{1}{2}(e+fx)))-\log(1+\cos(\frac{1}{2}(e+fx))-\sin(\frac{1}{2}(e+fx)))\sin(\frac{1}{2}(e+fx))+\log(1-\cos(\frac{1}{2}(e+fx))+\sin(\frac{1}{2}(e+fx)))\sin(\frac{1}{2}(e+fx))}{cf(-1+\sin(e+fx))\sqrt{a(1+\sin(e+fx))}}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[e + f\*x]/(Sqrt[a + a\*Sin[e + f\*x]]\*(c - c\*Sin[e + f\*x])),x]

[Out] (Cos[e + f\*x]\*(-1 + Cos[(e + f\*x)/2])\*(Log[1 + Cos[(e + f\*x)/2] - Sin[(e + f\*x)/2]] - Log[1 - Cos[(e + f\*x)/2] + Sin[(e + f\*x)/2]]) + (1 + I)\*(-1)^(3/4)\*ArcTanh[(1/2 + I/2)\*(-1)^(3/4)\*(-1 + Tan[(e + f\*x)/4])]\*(Cos[(e + f\*x)/2] - Sin[(e + f\*x)/2]) - Log[1 + Cos[(e + f\*x)/2] - Sin[(e + f\*x)/2]]\*Sin[(e + f\*x)/2] + Log[1 - Cos[(e + f\*x)/2] + Sin[(e + f\*x)/2]]\*Sin[(e + f\*x)/2])/(c\*f\*(-1 + Sin[e + f\*x])\*Sqrt[a\*(1 + Sin[e + f\*x])])

**Maple [A]**

time = 7.60, size = 124, normalized size = 1.03

method	result
default	$ \frac{(1+\sin(fx+e))\left(\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\sqrt{2}}{2\sqrt{a}}\right)\right)a^2\sqrt{a-a\sin(fx+e)}+2a^{\frac{5}{2}}-4\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}}{\sqrt{2}\sqrt{a}}\right)}{2ca^{\frac{5}{2}}\cos(fx+e)\sqrt{a+a\sin(fx+e)}f} $

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/sin(f\*x+e)/(c-c\*sin(f\*x+e))/(a+a\*sin(f\*x+e))^(1/2),x,method=\_RETURNVE  
RBOSE)



[Out]  $\frac{1}{2}*(1+\sin(f*x+e))*(2^{(1/2)}*\operatorname{arctanh}(1/2*(a-a*\sin(f*x+e))^{(1/2)}*2^{(1/2)}/a^{(1/2)})*a^2*(a-a*\sin(f*x+e))^{(1/2)}+2*a^{(5/2)}-4*\operatorname{arctanh}((a-a*\sin(f*x+e))^{(1/2)}/a^{(1/2)})*a^2*(a-a*\sin(f*x+e))^{(1/2)})/c/a^{(5/2)}/\cos(f*x+e)/(a+a*\sin(f*x+e))^{(1/2)}/f$

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sin(f*x+e)/(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")`

[Out] `-integrate(1/(sqrt(a*sin(f*x + e) + a)*(c*sin(f*x + e) - c)*sin(f*x + e)), x)`

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 359 vs. 2(110) = 220.

time = 0.42, size = 359, normalized size = 2.99

$$\frac{\sqrt{2} \sqrt{a} \cos(fx + e) \log\left(\frac{\cos(fx+e)^2 - \cos(fx+e) + 2}{\cos(fx+e)^2 - \cos(fx+e) + 2}\right) + 2 \sqrt{a} \cos(fx + e) \log\left(\frac{\cos(fx+e)^2 - \cos(fx+e) + 2}{\cos(fx+e)^2 - \cos(fx+e) + 2}\right)}{4 a c f \cos(fx + e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sin(f*x+e)/(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")`

[Out]  $\frac{1}{4}*(\sqrt{2}*\sqrt{a}*\cos(f*x + e)*\log(-(\cos(f*x + e))^2 - (\cos(f*x + e) - 2)*\sin(f*x + e) + 2*\sqrt{2}*\sqrt{a*\sin(f*x + e) + a}*(\cos(f*x + e) - \sin(f*x + e) + 1)/\sqrt{a} + 3*\cos(f*x + e) + 2)/(\cos(f*x + e)^2 - (\cos(f*x + e) + 2)*\sin(f*x + e) - \cos(f*x + e) - 2)) + 2*\sqrt{2}*\sqrt{a}*\cos(f*x + e)*\log((a*\cos(f*x + e))^3 - 7*a*\cos(f*x + e)^2 - 4*(\cos(f*x + e)^2 + (\cos(f*x + e) + 3)*\sin(f*x + e) - 2*\cos(f*x + e) - 3)*\sqrt{a*\sin(f*x + e) + a}*\sqrt{a} - 9*a*\cos(f*x + e) + (a*\cos(f*x + e)^2 + 8*a*\cos(f*x + e) - a)*\sin(f*x + e) - a)/(\cos(f*x + e)^3 + \cos(f*x + e)^2 + (\cos(f*x + e)^2 - 1)*\sin(f*x + e) - \cos(f*x + e) - 1)) + 4*\sqrt{2}*\sqrt{a*\sin(f*x + e) + a})/(a*c*f*\cos(f*x + e))$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sqrt{a \sin(e + fx) + a} \sin^2(e + fx) - \sqrt{a \sin(e + fx) + a} \sin(e + fx)} dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sin(f*x+e)/(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x)`

[Out] -Integral(1/(sqrt(a\*sin(e + f\*x) + a)\*sin(e + f\*x)\*\*2 - sqrt(a\*sin(e + f\*x) + a)\*sin(e + f\*x)), x)/c

**Giac [A]**

time = 0.52, size = 147, normalized size = 1.22

$$\frac{\sqrt{2} \left( \frac{2\sqrt{2} \log\left(\frac{|-2\sqrt{2} + 4\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)|}{2\sqrt{2} + 4\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)}\right)}{c} + \frac{\log(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) + 1)}{c} - \frac{\log(-\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) + 1)}{c} + \frac{2}{c\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)} \right)}{4\sqrt{a}f\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sin(f\*x+e)/(c-c\*sin(f\*x+e))/(a+a\*sin(f\*x+e))^(1/2),x, algorithm="giac")

[Out] -1/4\*sqrt(2)\*(2\*sqrt(2)\*log(abs(-2\*sqrt(2) + 4\*sin(-1/4\*pi + 1/2\*f\*x + 1/2\*e))/abs(2\*sqrt(2) + 4\*sin(-1/4\*pi + 1/2\*f\*x + 1/2\*e)))/c + log(sin(-1/4\*pi + 1/2\*f\*x + 1/2\*e) + 1)/c - log(-sin(-1/4\*pi + 1/2\*f\*x + 1/2\*e) + 1)/c + 2/(c\*sin(-1/4\*pi + 1/2\*f\*x + 1/2\*e)))/(sqrt(a)\*f\*sgn(cos(-1/4\*pi + 1/2\*f\*x + 1/2\*e)))

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin(e + fx) \sqrt{a + a \sin(e + fx)} (c - c \sin(e + fx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sin(e + f\*x)\*(a + a\*sin(e + f\*x))^(1/2)\*(c - c\*sin(e + f\*x))),x)

[Out] int(1/(sin(e + f\*x)\*(a + a\*sin(e + f\*x))^(1/2)\*(c - c\*sin(e + f\*x))), x)

$$3.15 \quad \int \frac{\sqrt{g \sin(e + fx)} \sqrt{a + a \sin(e + fx)}}{c - c \sin(e + fx)} dx$$

**Optimal.** Leaf size=103

$$\frac{2\sqrt{a} \sqrt{g} \tan^{-1} \left( \frac{\sqrt{a} \sqrt{g} \cos(e + fx)}{\sqrt{g \sin(e + fx)} \sqrt{a + a \sin(e + fx)}} \right)}{cf} + \frac{2 \sec(e + fx) \sqrt{g \sin(e + fx)} \sqrt{a + a \sin(e + fx)}}{cf}$$

[Out] 2\*arctan(cos(f\*x+e)\*a^(1/2)\*g^(1/2)/(g\*sin(f\*x+e))^(1/2)/(a+a\*sin(f\*x+e))^(1/2))\*a^(1/2)\*g^(1/2)/c/f+2\*sec(f\*x+e)\*(g\*sin(f\*x+e))^(1/2)\*(a+a\*sin(f\*x+e))^(1/2)/c/f

**Rubi [A]**

time = 0.31, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$ , Rules used = {3007, 2854, 211, 3009, 12, 30}

$$\frac{2\sqrt{a} \sqrt{g} \text{ArcTan} \left( \frac{\sqrt{a} \sqrt{g} \cos(e + fx)}{\sqrt{a \sin(e + fx) + a} \sqrt{g \sin(e + fx)}} \right)}{cf} + \frac{2 \sec(e + fx) \sqrt{a \sin(e + fx) + a} \sqrt{g \sin(e + fx)}}{cf}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[g\*Sin[e + f\*x]]\*Sqrt[a + a\*Sin[e + f\*x]])/(c - c\*Sin[e + f\*x]),x]

[Out] (2\*Sqrt[a]\*Sqrt[g]\*ArcTan[(Sqrt[a]\*Sqrt[g]\*Cos[e + f\*x])/(Sqrt[g\*Sin[e + f\*x]]\*Sqrt[a + a\*Sin[e + f\*x]])]/(c\*f) + (2\*Sec[e + f\*x]\*Sqrt[g\*Sin[e + f\*x]]\*Sqrt[a + a\*Sin[e + f\*x]])/(c\*f)

**Rule 12**

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

**Rule 30**

Int[(x\_)^(m\_), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

**Rule 211**

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

**Rule 2854**

Int[Sqrt[(a\_) + (b\_)\*sin[(e\_) + (f\_)\*(x\_)]]/Sqrt[(c\_) + (d\_)\*sin[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[-2\*(b/f), Subst[Int[1/(b + d\*x^2), x], x

```
, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

### Rule 3007

```
Int[(Sqrt[(g_)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_)*sin[(e_.) + (f_.)
)*(x_)]])/((c_) + (d_)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g/d, In
t[Sqrt[a + b*Sin[e + f*x]]/Sqrt[g*Sin[e + f*x]], x], x] - Dist[c*(g/d), Int
[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[g*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x],
x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && (EqQ[a^2 - b^
2, 0] || EqQ[c^2 - d^2, 0])
```

### Rule 3009

```
Int[Sqrt[(a_) + (b_)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[(g_)*sin[(e_.) + (f_.)
)*(x_)]*((c_) + (d_)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[-2*(b/f
), Subst[Int[1/(b*c + a*d + c*g*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[g*Sin[e
+ f*x]]*Sqrt[a + b*Sin[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

### Rubi steps

$$\int \frac{\sqrt{g \sin(e + fx)} \sqrt{a + a \sin(e + fx)}}{c - c \sin(e + fx)} dx = g \int \frac{\sqrt{a + a \sin(e + fx)}}{\sqrt{g \sin(e + fx)} (c - c \sin(e + fx))} dx - \frac{g \int \frac{\sqrt{a + a \sin(e + fx)}}{\sqrt{g \sin(e + fx)}} dx}{c}$$

$$= -\frac{(2ag) \text{Subst}\left(\int \frac{1}{cgx^2} dx, x, \frac{a \cos(e + fx)}{\sqrt{g \sin(e + fx)} \sqrt{a + a \sin(e + fx)}}\right)}{f}$$

$$= \frac{2\sqrt{a} \sqrt{g} \tan^{-1}\left(\frac{\sqrt{a} \sqrt{g} \cos(e + fx)}{\sqrt{g \sin(e + fx)} \sqrt{a + a \sin(e + fx)}}\right)}{cf}$$

$$= \frac{2\sqrt{a} \sqrt{g} \tan^{-1}\left(\frac{\sqrt{a} \sqrt{g} \cos(e + fx)}{\sqrt{g \sin(e + fx)} \sqrt{a + a \sin(e + fx)}}\right)}{cf} + \frac{2}{cf}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 0.63, size = 194, normalized size = 1.88

$$\frac{2e^{i(e+fx)} \left( 2(-1 + e^{2i(e+fx)}) - i(-i + e^{i(e+fx)}) \sqrt{-1 + e^{2i(e+fx)}} \tan^{-1}\left(\frac{\sqrt{-1 + e^{2i(e+fx)}}}{\sqrt{-1 + e^{2i(e+fx)}}}\right) - (-i + e^{i(e+fx)}) \sqrt{-1 + e^{2i(e+fx)}} \tanh^{-1}\left(\frac{e^{i(e+fx)}}{\sqrt{-1 + e^{2i(e+fx)}}}\right) \right) \sqrt{g \sin(e + fx)} \sqrt{a(1 + \sin(e + fx))}}{c(-1 + e^{4i(e+fx)}) f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[g*Sin[e + f*x]]*Sqrt[a + a*Sin[e + f*x]]/(c - c*Sin[e + f*x]),x]
```

```
[Out] (2*E^(I*(e + f*x))*(2*(-1 + E^((2*I)*(e + f*x))) - I*(-I + E^(I*(e + f*x))))*Sqrt[-1 + E^((2*I)*(e + f*x))]*ArcTan[Sqrt[-1 + E^((2*I)*(e + f*x))]] - (-I + E^(I*(e + f*x)))*Sqrt[-1 + E^((2*I)*(e + f*x))]*ArcTanh[E^(I*(e + f*x))/Sqrt[-1 + E^((2*I)*(e + f*x))]])*Sqrt[g*Sin[e + f*x]]*Sqrt[a*(1 + Sin[e + f*x])]/(c*(-1 + E^((4*I)*(e + f*x)))*f)
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 913 vs.  $2(87) = 174$ .

time = 28.70, size = 914, normalized size = 8.87

method	result
default	$\left( 4\sqrt{\frac{-1+\cos(fx+e)}{\sin(fx+e)}} \sqrt{2} \sin(fx+e) - \sin(fx+e) \ln \left( -\frac{\sqrt{\frac{-1+\cos(fx+e)}{\sin(fx+e)}} \sqrt{2} \sin(fx+e) + \sin(fx+e) - \cos(fx+e) + 1}{\sqrt{\frac{-1+\cos(fx+e)}{\sin(fx+e)}} \sqrt{2} \sin(fx+e) - \sin(fx+e) + \cos(fx+e) - 1} \right) - 4s \right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sin(f*x+e))^(1/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/c/f*(4*(-(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*2^(1/2)*sin(f*x+e)-sin(f*x+e))*ln(-((-(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*2^(1/2)*sin(f*x+e)+sin(f*x+e)-cos(f*x+e)+1)/((-(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*2^(1/2)*sin(f*x+e)-sin(f*x+e)+cos(f*x+e)-1))-4*sin(f*x+e)*arctan((-(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*2^(1/2)+1)-4*sin(f*x+e)*arctan((-(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*2^(1/2)-1)-sin(f*x+e)*ln(-((-(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*2^(1/2)*sin(f*x+e)-sin(f*x+e)+cos(f*x+e)-1)/((-(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*2^(1/2)*sin(f*x+e)+sin(f*x+e)-cos(f*x+e)+1))-cos(f*x+e)*ln(-((-(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*2^(1/2)*sin(f*x+e)+sin(f*x+e)-cos(f*x+e)+1)/((-(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*2^(1/2)*sin(f*x+e)-sin(f*x+e)+cos(f*x+e)-1))-4*cos(f*x+e)*arctan((-(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*2^(1/2)+1)-4*cos(f*x+e)*arctan((-(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*2^(1/2)-1)-cos(f*x+e)*ln(-((-(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*2^(1/2)*sin(f*x+e)-sin(f*x+e)+cos(f*x+e)-1)/((-(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*2^(1/2)*sin(f*x+e)+sin(f*x+e)-cos(f*x+e)+1))+ln(-((-(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*2^(1/2)*sin(f*x+e)+sin(f*x+e)-cos(f*x+e)+1)/((-(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*2^(1/2)*sin(f*x+e)-sin(f*x+e)+cos(f*x+e)-1))+4*arctan((-(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*2^(1/2)+1)+4*arctan((-(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*2^(1/2)-1)+ln(-((-(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*2^(1/2)*sin(f*x+e)-sin(f*x+e)+cos(f*x+e)-1)/((-(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*2^(1/2)*sin(f*x+e)+sin(f*x+e)-cos(f*x+e)+1)))*(g*sin(f*x+e))^(1/2)*2^(1/2)*sin(f*x+e)+sin(f*x+e)-cos(f*x+e)+1))
```

$(f*x+e)^{(1/2)}*(a*(1+\sin(f*x+e)))^{(1/2)}/\sin(f*x+e)/\cos(f*x+e)/(-(-1+\cos(f*x+e))/\sin(f*x+e))^{(1/2)}*2^{(1/2)}$

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*sin(f\*x+e))^(1/2)\*(a+a\*sin(f\*x+e))^(1/2)/(c-c\*sin(f\*x+e)),x, algorithm="maxima")

[Out] -integrate(sqrt(a\*sin(f\*x + e) + a)\*sqrt(g\*sin(f\*x + e))/(c\*sin(f\*x + e) - c), x)

**Fricas [A]**

time = 0.53, size = 480, normalized size = 4.66



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*sin(f\*x+e))^(1/2)\*(a+a\*sin(f\*x+e))^(1/2)/(c-c\*sin(f\*x+e)),x, algorithm="fricas")

[Out] [1/4\*(sqrt(-a\*g)\*cos(f\*x + e)\*log((128\*a\*g\*cos(f\*x + e)^5 - 128\*a\*g\*cos(f\*x + e)^4 - 416\*a\*g\*cos(f\*x + e)^3 + 128\*a\*g\*cos(f\*x + e)^2 + 289\*a\*g\*cos(f\*x + e) + 8\*(16\*cos(f\*x + e)^4 - 24\*cos(f\*x + e)^3 - 66\*cos(f\*x + e)^2 + (16\*cos(f\*x + e)^3 + 40\*cos(f\*x + e)^2 - 26\*cos(f\*x + e) - 51)\*sin(f\*x + e) + 25\*cos(f\*x + e) + 51)\*sqrt(-a\*g)\*sqrt(a\*sin(f\*x + e) + a)\*sqrt(g\*sin(f\*x + e)) + a\*g + (128\*a\*g\*cos(f\*x + e)^4 + 256\*a\*g\*cos(f\*x + e)^3 - 160\*a\*g\*cos(f\*x + e)^2 - 288\*a\*g\*cos(f\*x + e) + a\*g)\*sin(f\*x + e))/(cos(f\*x + e) + sin(f\*x + e) + 1)) + 8\*sqrt(a\*sin(f\*x + e) + a)\*sqrt(g\*sin(f\*x + e)))/(c\*f\*cos(f\*x + e)), -1/2\*(sqrt(a\*g)\*arctan(1/4\*sqrt(a\*g)\*(8\*cos(f\*x + e)^2 + 8\*sin(f\*x + e) - 9)\*sqrt(a\*sin(f\*x + e) + a)\*sqrt(g\*sin(f\*x + e)))/(2\*a\*g\*cos(f\*x + e)^3 + a\*g\*cos(f\*x + e)\*sin(f\*x + e) - 2\*a\*g\*cos(f\*x + e)))\*cos(f\*x + e) - 4\*sqrt(a\*sin(f\*x + e) + a)\*sqrt(g\*sin(f\*x + e)))/(c\*f\*cos(f\*x + e))]

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{g \sin(e + fx)} \sqrt{a \sin(e + fx) + a}}{\sin(e + fx) - 1} dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*sin(f\*x+e))\*\*(1/2)\*(a+a\*sin(f\*x+e))\*\*(1/2)/(c-c\*sin(f\*x+e)),x)

[Out]  $-\text{Integral}(\sqrt{g \sin(e + f x)} \sqrt{a \sin(e + f x) + a} / (\sin(e + f x) - 1), x) / c$

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*sin(f*x+e))^(1/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e)),x, algorithm="giac")`

[Out] Timed out

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{g \sin(e + f x)} \sqrt{a + a \sin(e + f x)}}{c - c \sin(e + f x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((g*sin(e + f*x))^(1/2)*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x)),x)`

[Out] `int(((g*sin(e + f*x))^(1/2)*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x)), x)`

$$3.16 \quad \int \frac{\sqrt{a + a \sin(e + fx)}}{\sqrt{g \sin(e + fx)} (c - c \sin(e + fx))} dx$$

Optimal. Leaf size=43

$$\frac{2 \sec(e + fx) \sqrt{g \sin(e + fx)} \sqrt{a + a \sin(e + fx)}}{c f g}$$

[Out]  $2 * \sec(f * x + e) * (g * \sin(f * x + e))^{(1/2)} * (a + a * \sin(f * x + e))^{(1/2)} / c / f / g$

Rubi [A]

time = 0.13, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$ , Rules used = {3009, 12, 30}

$$\frac{2 \sec(e + fx) \sqrt{a \sin(e + fx) + a} \sqrt{g \sin(e + fx)}}{c f g}$$

Antiderivative was successfully verified.

[In] `Int[Sqrt[a + a*Sin[e + f*x]]/(Sqrt[g*Sin[e + f*x]]*(c - c*Sin[e + f*x])),x]`

[Out] `(2*Sec[e + f*x]*Sqrt[g*Sin[e + f*x]]*Sqrt[a + a*Sin[e + f*x]])/(c*f*g)`

Rule 12

`Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]`

Rule 30

`Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]`

Rule 3009

`Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/(Sqrt[(g_)*sin[(e_) + (f_)*(x_)]]*((c_) + (d_)*sin[(e_) + (f_)*(x_)])), x_Symbol] := Dist[-2*(b/f), Subst[Int[1/(b*c + a*d + c*g*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[g*Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]`

Rubi steps



$$\int \frac{\sqrt{a + a \sin(e + fx)}}{\sqrt{g \sin(e + fx)} (c - c \sin(e + fx))} dx = \frac{(2a) \text{Subst} \left( \int \frac{1}{cgx^2} dx, x, \frac{a \cos(e+fx)}{\sqrt{g \sin(e + fx)} \sqrt{a + a \sin(e + fx)}} \right)}{f}$$

$$= \frac{(2a) \text{Subst} \left( \int \frac{1}{x^2} dx, x, \frac{a \cos(e+fx)}{\sqrt{g \sin(e + fx)} \sqrt{a + a \sin(e + fx)}} \right)}{cfg}$$

$$= \frac{2 \sec(e + fx) \sqrt{g \sin(e + fx)} \sqrt{a + a \sin(e + fx)}}{cfg}$$

**Mathematica [A]**

time = 0.15, size = 40, normalized size = 0.93

$$\frac{2\sqrt{a(1 + \sin(e + fx))} \tan(e + fx)}{cf \sqrt{g \sin(e + fx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + a\*Sin[e + f\*x]]/(Sqrt[g\*Sin[e + f\*x]]\*(c - c\*Sin[e + f\*x])),x]

[Out] (2\*Sqrt[a\*(1 + Sin[e + f\*x]])\*Tan[e + f\*x])/(c\*f\*Sqrt[g\*Sin[e + f\*x]])

**Maple [A]**

time = 0.29, size = 45, normalized size = 1.05

method	result	size
default	$\frac{2\sqrt{a(1 + \sin(fx + e))} \sin(fx + e)}{cf \sqrt{g \sin(fx + e)} \cos(fx + e)}$	45

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a\*sin(f\*x+e))^(1/2)/(c-c\*sin(f\*x+e))/(g\*sin(f\*x+e))^(1/2),x,method=\_RETURNVERBOSE)

[Out] 2/c/f\*(a\*(1+sin(f\*x+e)))^(1/2)\*sin(f\*x+e)/(g\*sin(f\*x+e))^(1/2)/cos(f\*x+e)

**Maxima [B]** Leaf count of result is larger than twice the leaf count of optimal. 331 vs. 2(42) = 84.

time = 0.54, size = 331, normalized size = 7.70

$$\frac{\left( \frac{\frac{1}{2}\sqrt{2}\sqrt{a}\sqrt{g}\sqrt{\sin(fx+e)} - \frac{\sqrt{2}\sqrt{a}\sqrt{g}\sqrt{\sin(fx+e)^2}}{(\cos(fx+e)+1)^2} \right) \sqrt{\frac{\sin(fx+e)}{\cos(fx+e)+1}} - 2 \frac{\left( \frac{1}{2}\sqrt{2}\sqrt{a}\sqrt{g}\sqrt{\sin(fx+e)} + \frac{\sqrt{2}\sqrt{a}\sqrt{g}\sqrt{\sin(fx+e)^2}}{(\cos(fx+e)+1)^2} \right) \sqrt{\frac{\sin(fx+e)}{\cos(fx+e)+1}}}{\frac{cg - \frac{2a \sin(fx+e)}{\cos(fx+e)+1}}{c\sqrt{g}}} - \frac{2\sqrt{2}\sqrt{a}\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)^{\frac{3}{2}} + \frac{1}{2}\sqrt{2}\sqrt{a}\sqrt{\sin(fx+e)}}{c\sqrt{g}} - \frac{2\sqrt{2}\sqrt{a}\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)^{\frac{3}{2}} - \frac{1}{2}\sqrt{2}\sqrt{a}\sqrt{\sin(fx+e)}}{c\sqrt{g}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*sin(f\*x+e))^(1/2)/(c-c\*sin(f\*x+e))/(g\*sin(f\*x+e))^(1/2),x, algorithm="maxima")

[Out] -1/12\*(4\*((3\*sqrt(2)\*sqrt(a)\*sqrt(g)\*sin(f\*x + e)/(cos(f\*x + e) + 1) - sqrt(2)\*sqrt(a)\*sqrt(g)\*sin(f\*x + e)^2/(cos(f\*x + e) + 1)^2)\*sqrt(sin(f\*x + e)/(cos(f\*x + e) + 1)) - 2\*(3\*sqrt(2)\*sqrt(a)\*sqrt(g)\*sin(f\*x + e)/(cos(f\*x + e) + 1) + sqrt(2)\*sqrt(a)\*sqrt(g)\*sin(f\*x + e)^2/(cos(f\*x + e) + 1)^2)/sqrt(sin(f\*x + e)/(cos(f\*x + e) + 1)))/(c\*g - c\*g\*sin(f\*x + e)/(cos(f\*x + e) + 1)) - (2\*sqrt(2)\*sqrt(a)\*(sin(f\*x + e)/(cos(f\*x + e) + 1))^(3/2) + 3\*sqrt(2)\*sqrt(a)\*sin(f\*x + e)/(cos(f\*x + e) + 1))/(c\*sqrt(g)) - (2\*sqrt(2)\*sqrt(a)\*(sin(f\*x + e)/(cos(f\*x + e) + 1))^(3/2) - 3\*sqrt(2)\*sqrt(a)\*sin(f\*x + e)/(cos(f\*x + e) + 1))/(c\*sqrt(g))/f

**Fricas** [A]

time = 0.36, size = 44, normalized size = 1.02

$$\frac{2 \sqrt{a \sin (f x+e)+a} \sqrt{g \sin (f x+e)}}{c f g \cos (f x+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*sin(f\*x+e))^(1/2)/(c-c\*sin(f\*x+e))/(g\*sin(f\*x+e))^(1/2),x, algorithm="fricas")

[Out] 2\*sqrt(a\*sin(f\*x + e) + a)\*sqrt(g\*sin(f\*x + e))/(c\*f\*g\*cos(f\*x + e))

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{a \sin (e+f x)+a}}{\sqrt{g \sin (e+f x)} \sin (e+f x)-\sqrt{g \sin (e+f x)}} d x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*sin(f\*x+e))\*\*(1/2)/(c-c\*sin(f\*x+e))/(g\*sin(f\*x+e))\*\*(1/2),x)

[Out] -Integral(sqrt(a\*sin(e + f\*x) + a)/(sqrt(g\*sin(e + f\*x))\*sin(e + f\*x) - sqrt(g\*sin(e + f\*x))), x)/c

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*sin(f\*x+e))^(1/2)/(c-c\*sin(f\*x+e))/(g\*sin(f\*x+e))^(1/2),x, algorithm="giac")

[Out] integrate(-sqrt(a\*sin(f\*x + e) + a)/((c\*sin(f\*x + e) - c)\*sqrt(g\*sin(f\*x + e))), x)

**Mupad [B]**

time = 12.94, size = 52, normalized size = 1.21

$$\frac{2 \sin(2e + 2fx) \sqrt{a(\sin(e + fx) + 1)}}{cf(\cos(2e + 2fx) + 1) \sqrt{g \sin(e + fx)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a\*sin(e + f\*x))^(1/2)/((g\*sin(e + f\*x))^(1/2)\*(c - c\*sin(e + f\*x))),x)

[Out] (2\*sin(2\*e + 2\*f\*x)\*(a\*(sin(e + f\*x) + 1))^(1/2))/(c\*f\*(cos(2\*e + 2\*f\*x) + 1)\*(g\*sin(e + f\*x))^(1/2))

$$3.17 \quad \int \frac{\sqrt{g \sin(e + fx)}}{\sqrt{a + a \sin(e + fx)} (c - c \sin(e + fx))} dx$$

Optimal. Leaf size=114

$$\frac{\sqrt{g} \tan^{-1} \left( \frac{\sqrt{a} \sqrt{g} \cos(e+fx)}{\sqrt{2} \sqrt{g \sin(e+fx)} \sqrt{a + a \sin(e+fx)}} \right)}{\sqrt{2} \sqrt{a} cf} + \frac{\sec(e+fx) \sqrt{g \sin(e+fx)} \sqrt{a + a \sin(e+fx)}}{acf}$$

[Out] 1/2\*arctan(1/2\*cos(f\*x+e)\*a^(1/2)\*g^(1/2)\*2^(1/2)/(g\*sin(f\*x+e))^(1/2)/(a+a\*sin(f\*x+e))^(1/2))\*g^(1/2)/c/f\*2^(1/2)/a^(1/2)+sec(f\*x+e)\*(g\*sin(f\*x+e))^(1/2)\*(a+a\*sin(f\*x+e))^(1/2)/a/c/f

Rubi [A]

time = 0.32, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$ , Rules used = {3015, 2861, 211, 3009, 12, 30}

$$\frac{\sqrt{g} \text{ArcTan} \left( \frac{\sqrt{a} \sqrt{g} \cos(e+fx)}{\sqrt{2} \sqrt{a \sin(e+fx) + a} \sqrt{g \sin(e+fx)}} \right)}{\sqrt{2} \sqrt{a} cf} + \frac{\sec(e+fx) \sqrt{a \sin(e+fx) + a} \sqrt{g \sin(e+fx)}}{acf}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[g\*Sin[e + f\*x]]/(Sqrt[a + a\*Sin[e + f\*x]]\*(c - c\*Sin[e + f\*x])),x]

[Out] (Sqrt[g]\*ArcTan[(Sqrt[a]\*Sqrt[g]\*Cos[e + f\*x])/(Sqrt[2]\*Sqrt[g\*Sin[e + f\*x]]\*Sqrt[a + a\*Sin[e + f\*x]])]/(Sqrt[2]\*Sqrt[a]\*c\*f) + (Sec[e + f\*x]\*Sqrt[g\*Sin[e + f\*x]]\*Sqrt[a + a\*Sin[e + f\*x]])/(a\*c\*f)

Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

Rule 30

Int[(x\_)^(m\_.), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 211

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)])], x_Symbol] := Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

### Rule 3009

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/(Sqrt[(g_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])], x_Symbol] := Dist[-2*(b/f), Subst[Int[1/(b*c + a*d + c*g*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[g*Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

### Rule 3015

```
Int[Sqrt[(g_)*sin[(e_) + (f_)*(x_)]]/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])], x_Symbol] := Dist[(-a)*(g/(b*c - a*d)), Int[1/(Sqrt[g*Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]]), x], x] + Dist[c*(g/(b*c - a*d)), Int[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[g*Sin[e + f*x]])*(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && (EqQ[a^2 - b^2, 0] || EqQ[c^2 - d^2, 0])
```

### Rubi steps

$$\int \frac{\sqrt{g \sin(e + fx)}}{\sqrt{a + a \sin(e + fx)} (c - c \sin(e + fx))} dx = \frac{g \int \frac{\sqrt{a + a \sin(e + fx)}}{\sqrt{g \sin(e + fx)} (c - c \sin(e + fx))} dx}{2a} - \frac{g \int \frac{1}{\sqrt{g \sin(e + fx)}} dx}{2a}$$

$$= -\frac{g \operatorname{Subst}\left(\int \frac{1}{c g x^2} dx, x, \frac{a \cos(e + fx)}{\sqrt{g \sin(e + fx)} \sqrt{a + a \sin(e + fx)}}\right)}{2a}$$

$$= \frac{\sqrt{g} \tan^{-1}\left(\frac{\sqrt{a} \sqrt{g} \cos(e + fx)}{\sqrt{2} \sqrt{g \sin(e + fx)} \sqrt{a + a \sin(e + fx)}}\right)}{\sqrt{2} \sqrt{a} c f}$$

$$= \frac{\sqrt{g} \tan^{-1}\left(\frac{\sqrt{a} \sqrt{g} \cos(e + fx)}{\sqrt{2} \sqrt{g \sin(e + fx)} \sqrt{a + a \sin(e + fx)}}\right)}{\sqrt{2} \sqrt{a} c f} + \dots$$

**Mathematica [A]**

time = 0.23, size = 133, normalized size = 1.17

$$\frac{\csc(2(e+fx))\sqrt{\sin(e+fx)}\sqrt{g\sin(e+fx)}\sqrt{a(1+\sin(e+fx))}\left(2\sqrt{c}\sqrt{\sin(e+fx)}-\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\right)\sqrt{c-c\sin(e+fx)}\right)}{ac^{3/2}f}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[g\*Sin[e + f\*x]]/(Sqrt[a + a\*Sin[e + f\*x]]\*(c - c\*Sin[e + f\*x])),x]

[Out] (Csc[2\*(e + f\*x)]\*Sqrt[Sin[e + f\*x]]\*Sqrt[g\*Sin[e + f\*x]]\*Sqrt[a\*(1 + Sin[e + f\*x])]\*(2\*Sqrt[c]\*Sqrt[Sin[e + f\*x]] - Sqrt[2]\*ArcTan[(Sqrt[2]\*Sqrt[c]\*Sqrt[Sin[e + f\*x]])/Sqrt[c - c\*Sin[e + f\*x]])\*Sqrt[c - c\*Sin[e + f\*x]])/(a\*c^(3/2)\*f)

**Maple [A]**

time = 0.32, size = 187, normalized size = 1.64

method	result
default	$\frac{\sqrt{g\sin(fx+e)}(-1+\cos(fx+e))\left(\sqrt{-\frac{-1+\cos(fx+e)}{\sin(fx+e)}}\cos(fx+e)-2\arctan\left(\sqrt{-\frac{-1+\cos(fx+e)}{\sin(fx+e)}}\right)\cos(fx+e)+\sqrt{-\frac{-1+\cos(fx+e)}{\sin(fx+e)}}\right)}{cf\sin(fx+e)\sqrt{a(1+\sin(fx+e))}\sqrt{-\frac{-1+\cos(fx+e)}{\sin(fx+e)}}(-1+\cos(fx+e))}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*sin(f\*x+e))^(1/2)/(c-c\*sin(f\*x+e))/(a+a\*sin(f\*x+e))^(1/2),x,method=\_RETURNVERBOSE)

[Out] -1/c/f\*(g\*sin(f\*x+e))^(1/2)\*(-1+cos(f\*x+e))\*((-(-1+cos(f\*x+e))/sin(f\*x+e))^(1/2)\*cos(f\*x+e)-2\*arctan((-(-1+cos(f\*x+e))/sin(f\*x+e))^(1/2))\*cos(f\*x+e)+(-(-1+cos(f\*x+e))/sin(f\*x+e))^(1/2)\*sin(f\*x+e)+(-(-1+cos(f\*x+e))/sin(f\*x+e))^(1/2))/sin(f\*x+e)/(a\*(1+sin(f\*x+e))^(1/2)/((-(-1+cos(f\*x+e))/sin(f\*x+e))^(1/2)/(-1+cos(f\*x+e)+sin(f\*x+e)))

**Maxima [B]** Leaf count of result is larger than twice the leaf count of optimal. 288 vs. 2(102) = 204.

time = 0.52, size = 288, normalized size = 2.53

$$\frac{4\sqrt{2}\sqrt{g}\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)^{\frac{3}{2}}}{\sqrt{a}c+\frac{\sqrt{a}c\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sqrt{a}c+\frac{\sqrt{a}c\sin(fx+e)}{\cos(fx+e)+1}}{\cos(fx+e)+1}}-\frac{2\sqrt{2}\sqrt{g}\arctan\left(\sqrt{\frac{\sin(fx+e)}{\cos(fx+e)+1}}\right)}{\sqrt{a}c}+\frac{\sqrt{2}\sqrt{g}\sqrt{\frac{\sin(fx+e)}{\cos(fx+e)+1}}+\sqrt{2}\sqrt{g}}{\sqrt{a}c+\frac{\sqrt{a}c\sin(fx+e)}{\cos(fx+e)+1}}+\frac{\sqrt{2}\sqrt{g}\sqrt{\frac{\sin(fx+e)}{\cos(fx+e)+1}}-\sqrt{2}\sqrt{g}}{\sqrt{a}c+\frac{\sqrt{a}c\sin(fx+e)}{\cos(fx+e)+1}}$$

2f

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*sin(f\*x+e))^(1/2)/(c-c\*sin(f\*x+e))/(a+a\*sin(f\*x+e))^(1/2),x, algorithm="maxima")

```
[Out] 1/2*(4*sqrt(2)*sqrt(g)*(sin(f*x + e)/(cos(f*x + e) + 1))^(3/2)/(sqrt(a)*c +
sqrt(a)*c*sin(f*x + e)/(cos(f*x + e) + 1) - (sqrt(a)*c + sqrt(a)*c*sin(f*x
+ e)/(cos(f*x + e) + 1))*sin(f*x + e)/(cos(f*x + e) + 1) - 2*sqrt(2)*sqrt
(g)*arctan(sqrt(sin(f*x + e)/(cos(f*x + e) + 1)))/(sqrt(a)*c) + (sqrt(2)*sq
rt(g)*sqrt(sin(f*x + e)/(cos(f*x + e) + 1)) + sqrt(2)*sqrt(g))/(sqrt(a)*c +
sqrt(a)*c*sin(f*x + e)/(cos(f*x + e) + 1)) + (sqrt(2)*sqrt(g)*sqrt(sin(f*x
+ e)/(cos(f*x + e) + 1)) - sqrt(2)*sqrt(g))/(sqrt(a)*c + sqrt(a)*c*sin(f*x
+ e)/(cos(f*x + e) + 1)))/f
```

**Fricas** [A]

time = 0.48, size = 416, normalized size = 3.65

$$\sqrt{2} \sqrt{-\frac{2}{g} \cos(fx + e)} \log \left( \frac{(1 + \sin(fx + e)) \sqrt{2} (1 + \sin(fx + e) + \cos(fx + e) - \cos(fx + e)) \sqrt{4 \sin^2(fx + e) + 4} + \sqrt{g \sin(fx + e)}}{\sin^2(fx + e) + \sin(fx + e) - \cos(fx + e) - 1} \sqrt{\frac{2}{g}} - 2 \sin(fx + e) - 2 \cos(fx + e) + 1 + 2 \sin(fx + e) \cos(fx + e)} \right) + 8 \sqrt{4 \sin^2(fx + e) + 4} \sqrt{g \sin(fx + e)} \sqrt{2} \sqrt{-\frac{2}{g} \cos(fx + e)} \operatorname{arctan} \left( \frac{\sqrt{2} \sqrt{4 \sin^2(fx + e) + 4} \sqrt{g \sin(fx + e)}}{1 + \sin(fx + e) - \cos(fx + e)} \sqrt{\frac{2}{g}} - 2 \sin(fx + e) - 2 \cos(fx + e) + 1 + 2 \sin(fx + e) \cos(fx + e)} \right) \cos(fx + e) - 4 \sqrt{4 \sin^2(fx + e) + 4} \sqrt{g \sin(fx + e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, a
lgorithm="fricas")
```

```
[Out] [1/8*(sqrt(2)*a*sqrt(-g/a)*cos(f*x + e)*log((17*g*cos(f*x + e)^3 + 4*sqrt(2)
)*(3*cos(f*x + e)^2 + (3*cos(f*x + e) + 4)*sin(f*x + e) - cos(f*x + e) - 4)
*sqrt(a*sin(f*x + e) + a)*sqrt(g*sin(f*x + e))*sqrt(-g/a) + 3*g*cos(f*x + e)
)^2 - 18*g*cos(f*x + e) + (17*g*cos(f*x + e)^2 + 14*g*cos(f*x + e) - 4*g)*s
in(f*x + e) - 4*g)/(cos(f*x + e)^3 + 3*cos(f*x + e)^2 + (cos(f*x + e)^2 - 2
*cos(f*x + e) - 4)*sin(f*x + e) - 2*cos(f*x + e) - 4)) + 8*sqrt(a*sin(f*x +
e) + a)*sqrt(g*sin(f*x + e)))/(a*c*f*cos(f*x + e)), -1/4*(sqrt(2)*a*sqrt(g
/a)*arctan(1/4*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(g*sin(f*x + e))*sqrt(g
/a)*(3*sin(f*x + e) - 1)/(g*cos(f*x + e)*sin(f*x + e))*cos(f*x + e) - 4*sq
rt(a*sin(f*x + e) + a)*sqrt(g*sin(f*x + e)))/(a*c*f*cos(f*x + e))]
```

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{g \sin(e + fx)}}{\sqrt{a \sin(e + fx) + a} \sin(e + fx) - \sqrt{a \sin(e + fx) + a}} dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] -Integral(sqrt(g*sin(e + f*x))/(sqrt(a*sin(e + f*x) + a)*sin(e + f*x) - sq
r t(a*sin(e + f*x) + a)), x)/c
```

**Giac** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, a
lgorithm="giac")
```

```
[Out] Timed out
```

**Mupad [F]**

```
time = 0.00, size = -1, normalized size = -0.01
```

$$\int \frac{\sqrt{g \sin(e + f x)}}{\sqrt{a + a \sin(e + f x)} (c - c \sin(e + f x))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sin(e + f*x))^(1/2)/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))
),x)
```

```
[Out] int((g*sin(e + f*x))^(1/2)/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))
), x)
```



$$3.18 \quad \int \frac{1}{\sqrt{g \sin(e + fx)} \sqrt{a + a \sin(e + fx)} (c - c \sin(e + fx))} dx$$

**Optimal.** Leaf size=118

$$\frac{\tan^{-1}\left(\frac{\sqrt{a} \sqrt{g} \cos(e+fx)}{\sqrt{2} \sqrt{g \sin(e+fx)} \sqrt{a+a \sin(e+fx)}}\right)}{\sqrt{2} \sqrt{a} c f \sqrt{g}} + \frac{\sec(e+fx) \sqrt{g \sin(e+fx)} \sqrt{a+a \sin(e+fx)}}{a c f g}$$

[Out] -1/2\*arctan(1/2\*cos(f\*x+e)\*a^(1/2)\*g^(1/2)\*2^(1/2)/(g\*sin(f\*x+e))^(1/2)/(a+a\*sin(f\*x+e))^(1/2))/c/f\*2^(1/2)/a^(1/2)/g^(1/2)+sec(f\*x+e)\*(g\*sin(f\*x+e))^(1/2)\*(a+a\*sin(f\*x+e))^(1/2)/a/c/f/g

**Rubi [A]**

time = 0.32, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$ , Rules used = {3017, 2861, 211, 3009, 12, 30}

$$\frac{\sec(e+fx) \sqrt{a \sin(e+fx) + a} \sqrt{g \sin(e+fx)}}{a c f g} - \frac{\text{ArcTan}\left(\frac{\sqrt{a} \sqrt{g} \cos(e+fx)}{\sqrt{2} \sqrt{a \sin(e+fx) + a} \sqrt{g \sin(e+fx)}}\right)}{\sqrt{2} \sqrt{a} c f \sqrt{g}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[g\*Sin[e + f\*x]]\*Sqrt[a + a\*Sin[e + f\*x]]\*(c - c\*Sin[e + f\*x])), x]

[Out] -(ArcTan[(Sqrt[a]\*Sqrt[g]\*Cos[e + f\*x])/(Sqrt[2]\*Sqrt[g\*Sin[e + f\*x]]\*Sqrt[a + a\*Sin[e + f\*x]])]/(Sqrt[2]\*Sqrt[a]\*c\*f\*Sqrt[g])) + (Sec[e + f\*x]\*Sqrt[g\*Sin[e + f\*x]]\*Sqrt[a + a\*Sin[e + f\*x]])/(a\*c\*f\*g)

**Rule 12**

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

**Rule 30**

Int[(x\_)^(m\_), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

**Rule 211**

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

**Rule 2861**

```
Int[1/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)])], x_Symbol] := Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

### Rule 3009

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/(Sqrt[(g_)*sin[(e_) + (f_)*(x_)]]*((c_) + (d_)*sin[(e_) + (f_)*(x_)])), x_Symbol] := Dist[-2*(b/f), Subst[Int[1/(b*c + a*d + c*g*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[g*Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]])]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

### Rule 3017

```
Int[1/(Sqrt[(g_)*sin[(e_) + (f_)*(x_)]]*Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((c_) + (d_)*sin[(e_) + (f_)*(x_)])), x_Symbol] := Dist[b/(b*c - a*d), Int[1/(Sqrt[g*Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]]), x], x] - Dist[d/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[g*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && (EqQ[a^2 - b^2, 0] || EqQ[c^2 - d^2, 0])
```

### Rubi steps

$$\int \frac{1}{\sqrt{g \sin(e+fx)} \sqrt{a+a \sin(e+fx)} (c-c \sin(e+fx))} dx = \frac{\int \frac{\sqrt{a+a \sin(e+fx)}}{\sqrt{g \sin(e+fx)} (c-c \sin(e+fx))} dx}{2a} + \frac{\int \frac{1}{\sqrt{g \sin(e+fx)}} dx}{2a}$$

$$= -\frac{f}{\sqrt{2} \sqrt{a} c f \sqrt{g}} \tan^{-1} \left( \frac{\sqrt{a} \sqrt{g} \cos(e+fx)}{\sqrt{2} \sqrt{g \sin(e+fx)} \sqrt{a+a \sin(e+fx)}} \right)$$

$$= -\frac{f}{\sqrt{2} \sqrt{a} c f \sqrt{g}} \tan^{-1} \left( \frac{\sqrt{a} \sqrt{g} \cos(e+fx)}{\sqrt{2} \sqrt{g \sin(e+fx)} \sqrt{a+a \sin(e+fx)}} \right)$$

Mathematica [A]

time = 0.19, size = 132, normalized size = 1.12

$$\frac{\csc(2(e+fx)) \sin^{\frac{3}{2}}(e+fx) \sqrt{a(1+\sin(e+fx))} \left( 2\sqrt{c} \sqrt{\sin(e+fx)} + \sqrt{2} \tan^{-1} \left( \frac{\sqrt{2} \sqrt{c} \sqrt{\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}} \right) \sqrt{c-c\sin(e+fx)} \right)}{ac^{3/2} f \sqrt{g \sin(e+fx)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[g\*Sin[e + f\*x]]\*Sqrt[a + a\*Sin[e + f\*x]]\*(c - c\*Sin[e + f\*x])), x]

[Out] (Csc[2\*(e + f\*x)]\*Sin[e + f\*x]^(3/2)\*Sqrt[a\*(1 + Sin[e + f\*x])]\*(2\*Sqrt[c]\*Sqrt[Sin[e + f\*x]] + Sqrt[2]\*ArcTan[(Sqrt[2]\*Sqrt[c]\*Sqrt[Sin[e + f\*x]])/Sqrt[c - c\*Sin[e + f\*x]])\*Sqrt[c - c\*Sin[e + f\*x]])/(a\*c^(3/2)\*f\*Sqrt[g\*Sin[e + f\*x]])

**Maple [A]**

time = 0.32, size = 117, normalized size = 0.99

method	result	size
default	$\frac{\left( 2\sqrt{-\frac{-1+\cos(fx+e)}{\sin(fx+e)}} \arctan\left(\sqrt{-\frac{-1+\cos(fx+e)}{\sin(fx+e)}}\right) \cos(fx+e) - \cos(fx+e) + \sin(fx+e) + 1 \right) \sin(fx+e)}{cf(-1+\cos(fx+e)+\sin(fx+e)) \sqrt{g \sin(fx+e)} \sqrt{a(1+\sin(fx+e))}}$	117

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c-c\*sin(f\*x+e))/(g\*sin(f\*x+e))^(1/2)/(a+a\*sin(f\*x+e))^(1/2), x, method = \_RETURNVERBOSE)

[Out] 1/c/f\*(2\*(-(-1+cos(f\*x+e))/sin(f\*x+e))^(1/2)\*arctan((-(-1+cos(f\*x+e))/sin(f\*x+e))^(1/2))\*cos(f\*x+e)-cos(f\*x+e)+sin(f\*x+e)+1)\*sin(f\*x+e)/(-1+cos(f\*x+e)+sin(f\*x+e))/(g\*sin(f\*x+e))^(1/2)/(a\*(1+sin(f\*x+e)))^(1/2)

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c-c\*sin(f\*x+e))/(g\*sin(f\*x+e))^(1/2)/(a+a\*sin(f\*x+e))^(1/2), x, algorithm="maxima")

[Out] -integrate(1/(sqrt(a\*sin(f\*x + e) + a)\*(c\*sin(f\*x + e) - c)\*sqrt(g\*sin(f\*x + e))), x)

**Fricas [A]**

time = 0.51, size = 422, normalized size = 3.58

$$\frac{\sqrt{2} \operatorname{arctan}\left(\frac{\sqrt{2} \sqrt{a \sin(fx+e) + a} \sqrt{g \sin(fx+e)}}{\sqrt{c - c \sin(fx+e)}}\right) \cos(fx+e) + 4 \sqrt{a \sin(fx+e) + a} \sqrt{g \sin(fx+e)}}{8 a c f g \cos(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-c*sin(f*x+e))/(g*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x,
algorithm="fricas")
```

```
[Out] [1/8*(sqrt(2)*a*g*sqrt(-1/(a*g))*cos(f*x + e)*log(-(4*sqrt(2)*(3*cos(f*x +
e)^2 + (3*cos(f*x + e) + 4)*sin(f*x + e) - cos(f*x + e) - 4)*sqrt(a*sin(f*x
+ e) + a)*sqrt(g*sin(f*x + e))*sqrt(-1/(a*g)) - 17*cos(f*x + e)^3 - 3*cos(
f*x + e)^2 - (17*cos(f*x + e)^2 + 14*cos(f*x + e) - 4)*sin(f*x + e) + 18*co
s(f*x + e) + 4)/(cos(f*x + e)^3 + 3*cos(f*x + e)^2 + (cos(f*x + e)^2 - 2*co
s(f*x + e) - 4)*sin(f*x + e) - 2*cos(f*x + e) - 4)) + 8*sqrt(a*sin(f*x + e)
+ a)*sqrt(g*sin(f*x + e)))/(a*c*f*g*cos(f*x + e)), 1/4*(sqrt(2)*a*g*sqrt(1
/(a*g))*arctan(1/4*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(g*sin(f*x + e))*sq
rt(1/(a*g))*(3*sin(f*x + e) - 1)/(cos(f*x + e)*sin(f*x + e)))*cos(f*x + e)
+ 4*sqrt(a*sin(f*x + e) + a)*sqrt(g*sin(f*x + e)))/(a*c*f*g*cos(f*x + e)]]
```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{g \sin(e + fx)} \sqrt{a \sin(e + fx) + a} \sin(e + fx) - \sqrt{g \sin(e + fx)} \sqrt{a \sin(e + fx) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-c*sin(f*x+e))/(g*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),
x)
```

```
[Out] -Integral(1/(sqrt(g*sin(e + f*x))*sqrt(a*sin(e + f*x) + a)*sin(e + f*x) - s
qrt(g*sin(e + f*x))*sqrt(a*sin(e + f*x) + a)), x)/c
```

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-c*sin(f*x+e))/(g*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x,
algorithm="giac")
```

```
[Out] Timed out
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{g \sin(e + fx)} \sqrt{a + a \sin(e + fx)} (c - c \sin(e + fx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((g*sin(e + f*x))^(1/2)*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x  
))),x)
```

```
[Out] int(1/((g*sin(e + f*x))^(1/2)*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x  
))), x)
```

### 3.19 $\int \csc(e+fx) \sqrt{a+a\sin(e+fx)} \sqrt{c-c\sin(e+fx)}$

Optimal. Leaf size=46

$$\frac{\log(\sin(e+fx)) \sec(e+fx) \sqrt{a+a\sin(e+fx)} \sqrt{c-c\sin(e+fx)}}{f}$$

[Out]  $\ln(\sin(f*x+e))*\sec(f*x+e)*(a+a*\sin(f*x+e))^{(1/2)}*(c-c*\sin(f*x+e))^{(1/2)}/f$

Rubi [A]

time = 0.11, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$ , Rules used = {3027, 3556}

$$\frac{\sec(e+fx) \sqrt{a\sin(e+fx)+a} \sqrt{c-c\sin(e+fx)} \log(\sin(e+fx))}{f}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Csc}[e+f*x]*\text{Sqrt}[a+a*\text{Sin}[e+f*x]]*\text{Sqrt}[c-c*\text{Sin}[e+f*x]],x]$

[Out]  $(\text{Log}[\text{Sin}[e+f*x]]*\text{Sec}[e+f*x]*\text{Sqrt}[a+a*\text{Sin}[e+f*x]]*\text{Sqrt}[c-c*\text{Sin}[e+f*x]])/f$

Rule 3027

$\text{Int}[(\text{Sqrt}[(a_)+(b_)*\sin[(e_)+(f_)*(x_)]]*\text{Sqrt}[(c_)+(d_)*\sin[(e_)+(f_)*(x_)]])/\sin[(e_)+(f_)*(x_)], x\_Symbol] \rightarrow \text{Dist}[\text{Sqrt}[a+b*\text{Sin}[e+f*x]]*(\text{Sqrt}[c+d*\text{Sin}[e+f*x]]/\text{Cos}[e+f*x]), \text{Int}[\text{Cot}[e+f*x], x], x] /;$   $\text{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{EqQ}[c^2 - d^2, 0]$

Rule 3556

$\text{Int}[\tan[(c_)+(d_)*(x_)], x\_Symbol] \rightarrow \text{Simp}[-\text{Log}[\text{RemoveContent}[\text{Cos}[c+d*x], x]]/d, x] /;$   $\text{FreeQ}\{c, d\}, x]$

Rubi steps

$$\int \csc(e+fx) \sqrt{a+a\sin(e+fx)} \sqrt{c-c\sin(e+fx)} dx = \left( \sec(e+fx) \sqrt{a+a\sin(e+fx)} \sqrt{c-c\sin(e+fx)} \right) \\ = \frac{\log(\sin(e+fx)) \sec(e+fx) \sqrt{a+a\sin(e+fx)} \sqrt{c-c\sin(e+fx)}}{f}$$

**Mathematica [A]**

time = 0.07, size = 62, normalized size = 1.35

$$\frac{(\log(\cos(\frac{1}{2}(e+fx))) + \log(\sin(\frac{1}{2}(e+fx)))) \sec(e+fx) \sqrt{a(1+\sin(e+fx))} \sqrt{c-c\sin(e+fx)}}{f}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[e + f\*x]\*Sqrt[a + a\*Sin[e + f\*x]]\*Sqrt[c - c\*Sin[e + f\*x]],x]

[Out] ((Log[Cos[(e + f\*x)/2]] + Log[Sin[(e + f\*x)/2]])\*Sec[e + f\*x]\*Sqrt[a\*(1 + Sin[e + f\*x])]\*Sqrt[c - c\*Sin[e + f\*x]])/f

**Maple [A]**

time = 2.62, size = 74, normalized size = 1.61

method	result	size
default	$-\frac{\left(\ln\left(\frac{2}{1+\cos(fx+e)}\right) - \ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)\right) \sqrt{a(1+\sin(fx+e))} \sqrt{-c(\sin(fx+e)-1)}}{f \cos(fx+e)}$	74

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a\*sin(f\*x+e))^(1/2)\*(c-c\*sin(f\*x+e))^(1/2)/sin(f\*x+e),x,method=\_RETURNVERBOSE)

[Out] -1/f\*(ln(2/(1+cos(f\*x+e)))-ln(-(-1+cos(f\*x+e))/sin(f\*x+e)))\*(a\*(1+sin(f\*x+e)))^(1/2)\*(-c\*(sin(f\*x+e)-1))^(1/2)/cos(f\*x+e)

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*sin(f\*x+e))^(1/2)\*(c-c\*sin(f\*x+e))^(1/2)/sin(f\*x+e),x, algorithm="maxima")

[Out] integrate(sqrt(a\*sin(f\*x + e) + a)\*sqrt(-c\*sin(f\*x + e) + c)/sin(f\*x + e), x)

**Fricas [A]**

time = 0.57, size = 216, normalized size = 4.70

$$\left[ \frac{\sqrt{ac} \log\left(\frac{4(256\operatorname{ac}\cos(fx+e)^3 - 512\operatorname{ac}\cos(fx+e)^2 + 387\operatorname{ac}\cos(fx+e) + 256\cos(fx+e)^4 - 512\cos(fx+e)^2 + 175))\sqrt{ac}\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}}{\cos(fx+e)^2 - \cos(fx+e)}\right)}{2f}, -\frac{\sqrt{-ac} \arctan\left(\frac{\sqrt{-ac}(16\cos(fx+e)^2 - 7)\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}}{16\operatorname{ac}\cos(fx+e)^3 - 25\operatorname{ac}\cos(fx+e)}\right)}{f} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*sin(f\*x+e))^(1/2)\*(c-c\*sin(f\*x+e))^(1/2)/sin(f\*x+e),x, algorithm="fricas")

[Out] [1/2\*sqrt(a\*c)\*log(4\*(256\*a\*c\*cos(f\*x + e)^5 - 512\*a\*c\*cos(f\*x + e)^3 + 337\*a\*c\*cos(f\*x + e) + (256\*cos(f\*x + e)^4 - 512\*cos(f\*x + e)^2 + 175)\*sqrt(a\*c)\*sqrt(a\*sin(f\*x + e) + a)\*sqrt(-c\*sin(f\*x + e) + c))/(cos(f\*x + e)^3 - cos(f\*x + e)))/f, -sqrt(-a\*c)\*arctan(sqrt(-a\*c)\*(16\*cos(f\*x + e)^2 - 7)\*sqrt(a\*sin(f\*x + e) + a)\*sqrt(-c\*sin(f\*x + e) + c)/(16\*a\*c\*cos(f\*x + e)^3 - 25\*a\*c\*cos(f\*x + e)))/f]

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a(\sin(e+fx)+1)} \sqrt{-c(\sin(e+fx)-1)}}{\sin(e+fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*sin(f\*x+e))\*\*(1/2)\*(c-c\*sin(f\*x+e))\*\*(1/2)/sin(f\*x+e),x)

[Out] Integral(sqrt(a\*(sin(e + f\*x) + 1))\*sqrt(-c\*(sin(e + f\*x) - 1))/sin(e + f\*x), x)

**Giac [A]**

time = 0.46, size = 60, normalized size = 1.30

$$\frac{\sqrt{a} \sqrt{c} \log\left(\left|2 \cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^2 - 1\right|\right) \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*sin(f\*x+e))^(1/2)\*(c-c\*sin(f\*x+e))^(1/2)/sin(f\*x+e),x, algorithm="giac")

[Out] -sqrt(a)\*sqrt(c)\*log(abs(2\*cos(-1/4\*pi + 1/2\*f\*x + 1/2\*e)^2 - 1))\*sgn(cos(-1/4\*pi + 1/2\*f\*x + 1/2\*e))\*sgn(sin(-1/4\*pi + 1/2\*f\*x + 1/2\*e))/f

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a + a \sin(e + f x)} \sqrt{c - c \sin(e + f x)}}{\sin(e + f x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + a\*sin(e + f\*x))^(1/2)\*(c - c\*sin(e + f\*x))^(1/2))/sin(e + f\*x),x)

[Out] int(((a + a\*sin(e + f\*x))^(1/2)\*(c - c\*sin(e + f\*x))^(1/2))/sin(e + f\*x), x)



$$3.20 \quad \int \frac{\csc(e+fx) \sqrt{a + a \sin(e + fx)}}{\sqrt{c - c \sin(e + fx)}} dx$$

**Optimal.** Leaf size=102

$$-\frac{a \cos(e + fx) \log(1 - \sin(e + fx))}{f \sqrt{a + a \sin(e + fx)} \sqrt{c - c \sin(e + fx)}} + \frac{\log(\sin(e + fx)) \sec(e + fx) \sqrt{a + a \sin(e + fx)} \sqrt{c - c \sin(e + fx)}}{cf}$$

[Out] -a\*cos(f\*x+e)\*ln(1-sin(f\*x+e))/f/(a+a\*sin(f\*x+e))^(1/2)/(c-c\*sin(f\*x+e))^(1/2)+ln(sin(f\*x+e))\*sec(f\*x+e)\*(a+a\*sin(f\*x+e))^(1/2)\*(c-c\*sin(f\*x+e))^(1/2)/c/f

**Rubi [A]**

time = 0.29, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {3021, 2816, 2746, 31, 3027, 3556}

$$\frac{\sec(e + fx) \sqrt{a \sin(e + fx) + a} \sqrt{c - c \sin(e + fx)} \log(\sin(e + fx))}{cf} - \frac{a \cos(e + fx) \log(1 - \sin(e + fx))}{f \sqrt{a \sin(e + fx) + a} \sqrt{c - c \sin(e + fx)}}$$

Antiderivative was successfully verified.

[In] Int[(Csc[e + f\*x]\*Sqrt[a + a\*Sin[e + f\*x]])/Sqrt[c - c\*Sin[e + f\*x]],x]

[Out] -((a\*Cos[e + f\*x]\*Log[1 - Sin[e + f\*x]])/(f\*Sqrt[a + a\*Sin[e + f\*x]]\*Sqrt[c - c\*Sin[e + f\*x]]) + (Log[Sin[e + f\*x]]\*Sec[e + f\*x]\*Sqrt[a + a\*Sin[e + f\*x]]\*Sqrt[c - c\*Sin[e + f\*x]])/(c\*f)

**Rule 31**

Int[((a\_) + (b\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

**Rule 2746**

Int[cos[(e\_) + (f\_.)\*(x\_)]^(p\_.)\*((a\_) + (b\_.)\*sin[(e\_) + (f\_.)\*(x\_)])^(m\_.), x\_Symbol] := Dist[1/(b^p\*f), Subst[Int[(a + x)^(m + (p - 1)/2)\*(a - x)^(n - (p - 1)/2), x], x, b\*Sin[e + f\*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2])

**Rule 2816**

Int[Sqrt[(a\_) + (b\_.)\*sin[(e\_) + (f\_.)\*(x\_)]]/Sqrt[(c\_) + (d\_.)\*sin[(e\_) + (f\_.)\*(x\_)]], x\_Symbol] := Dist[a\*c\*(Cos[e + f\*x]/(Sqrt[a + b\*Sin[e + f\*x]]\*Sqrt[c + d\*Sin[e + f\*x]]), Int[Cos[e + f\*x]/(c + d\*Sin[e + f\*x]), x], x

] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 - b^2, 0]

### Rule 3021

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/(sin[(e_) + (f_)*(x_)]*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[-d/c, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/c, Int[Sqrt[a + b*Sin[e + f*x]]*(Sqrt[c + d*Sin[e + f*x]]/Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[b*c + a*d, 0]
```

### Rule 3027

```
Int[(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]/sin[(e_) + (f_)*(x_)], x_Symbol] := Dist[Sqrt[a + b*Sin[e + f*x]]*(Sqrt[c + d*Sin[e + f*x]]/Cos[e + f*x]), Int[Cot[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]
```

### Rule 3556

```
Int[tan[(c_) + (d_)*(x_)], x_Symbol] := Simp[-Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

### Rubi steps

$$\begin{aligned} \int \frac{\csc(e+fx)\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}} dx &= \frac{\int \csc(e+fx)\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)} dx}{c} + \int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}} dx \\ &= \frac{(a\cos(e+fx)) \int \frac{\cos(e+fx)}{c-c\sin(e+fx)} dx}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{(\sec(e+fx)\sqrt{a+a\sin(e+fx)})}{\sqrt{c-c\sin(e+fx)}} \\ &= \frac{\log(\sin(e+fx))\sec(e+fx)\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}{cf} \\ &= -\frac{a\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{\log(\sin(e+fx))\sec(e+fx)}{\sqrt{c-c\sin(e+fx)}} \end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 0.98, size = 145, normalized size = 1.42

$$\frac{\sqrt{2}(1+ie^{i(e+fx)})\left(\log(-1+e^{i(e+fx)})-2\log((-i+e^{i(e+fx)})f)+\log((1+e^{i(e+fx)})f)\right)\sqrt{a(1+\sin(e+fx))}}{\sqrt{ice^{-i(e+fx)}(-i+e^{i(e+fx)})^2}(i+e^{i(e+fx)})f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/Sqrt[c - c*Sin[e + f*x]],
x]
```

```
[Out] (Sqrt[2]*(1 + I*E^(I*(e + f*x)))*(Log[-1 + E^(I*(e + f*x))] - 2*Log[(-I + E
^(I*(e + f*x))]*f] + Log[(1 + E^(I*(e + f*x)))*f])*Sqrt[a*(1 + Sin[e + f*x]
)))/(Sqrt[(I*c*(-I + E^(I*(e + f*x)))^2)/E^(I*(e + f*x))]*(I + E^(I*(e + f*
x)))*f)
```

**Maple [A]**

time = 0.26, size = 111, normalized size = 1.09

method	result	size
default	$\frac{\sqrt{a(1 + \sin(fx + e))} (-1 + \cos(fx + e) + \sin(fx + e)) \left( 2 \ln \left( -\frac{-1 + \cos(fx + e) + \sin(fx + e)}{\sin(fx + e)} \right) - \ln \left( -\frac{-1 + \cos(fx + e)}{\sin(fx + e)} \right) \right)}{f(-1 + \cos(fx + e) - \sin(fx + e)) \sqrt{-c(\sin(fx + e) - 1)}}$	111

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)/sin(f*x+e)/(c-c*sin(f*x+e))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/f*(a*(1+sin(f*x+e))^(1/2)*(-1+cos(f*x+e)+sin(f*x+e))*(2*ln(-(-1+cos(f*x+
e)+sin(f*x+e))/sin(f*x+e))-ln(-(-1+cos(f*x+e))/sin(f*x+e)))/(-1+cos(f*x+e)-
sin(f*x+e))/(-c*(sin(f*x+e)-1))^(1/2)
```

**Maxima [A]**

time = 0.51, size = 63, normalized size = 0.62

$$\frac{2\sqrt{a} \log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{\sqrt{c}} - \frac{\sqrt{a} \log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{\sqrt{c}}$$

$f$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/sin(f*x+e)/(c-c*sin(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] (2*sqrt(a)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/sqrt(c) - sqrt(a)*log(s
in(f*x + e)/(cos(f*x + e) + 1))/sqrt(c))/f
```

**Fricas [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*sin(f\*x+e))^(1/2)/sin(f\*x+e)/(c-c\*sin(f\*x+e))^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(a\*sin(f\*x + e) + a)\*sqrt(-c\*sin(f\*x + e) + c)/(c\*cos(f\*x + e)^2 + c\*sin(f\*x + e) - c), x)

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a(\sin(e+fx)+1)}}{\sqrt{-c(\sin(e+fx)-1)} \sin(e+fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*sin(f\*x+e))\*\*(1/2)/sin(f\*x+e)/(c-c\*sin(f\*x+e))\*\*(1/2),x)

[Out] Integral(sqrt(a\*(sin(e + f\*x) + 1))/(sqrt(-c\*(sin(e + f\*x) - 1))\*sin(e + f\*x)), x)

**Giac [A]**

time = 0.44, size = 1, normalized size = 0.01

0

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*sin(f\*x+e))^(1/2)/sin(f\*x+e)/(c-c\*sin(f\*x+e))^(1/2),x, algorithm="giac")

[Out] 0

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+a \sin(e+fx)}}{\sin(e+fx) \sqrt{c-c \sin(e+fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a\*sin(e + f\*x))^(1/2)/(sin(e + f\*x)\*(c - c\*sin(e + f\*x))^(1/2)),x)

[Out] int((a + a\*sin(e + f\*x))^(1/2)/(sin(e + f\*x)\*(c - c\*sin(e + f\*x))^(1/2)), x)

$$3.21 \quad \int \frac{\csc(e+fx) \sqrt{c - c \sin(e + fx)}}{\sqrt{a + a \sin(e + fx)}} dx$$

**Optimal.** Leaf size=100

$$-\frac{c \cos(e + fx) \log(1 + \sin(e + fx))}{f \sqrt{a + a \sin(e + fx)} \sqrt{c - c \sin(e + fx)}} + \frac{\log(\sin(e + fx)) \sec(e + fx) \sqrt{a + a \sin(e + fx)} \sqrt{c - c \sin(e + fx)}}{af}$$

[Out] -c\*cos(f\*x+e)\*ln(1+sin(f\*x+e))/f/(a+a\*sin(f\*x+e))^(1/2)/(c-c\*sin(f\*x+e))^(1/2)+ln(sin(f\*x+e))\*sec(f\*x+e)\*(a+a\*sin(f\*x+e))^(1/2)\*(c-c\*sin(f\*x+e))^(1/2)/a/f

**Rubi [A]**

time = 0.30, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {3021, 2816, 2746, 31, 3027, 3556}

$$\frac{\sec(e + fx) \sqrt{a \sin(e + fx) + a} \sqrt{c - c \sin(e + fx)} \log(\sin(e + fx))}{af} - \frac{c \cos(e + fx) \log(\sin(e + fx) + 1)}{f \sqrt{a \sin(e + fx) + a} \sqrt{c - c \sin(e + fx)}}$$

Antiderivative was successfully verified.

[In] Int[(Csc[e + f\*x]\*Sqrt[c - c\*Sin[e + f\*x]])/Sqrt[a + a\*Sin[e + f\*x]],x]

[Out] -((c\*cos[e + f\*x]\*Log[1 + Sin[e + f\*x]])/(f\*Sqrt[a + a\*Sin[e + f\*x]]\*Sqrt[c - c\*Sin[e + f\*x]])) + (Log[Sin[e + f\*x]]\*Sec[e + f\*x]\*Sqrt[a + a\*Sin[e + f\*x]]\*Sqrt[c - c\*Sin[e + f\*x]])/(a\*f)

**Rule 31**

Int[((a\_) + (b\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

**Rule 2746**

Int[cos[(e\_) + (f\_.)\*(x\_)]^(p\_.)\*((a\_) + (b\_.)\*sin[(e\_) + (f\_.)\*(x\_)])^(m\_.), x\_Symbol] := Dist[1/(b^p\*f), Subst[Int[(a + x)^(m + (p - 1)/2)\*(a - x)^(p - 1/2), x], x, b\*Sin[e + f\*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2])

**Rule 2816**

Int[Sqrt[(a\_) + (b\_.)\*sin[(e\_) + (f\_.)\*(x\_)]]/Sqrt[(c\_) + (d\_.)\*sin[(e\_) + (f\_.)\*(x\_)]], x\_Symbol] := Dist[a\*c\*(Cos[e + f\*x]/(Sqrt[a + b\*Sin[e + f\*x]]\*Sqrt[c + d\*Sin[e + f\*x]])), Int[Cos[e + f\*x]/(c + d\*Sin[e + f\*x]), x], x

] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b\*c + a\*d, 0] && EqQ[a^2 - b^2, 0]

### Rule 3021

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/(sin[(e_) + (f_)*(x_)]*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[-d/c, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/c, Int[Sqrt[a + b*Sin[e + f*x]]*(Sqrt[c + d*Sin[e + f*x]]/Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[b*c + a*d, 0]
```

### Rule 3027

```
Int[(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]/sin[(e_) + (f_)*(x_)], x_Symbol] := Dist[Sqrt[a + b*Sin[e + f*x]]*(Sqrt[c + d*Sin[e + f*x]]/Cos[e + f*x]), Int[Cot[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]
```

### Rule 3556

```
Int[tan[(c_) + (d_)*(x_)], x_Symbol] := Simp[-Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

### Rubi steps

$$\begin{aligned} \int \frac{\csc(e+fx)\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}} dx &= \frac{\int \csc(e+fx)\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)} dx}{a} - \int \frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}} dx \\ &= -\frac{(ac\cos(e+fx))\int \frac{\cos(e+fx)}{a+a\sin(e+fx)} dx}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{(\sec(e+fx)\sqrt{a+a\sin(e+fx)})}{\sqrt{c-c\sin(e+fx)}} \\ &= \frac{\log(\sin(e+fx))\sec(e+fx)\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}{af} \\ &= -\frac{c\cos(e+fx)\log(1+\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{\log(\sin(e+fx))\sec(e+fx)}{\sqrt{c-c\sin(e+fx)}} \end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 1.00, size = 148, normalized size = 1.48

$$\frac{\sqrt{2}(1-ie^{i(e+fx)})\left(\log((-1+e^{i(e+fx)})f)-2\log((i+e^{i(e+fx)})f)+\log((1+e^{i(e+fx)})f)\right)\sqrt{c-c\sin(e+fx)}}{(-i+e^{i(e+fx)})\sqrt{-iae^{-i(e+fx)}(i+e^{i(e+fx)})^2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/Sqrt[a + a*Sin[e + f*x]],
x]
```

```
[Out] (Sqrt[2]*(1 - I*E^(I*(e + f*x)))*(Log[(-1 + E^(I*(e + f*x)))*f] - 2*Log[(I
+ E^(I*(e + f*x)))*f] + Log[(1 + E^(I*(e + f*x)))*f])*Sqrt[c - c*Sin[e + f*
x]])/((-I + E^(I*(e + f*x)))*Sqrt[((-I)*a*(I + E^(I*(e + f*x)))^2]/E^(I*(e
+ f*x)))*f)
```

**Maple [A]**

time = 0.25, size = 113, normalized size = 1.13

method	result	size
default	$\frac{(-1+\cos(fx+e)-\sin(fx+e))\sqrt{-c(\sin(fx+e)-1)}\left(2\ln\left(-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}\right)-\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)\right)}{f\sqrt{a(1+\sin(fx+e))}(-1+\cos(fx+e)+\sin(fx+e))}$	113

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(1/2)/sin(f*x+e)/(a+a*sin(f*x+e))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/f*(-1+cos(f*x+e)-sin(f*x+e))*(-c*(sin(f*x+e)-1))^(1/2)*(2*ln(-(-1+cos(f*x
+e)-sin(f*x+e))/sin(f*x+e))-ln(-(-1+cos(f*x+e))/sin(f*x+e)))/(a*(1+sin(f*x+
e)))^(1/2)/(-1+cos(f*x+e)+sin(f*x+e))
```

**Maxima [A]**

time = 0.54, size = 63, normalized size = 0.63

$$\frac{2\sqrt{c}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{\sqrt{a}} - \frac{\sqrt{c}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{\sqrt{a}}$$

$f$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(1/2)/sin(f*x+e)/(a+a*sin(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] (2*sqrt(c)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/sqrt(a) - sqrt(c)*log(s
in(f*x + e)/(cos(f*x + e) + 1))/sqrt(a))/f
```

**Fricas [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c\*sin(f\*x+e))^(1/2)/sin(f\*x+e)/(a+a\*sin(f\*x+e))^(1/2),x, algorithm="fricas")

[Out] integral(-sqrt(a\*sin(f\*x + e) + a)\*sqrt(-c\*sin(f\*x + e) + c)/(a\*cos(f\*x + e)^2 - a\*sin(f\*x + e) - a), x)

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c(\sin(e + fx) - 1)}}{\sqrt{a(\sin(e + fx) + 1)} \sin(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c\*sin(f\*x+e))\*\*(1/2)/sin(f\*x+e)/(a+a\*sin(f\*x+e))\*\*(1/2),x)

[Out] Integral(sqrt(-c\*(sin(e + f\*x) - 1))/(sqrt(a\*(sin(e + f\*x) + 1))\*sin(e + f\*x)), x)

**Giac [A]**

time = 0.47, size = 117, normalized size = 1.17

$$\frac{\sqrt{2} \sqrt{a} \sqrt{c} \left( \frac{\sqrt{2} \log(-\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)^2 + 1)}{\operatorname{asgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))} - \frac{\sqrt{2} \log(|2 \sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)^2 - 1|)}{\operatorname{asgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))} \right) \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c\*sin(f\*x+e))^(1/2)/sin(f\*x+e)/(a+a\*sin(f\*x+e))^(1/2),x, algorithm="giac")

[Out] 1/2\*sqrt(2)\*sqrt(a)\*sqrt(c)\*(sqrt(2)\*log(-sin(-1/4\*pi + 1/2\*f\*x + 1/2\*e)^2 + 1)/(a\*sgn(cos(-1/4\*pi + 1/2\*f\*x + 1/2\*e))) - sqrt(2)\*log(abs(2\*sin(-1/4\*pi + 1/2\*f\*x + 1/2\*e)^2 - 1))/(a\*sgn(cos(-1/4\*pi + 1/2\*f\*x + 1/2\*e))))\*sgn(sin(-1/4\*pi + 1/2\*f\*x + 1/2\*e))/f

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c - c \sin(e + fx)}}{\sin(e + fx) \sqrt{a + a \sin(e + fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c - c\*sin(e + f\*x))^(1/2)/(sin(e + f\*x)\*(a + a\*sin(e + f\*x))^(1/2)),x)

[Out] int((c - c\*sin(e + f\*x))^(1/2)/(sin(e + f\*x)\*(a + a\*sin(e + f\*x))^(1/2)), x)



$$3.22 \quad \int \frac{\csc(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} dx$$

Optimal. Leaf size=46

$$\frac{\cos(e+fx)\log(\tan(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

[Out]  $\cos(f*x+e)*\ln(\tan(f*x+e))/f/(a+a*\sin(f*x+e))^{(1/2)}/(c-c*\sin(f*x+e))^{(1/2)}$

Rubi [A]

time = 0.12, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$ , Rules used = {3025, 2700, 29}

$$\frac{\cos(e+fx)\log(\tan(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Csc}[e + f*x]/(\text{Sqrt}[a + a*\text{Sin}[e + f*x]]*\text{Sqrt}[c - c*\text{Sin}[e + f*x]]),x]$

[Out]  $(\text{Cos}[e + f*x]*\text{Log}[\text{Tan}[e + f*x]])/(f*\text{Sqrt}[a + a*\text{Sin}[e + f*x]]*\text{Sqrt}[c - c*\text{Sin}[e + f*x]])$

Rule 29

$\text{Int}[(x_{-})^{(-1)}, x_{\text{Symbol}}] \text{ :> Simp}[\text{Log}[x], x]$

Rule 2700

$\text{Int}[\text{csc}[(e_{-}) + (f_{-})*(x_{-})]^{(m_{-})}*\text{sec}[(e_{-}) + (f_{-})*(x_{-})]^{(n_{-})}, x_{\text{Symbol}}] \text{ :> Dist}[1/f, \text{Subst}[\text{Int}[(1 + x^2)^{((m + n)/2 - 1)}/x^m, x], x, \text{Tan}[e + f*x]], x] \text{ /; FreeQ}\{e, f\}, x] \ \&\& \ \text{IntegersQ}[m, n, (m + n)/2]$

Rule 3025

$\text{Int}[1/(\sin[(e_{-}) + (f_{-})*(x_{-})]*\text{Sqrt}[(a_{-}) + (b_{-})*\sin[(e_{-}) + (f_{-})*(x_{-})]]*\text{Sqrt}[(c_{-}) + (d_{-})*\sin[(e_{-}) + (f_{-})*(x_{-})]]), x_{\text{Symbol}}] \text{ :> Dist}[\text{Cos}[e + f*x]/(\text{Sqrt}[a + b*\text{Sin}[e + f*x]]*\text{Sqrt}[c + d*\text{Sin}[e + f*x]]), \text{Int}[1/(\text{Cos}[e + f*x]*\text{Sin}[e + f*x]), x], x] \text{ /; FreeQ}\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{EqQ}[c^2 - d^2, 0]$

Rubi steps

$$\int \frac{\csc(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} dx = \frac{\cos(e+fx) \int \csc(e+fx) \sec(e+fx) dx}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{\cos(e+fx) \text{Subst}\left(\int \frac{1}{x} dx, x, \tan(e+fx)\right)}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{\cos(e+fx) \log(\tan(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

**Mathematica [A]**

time = 0.13, size = 63, normalized size = 1.37

$$\frac{(\log(\cos(e+fx)) - \log(\sin(e+fx))) \sec(e+fx) \sqrt{a(1+\sin(e+fx))} \sqrt{c-c\sin(e+fx)}}{acf}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[e + f\*x]/(Sqrt[a + a\*Sin[e + f\*x]]\*Sqrt[c - c\*Sin[e + f\*x]]), x]

[Out] -(((Log[Cos[e + f\*x]] - Log[Sin[e + f\*x]])\*Sec[e + f\*x]\*Sqrt[a\*(1 + Sin[e + f\*x]])\*Sqrt[c - c\*Sin[e + f\*x]])/(a\*c\*f))

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 110 vs. 2(42) = 84.

time = 0.25, size = 111, normalized size = 2.41

method	result	size
default	$-\frac{\left(\ln\left(\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)+\ln\left(\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}\right)-\ln\left(\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)\right)\cos(fx+e)}{f\sqrt{a(1+\sin(fx+e))}\sqrt{-c(\sin(fx+e)-1)}}$	111

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/sin(f\*x+e)/(a+a\*sin(f\*x+e))^(1/2)/(c-c\*sin(f\*x+e))^(1/2),x,method=\_RETURNVERBOSE)

[Out] -1/f\*(ln(-(-1+cos(f\*x+e)+sin(f\*x+e))/sin(f\*x+e))+ln(-(-1+cos(f\*x+e)-sin(f\*x+e))/sin(f\*x+e))-ln(-(-1+cos(f\*x+e))/sin(f\*x+e)))\*cos(f\*x+e)/(a\*(1+sin(f\*x+e)))^(1/2)/(-c\*(sin(f\*x+e)-1))^(1/2)

**Maxima [C]** Result contains complex when optimal does not.

time = 0.64, size = 202, normalized size = 4.39

$$\frac{(-1)^4 \cos(2fx+2e) \cosh(4\pi \sin(2fx+2e)) \log\left(\frac{16(\cos(2fx+2e)^2 + \sin(2fx+2e)^2 + 2\cos(2fx+2e)+1)}{ac|e^{2i(fx+2e)}-1|^2}\right) - 2i(-1)^4 \cos(2fx+2e) \arctan\left(\frac{4\sin(2fx+2e)}{\sqrt{a}\sqrt{c|e^{2i(fx+2e)}-1|}}, \frac{4(\cos(2fx+2e)+1)}{\sqrt{a}\sqrt{c|e^{2i(fx+2e)}-1|}}\right) \sinh(4\pi \sin(2fx+2e))}{2\sqrt{a}\sqrt{c}f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sin(f\*x+e)/(a+a\*sin(f\*x+e))^(1/2)/(c-c\*sin(f\*x+e))^(1/2),x, algorithm="maxima")

[Out] 
$$-1/2*((-1)^{(4*\cos(2*f*x + 2*e))*\cosh(4*\pi*\sin(2*f*x + 2*e))*\log(16*(\cos(2*f*x + 2*e)^2 + \sin(2*f*x + 2*e)^2 + 2*\cos(2*f*x + 2*e) + 1)/(a*c*\abs(e^{(2*I*f*x + 2*I*e)} - 1)^2)) - 2*I*(-1)^{(4*\cos(2*f*x + 2*e))*\arctan(2*\sin(2*f*x + 2*e)/(\sqrt{a}*\sqrt{c}*\abs(e^{(2*I*f*x + 2*I*e)} - 1))), 4*(\cos(2*f*x + 2*e) + 1)/(\sqrt{a}*\sqrt{c}*\abs(e^{(2*I*f*x + 2*I*e)} - 1)))*\sinh(4*\pi*\sin(2*f*x + 2*e)))/(\sqrt{a}*\sqrt{c}*f)$$

**Fricas** [A]

time = 0.48, size = 205, normalized size = 4.46

$$\left[ \frac{\sqrt{ac} \log\left(\frac{4(2ac\cos(fx+e)^5 - 2ac\cos(fx+e)^3 + ac\cos(fx+e) - \sqrt{ac}(2\cos(fx+e)^2 - 1))\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}}{\cos(fx+e)^5 - \cos(fx+e)^3}\right)}{2acf}, \frac{\sqrt{-ac} \arctan\left(\frac{\sqrt{-ac}\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}}{2ac\cos(fx+e)^3 - ac\cos(fx+e)}\right)}{acf} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sin(f\*x+e)/(a+a\*sin(f\*x+e))^(1/2)/(c-c\*sin(f\*x+e))^(1/2),x, algorithm="fricas")

[Out] 
$$[1/2*\sqrt{a*c}*\log(-4*(2*a*c*\cos(f*x + e)^5 - 2*a*c*\cos(f*x + e)^3 + a*c*\cos(f*x + e) - \sqrt{a*c}*(2*\cos(f*x + e)^2 - 1)*\sqrt{a*\sin(f*x + e) + a}*\sqrt{-c*\sin(f*x + e) + c}))/(\cos(f*x + e)^5 - \cos(f*x + e)^3))/(\sqrt{a*c}*f), \sqrt{-a*c}*\arctan(\sqrt{-a*c}*\sqrt{a*\sin(f*x + e) + a}*\sqrt{-c*\sin(f*x + e) + c}/(2*a*c*\cos(f*x + e)^3 - a*c*\cos(f*x + e)))/(\sqrt{a*c}*f)]$$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a(\sin(e + fx) + 1)} \sqrt{-c(\sin(e + fx) - 1)} \sin(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sin(f\*x+e)/(a+a\*sin(f\*x+e))^(1/2)/(c-c\*sin(f\*x+e))^(1/2),x)

[Out] Integral(1/(sqrt(a\*(sin(e + f\*x) + 1))\*sqrt(-c\*(sin(e + f\*x) - 1))\*sin(e + f\*x)), x)

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(46) = 92.

time = 0.52, size = 187, normalized size = 4.07

$$\frac{\sqrt{a} \sqrt{c} \left( \frac{\log\left(-\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^2 + 1\right)}{\operatorname{acsgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)} - \frac{2 \log\left(\left|2 \cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^2 - 1\right|\right)}{\operatorname{acsgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)} + \frac{2 \log\left(\left|\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right|\right)}{\operatorname{acsgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)} \right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(f*x+e)/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(a)*sqrt(c)*(log(-cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 1)/(a*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) - 2*log(abs(2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1))/(a*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + 2*log(abs(cos(-1/4*pi + 1/2*f*x + 1/2*e)))/(a*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sin(e + f x) \sqrt{a + a \sin(e + f x)} \sqrt{c - c \sin(e + f x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(1/2)), x)
```

```
[Out] int(1/(sin(e + f*x)*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(1/2)), x)
```

$$3.23 \quad \int \frac{\csc(e+fx) \sqrt{a + a \sin(e + fx)}}{c+d \sin(e+fx)} dx$$

**Optimal.** Leaf size=105

$$\frac{2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{a + a \sin(e + fx)}}\right)}{cf} + \frac{2\sqrt{a} \sqrt{d} \tanh^{-1}\left(\frac{\sqrt{a} \sqrt{d} \cos(e+fx)}{\sqrt{c+d} \sqrt{a + a \sin(e + fx)}}\right)}{c\sqrt{c+d} f}$$

[Out]  $-2*\operatorname{arctanh}(\cos(f*x+e)*a^{(1/2)/(a+a*\sin(f*x+e))^{(1/2)}}*a^{(1/2)}/c/f+2*\operatorname{arctanh}(\cos(f*x+e)*a^{(1/2)*d^{(1/2)/(c+d)^{(1/2)/(a+a*\sin(f*x+e))^{(1/2)}}*a^{(1/2)*d^{(1/2)}/c/f/(c+d)^{(1/2)}}))$

**Rubi [A]**

time = 0.20, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$ , Rules used = {3013, 2852, 212, 214}

$$\frac{2\sqrt{a} \sqrt{d} \tanh^{-1}\left(\frac{\sqrt{a} \sqrt{d} \cos(e+fx)}{\sqrt{c+d} \sqrt{a \sin(e + fx) + a}}\right)}{cf\sqrt{c+d}} - \frac{2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{a \sin(e + fx) + a}}\right)}{cf}$$

Antiderivative was successfully verified.

[In] `Int[(Csc[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c + d*Sin[e + f*x]),x]`

[Out]  $(-2*\operatorname{Sqrt}[a]*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Cos}[e + f*x])/\operatorname{Sqrt}[a + a*\operatorname{Sin}[e + f*x]])]/(c*f) + (2*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[d]*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[d]*\operatorname{Cos}[e + f*x])/(\operatorname{Sqrt}[c + d]*\operatorname{Sqrt}[a + a*\operatorname{Sin}[e + f*x]])]/(c*\operatorname{Sqrt}[c + d]*f)$

Rule 212

`Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

Rule 214

`Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

Rule 2852

`Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x])], x] /; FreeQ[{a, b, c, d,`

`e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]`

### Rule 3013

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/(sin[(e_) + (f_)*(x_)]*((c_) + (d_)*sin[(e_) + (f_)*(x_)])), x_Symbol] := Dist[1/c, Int[Sqrt[a + b*Sin[e + f*x]]/Sin[e + f*x], x], x] - Dist[d/c, Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

### Rubi steps

$$\int \frac{\csc(e+fx)\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)} dx = \frac{\int \csc(e+fx)\sqrt{a+a\sin(e+fx)} dx}{c} - \frac{d \int \frac{\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)} dx}{c}$$

$$= -\frac{(2a)\text{Subst}\left(\int \frac{1}{a-x^2} dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{cf} + \frac{(2ad)\text{Subst}\left(\int \frac{1}{c+dx} dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{c}$$

$$= -\frac{2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{cf} + \frac{2\sqrt{a}\sqrt{d} \tanh^{-1}\left(\frac{\sqrt{c+d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{c\sqrt{c+d}}$$

**Mathematica [C]** Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 23.96, size = 746, normalized size = 7.10

---

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c + d*Sin[e + f*x]),x]
```

```
[Out] ((-1/8 + I/8)*((4 + 4*I)*Sqrt[c + d]*(Log[1 + Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[1 - Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) + Sqrt[d]*RootSum[-d + (2*I)*c*E^(I*e)*#1^2 + d*E^((2*I)*e)*#1^4 & , ((1 + I)*d*Sqrt[E^((-I)*e)]*f*x - (2 - 2*I)*d*Sqrt[E^((-I)*e)]*Log[E^((I/2)*f*x) - #1] - I*Sqrt[d]*Sqrt[c + d]*f*x*#1 + 2*Sqrt[d]*Sqrt[c + d]*Log[E^((I/2)*f*x) - #1]*#1 + ((1 - I)*c*f*x*#1^2)/Sqrt[E^((-I)*e)] + ((2 + 2*I)*c*Log[E^((I/2)*f*x) - #1]*#1^2)/Sqrt[E^((-I)*e)] - Sqrt[d]*Sqrt[c + d]*E^(I*e)*f*x*#1^3 - (2*I)*Sqrt[d]*Sqrt[c + d]*E^(I*e)*Log[E^((I/2)*f*x) - #1]*#1^3)/((-I)*d - c*E^(I*e)*#1^2) & ]*(Cos[e/2] + I*Sin[e/2]) + Sqrt[d]*RootSum[-d + (2*I)*c*E^(I*e)*#1^2 + d*E^((2*I)*e)*#1^4 & , ((1 - I)*d*Sqrt[E^((-I)*e)]*f*x + (2 + 2*I)*d*Sqrt[c + d]*Sqrt[E^((-I)*e)]*Log[E^((I/2)*f*x) - #1] + I*Sqrt[d]*Sqrt[c + d]*Log[E^((I/2)*f*x) - #1]*#1 + ((1 - I)*c*f*x*#1^2)/Sqrt[E^((-I)*e)] + ((2 + 2*I)*c*Log[E^((I/2)*f*x) - #1]*#1^2)/Sqrt[E^((-I)*e)] - Sqrt[d]*Sqrt[c + d]*E^(I*e)*f*x*#1^3 - (2*I)*Sqrt[d]*Sqrt[c + d]*E^(I*e)*Log[E^((I/2)*f*x) - #1]*#1^3)/((-I)*d - c*E^(I*e)*#1^2) & ]*(Cos[e/2] + I*Sin[e/2])
```

$$E^{(-I)e}] * \text{Log}[E^{(I/2)f*x} - \#1] + \text{Sqrt}[d] * \text{Sqrt}[c + d] * f * x * \#1 + (2 * I) * \text{Sqrt}[d] * \text{Sqrt}[c + d] * \text{Log}[E^{(I/2)f*x} - \#1] * \#1 - ((1 + I) * c * f * x * \#1^2) / \text{Sqrt}[E^{(-I)e}] + ((2 - 2 * I) * c * \text{Log}[E^{(I/2)f*x} - \#1] * \#1^2) / \text{Sqrt}[E^{(-I)e}] - I * \text{Sqrt}[d] * \text{Sqrt}[c + d] * E^{Ie} * f * x * \#1^3 + 2 * \text{Sqrt}[d] * \text{Sqrt}[c + d] * E^{Ie} * \text{Log}[E^{(I/2)f*x} - \#1] * \#1^3 / (d - I * c * E^{Ie} * \#1^2) * (\text{Cos}[e/2] + I * \text{Sin}[e/2]) * \text{Sqrt}[a * (1 + \text{Sin}[e + f * x])] / (c * \text{Sqrt}[c + d] * f * (\text{Cos}[(e + f * x)/2] + \text{Sin}[(e + f * x)/2]))$$

**Maple [A]**

time = 9.70, size = 120, normalized size = 1.14

method	result
default	$\frac{2^{(1+\sin(fx+e))} \sqrt{-a(\sin(fx+e)-1)} \left( d \operatorname{arctanh} \left( \frac{\sqrt{-a(\sin(fx+e)-1)} d}{\sqrt{a(c+d)} d} \right) a^{\frac{3}{2}} - \operatorname{arctanh} \left( \frac{\sqrt{-a(\sin(fx+e)-1)} d}{\sqrt{a(c+d)} d} \right) \right)}{\sqrt{a} c \sqrt{a(c+d)} d \cos(fx+e) \sqrt{a+a \sin(fx+e)} f}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+a*sin(f*x+e))^(1/2)/sin(f*x+e)/(c+d*sin(f*x+e)),x,method=_RETURNVERBOSE)`

[Out]  $2/a^{(1/2)} * (1 + \sin(f*x+e)) * (-a * (\sin(f*x+e) - 1))^{(1/2)} * (d * \operatorname{arctanh}((-a * (\sin(f*x+e) - 1))^{(1/2)} * d / (a * (c+d) * d)) * a^{(3/2)} - \operatorname{arctanh}((-a * (\sin(f*x+e) - 1))^{(1/2)} * d / (a * (c+d) * d))) / c / (a * (c+d) * d)^{(1/2)} / \cos(f*x+e) / (a + a * \sin(f*x+e))^{(1/2)} / f$

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(f*x+e))^(1/2)/sin(f*x+e)/(c+d*sin(f*x+e)),x, algorithm="maxima")`

[Out] `integrate(sqrt(a*sin(f*x + e) + a)/((d*sin(f*x + e) + c)*sin(f*x + e)), x)`

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 261 vs. 2(89) = 178.

time = 0.76, size = 833, normalized size = 7.93

$\frac{2^{(1+\sin(fx+e))} \sqrt{-a(\sin(fx+e)-1)} \left( d \operatorname{arctanh} \left( \frac{\sqrt{-a(\sin(fx+e)-1)} d}{\sqrt{a(c+d)} d} \right) a^{\frac{3}{2}} - \operatorname{arctanh} \left( \frac{\sqrt{-a(\sin(fx+e)-1)} d}{\sqrt{a(c+d)} d} \right) \right)}{\sqrt{a} c \sqrt{a(c+d)} d \cos(fx+e) \sqrt{a+a \sin(fx+e)} f}$
---

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(f*x+e))^(1/2)/sin(f*x+e)/(c+d*sin(f*x+e)),x, algorithm="fricas")`

```
[Out] [1/2*(sqrt(a*d/(c + d))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2
- (6*a*c*d + 7*a*d^2)*cos(f*x + e)^2 + 4*((c*d + d^2)*cos(f*x + e)^2 - c^2
- 4*c*d - 3*d^2 - (c^2 + 3*c*d + 2*d^2)*cos(f*x + e) + (c^2 + 4*c*d + 3*d^
2 + (c*d + d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(a
*d/(c + d)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^2*cos(f*x + e
)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x + e))*sin(f*x
+ e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d
^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) -
c^2 - 2*c*d - d^2)*sin(f*x + e))) + sqrt(a)*log((a*cos(f*x + e)^3 - 7*a*cos
(f*x + e)^2 - 4*(cos(f*x + e)^2 + (cos(f*x + e) + 3)*sin(f*x + e) - 2*cos(f
*x + e) - 3)*sqrt(a*sin(f*x + e) + a)*sqrt(a) - 9*a*cos(f*x + e) + (a*cos(f
*x + e)^2 + 8*a*cos(f*x + e) - a)*sin(f*x + e) - a)/(cos(f*x + e)^3 + cos(f
*x + e)^2 + (cos(f*x + e)^2 - 1)*sin(f*x + e) - cos(f*x + e) - 1)))/(c*f),
1/2*(2*sqrt(-a*d/(c + d))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x +
e) - c - 2*d)*sqrt(-a*d/(c + d))/(a*d*cos(f*x + e))) + sqrt(a)*log((a*cos(f
*x + e)^3 - 7*a*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 + (cos(f*x + e) + 3)*sin
(f*x + e) - 2*cos(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a)*sqrt(a) - 9*a*cos(
f*x + e) + (a*cos(f*x + e)^2 + 8*a*cos(f*x + e) - a)*sin(f*x + e) - a)/(cos
(f*x + e)^3 + cos(f*x + e)^2 + (cos(f*x + e)^2 - 1)*sin(f*x + e) - cos(f*x
+ e) - 1)))/(c*f)]
```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a(\sin(e + fx) + 1)}}{(c + d \sin(e + fx)) \sin(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)/sin(f*x+e)/(c+d*sin(f*x+e)),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))/((c + d*sin(e + f*x))*sin(e + f*x)), x)
```

**Giac [A]**

time = 0.49, size = 148, normalized size = 1.41

$$\frac{\sqrt{2} \left( \frac{2\sqrt{2} \operatorname{arctan}\left(\frac{\sqrt{2} d \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)}{\sqrt{-cd - d^2}}\right) \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}{\sqrt{-cd - d^2} c} - \frac{\sqrt{2} \log\left(\frac{-2\sqrt{2} + 4 \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)}{2\sqrt{2} + 4 \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)}\right) \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}{c} \right) \sqrt{a}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/sin(f*x+e)/(c+d*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] 1/2*sqrt(2)*(2*sqrt(2)*d*arctan(sqrt(2)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)/sq
rt(-c*d - d^2))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))/(sqrt(-c*d - d^2)*c) -
```



```
sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*f*x + 1/2*e))/abs(2*sqrt(2)
) + 4*sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))/
c)*sqrt(a)/f
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a + a \sin(e + f x)}}{\sin(e + f x) (c + d \sin(e + f x))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(1/2)/(sin(e + f*x)*(c + d*sin(e + f*x))),x)
```

```
[Out] int((a + a*sin(e + f*x))^(1/2)/(sin(e + f*x)*(c + d*sin(e + f*x))), x)
```

$$3.24 \quad \int \frac{\csc(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))} dx$$

Optimal. Leaf size=165

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a} cf} + \frac{\sqrt{2} \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{2} \sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a} (c-d)f} - \frac{2d^{3/2} \tanh^{-1}\left(\frac{\sqrt{a} \sqrt{c}}{\sqrt{c+d} \sqrt{a-c}}\right)}{\sqrt{a} c(c-d)\sqrt{c}}$$

[Out]  $-2*\operatorname{arctanh}(\cos(f*x+e)*a^{(1/2)}/(a+a*\sin(f*x+e))^{(1/2)})/c/f/a^{(1/2)}+\operatorname{arctanh}(1/2*\cos(f*x+e)*a^{(1/2)}*2^{(1/2)}/(a+a*\sin(f*x+e))^{(1/2)})*2^{(1/2)}/(c-d)/f/a^{(1/2)}-2*d^{(3/2)}*\operatorname{arctanh}(\cos(f*x+e)*a^{(1/2)}*d^{(1/2)}/(c+d)^{(1/2)}/(a+a*\sin(f*x+e))^{(1/2)})/c/(c-d)/f/a^{(1/2)}/(c+d)^{(1/2)}$

Rubi [A]

time = 0.31, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {3019, 2852, 214, 3064, 2728, 212}

$$\frac{2d^{3/2} \tanh^{-1}\left(\frac{\sqrt{a} \sqrt{d} \cos(e+fx)}{\sqrt{c+d} \sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a} cf(c-d)\sqrt{c+d}} + \frac{\sqrt{2} \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{2} \sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a} f(c-d)} - \frac{2 \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a} cf}$$

Antiderivative was successfully verified.

[In] Int[Csc[e + f\*x]/(Sqrt[a + a\*Sin[e + f\*x]]\*(c + d\*Sin[e + f\*x])),x]

[Out]  $(-2*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Cos}[e+f*x])/(\operatorname{Sqrt}[a+a*\operatorname{Sin}[e+f*x]])]/(\operatorname{Sqrt}[a]*c*f) + (\operatorname{Sqrt}[2]*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Cos}[e+f*x])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a+a*\operatorname{Sin}[e+f*x]])])/(\operatorname{Sqrt}[a]*(c-d)*f) - (2*d^{(3/2)}*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[d]*\operatorname{Cos}[e+f*x])/(\operatorname{Sqrt}[c+d]*\operatorname{Sqrt}[a+a*\operatorname{Sin}[e+f*x]])])/(\operatorname{Sqrt}[a]*c*(c-d)*\operatorname{Sqrt}[c+d]*f))$

Rule 212

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 214

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-a/b, 2]/a)\*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 2728

Int[1/Sqrt[(a\_) + (b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)]], x\_Symbol] := Dist[-2/d, Subst[Int[1/(2\*a - x^2), x], x, b\*(Cos[c + d\*x]/Sqrt[a + b\*Sin[c + d\*x])]],

x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

### Rule 2852

Int[Sqrt[(a\_) + (b\_)\*sin[(e\_) + (f\_)\*(x\_)]]/((c\_) + (d\_)\*sin[(e\_) + (f\_)\*(x\_)]), x\_Symbol] :> Dist[-2\*(b/f), Subst[Int[1/(b\*c + a\*d - d\*x^2), x], x, b\*(Cos[e + f\*x]/Sqrt[a + b\*Sin[e + f\*x])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

### Rule 3019

Int[1/(sin[(e\_) + (f\_)\*(x\_)]\*Sqrt[(a\_) + (b\_)\*sin[(e\_) + (f\_)\*(x\_)]]\*(c\_) + (d\_)\*sin[(e\_) + (f\_)\*(x\_)]), x\_Symbol] :> Dist[d^2/(c\*(b\*c - a\*d)), Int[Sqrt[a + b\*Sin[e + f\*x]]/(c + d\*Sin[e + f\*x]), x], x] + Dist[1/(c\*(b\*c - a\*d)), Int[(b\*c - a\*d - b\*d\*Sin[e + f\*x])/(Sin[e + f\*x]\*Sqrt[a + b\*Sin[e + f\*x])], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 - b^2, 0]

### Rule 3064

Int[((A\_) + (B\_)\*sin[(e\_) + (f\_)\*(x\_)])/(Sqrt[(a\_) + (b\_)\*sin[(e\_) + (f\_)\*(x\_)]]\*(c\_) + (d\_)\*sin[(e\_) + (f\_)\*(x\_)]), x\_Symbol] :> Dist[(A\*b - a\*B)/(b\*c - a\*d), Int[1/Sqrt[a + b\*Sin[e + f\*x]], x], x] + Dist[(B\*c - A\*d)/(b\*c - a\*d), Int[Sqrt[a + b\*Sin[e + f\*x]]/(c + d\*Sin[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

### Rubi steps

$$\begin{aligned}
 \int \frac{\csc(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))} dx &= \frac{\int \frac{\csc(e+fx)(ac-ad-ad\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}} dx}{ac(c-d)} + \frac{d^2 \int \frac{\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}}{ac(c-d)} \\
 &= \frac{\int \csc(e+fx)\sqrt{a+a\sin(e+fx)} dx}{ac} - \frac{\int \frac{1}{\sqrt{a+a\sin(e+fx)}} dx}{c-d} \\
 &= -\frac{2d^{3/2} \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}c(c-d)\sqrt{c+d}f} - \frac{2\text{Subst}}{\sqrt{a}c} \\
 &= -\frac{2 \tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}cf} + \frac{\sqrt{2} \tanh^{-1}\left(\frac{\sqrt{2}}{\sqrt{a}}\right)}{\sqrt{a}}
 \end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.  
time = 1.58, size = 331, normalized size = 2.01

$$\frac{(2+d)\sqrt{c+d}\sqrt{a}\operatorname{arctanh}\left(\frac{1+(-1)^{3/4}(-1+\sin(fx+e))}{1+(-1)^{3/4}(-1+\sin(fx+e))}\right)+(-d)\sqrt{c+d}\sqrt{a}\operatorname{arctanh}\left(\frac{1+(-1)^{3/4}(-1+\sin(fx+e))}{1+(-1)^{3/4}(-1+\sin(fx+e))}\right)-(-d)\sqrt{c+d}\sqrt{a}\operatorname{arctanh}\left(\frac{1+(-1)^{3/4}(-1+\sin(fx+e))}{1+(-1)^{3/4}(-1+\sin(fx+e))}\right)+(-d)\sqrt{c+d}\sqrt{a}\operatorname{arctanh}\left(\frac{1+(-1)^{3/4}(-1+\sin(fx+e))}{1+(-1)^{3/4}(-1+\sin(fx+e))}\right)}{d\sqrt{c+d}\sqrt{a}\sqrt{1+\sin(fx+e)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])),x]
[Out] -((((2 + 2*I)*(-1)^(3/4)*c*Sqrt[c + d]*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])] + (c - d)*Sqrt[c + d]*Log[1 + Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - c*Sqrt[c + d]*Log[1 - Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + d*Sqrt[c + d]*Log[1 - Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + d^(3/2)*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] + Sqrt[d]*Cos[(e + f*x)/2] - Sqrt[d]*Sin[(e + f*x)/2])] - d^(3/2)*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] - Sqrt[d]*Cos[(e + f*x)/2] + Sqrt[d]*Sin[(e + f*x)/2])])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])/(c*(c - d)*Sqrt[c + d]*f*Sqrt[a*(1 + Sin[e + f*x])])
```

**Maple [A]**  
time = 10.94, size = 208, normalized size = 1.26

method	result
default	$\frac{(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\left(2d^2\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}}{\sqrt{acd+ad^2}}\right)^a a^{\frac{5}{2}}-\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}}{\sqrt{acd+ad^2}}\right)\right)}{a^{\frac{5}{2}}c(c-d)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(f*x+e)/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x,method=_RETURNVE
RBOSE)
[Out] -(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(2*d^2*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)-2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c*a^2*(a*(c+d)*d)^(1/2)+2*arctanh((a-a*sin(f*x+e))^(1/2)/a^(1/2))*a^2*(a*(c+d)*d)^(1/2)*c-2*arctanh((a-a*sin(f*x+e))^(1/2)/a^(1/2))*a^2*(a*(c+d)*d)^(1/2)*d)/a^(5/2)/c/(c-d)/(a*(c+d)*d)^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**  
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(f*x+e)/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(1/(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)*sin(f*x + e)), x
)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 410 vs. 2(142) = 284.

time = 1.74, size = 1118, normalized size = 6.78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(f*x+e)/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algorithm
="fricas")
```

```
[Out] [-1/2*(a*d*sqrt(d/(a*c + a*d))*log((d^2*cos(f*x + e)^3 - (6*c*d + 7*d^2)*co
s(f*x + e)^2 - c^2 - 2*c*d - d^2 + 4*((c*d + d^2)*cos(f*x + e)^2 - c^2 - 4*
c*d - 3*d^2 - (c^2 + 3*c*d + 2*d^2)*cos(f*x + e) + (c^2 + 4*c*d + 3*d^2 + (
c*d + d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d/(a*c
+ a*d)) - (c^2 + 8*c*d + 9*d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - c^2 -
2*c*d - d^2 + 2*(3*c*d + 4*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x +
e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(
f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*si
n(f*x + e))] + sqrt(2)*sqrt(a)*c*log(-(cos(f*x + e)^2 - (cos(f*x + e) - 2)*
sin(f*x + e) - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e) - sin(f*x +
e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)
*sin(f*x + e) - cos(f*x + e) - 2)) - sqrt(a)*(c - d)*log((a*cos(f*x + e)^3
- 7*a*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 + (cos(f*x + e) + 3)*sin(f*x + e)
- 2*cos(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a)*sqrt(a) - 9*a*cos(f*x + e) +
(a*cos(f*x + e)^2 + 8*a*cos(f*x + e) - a)*sin(f*x + e) - a)/(cos(f*x + e)^
3 + cos(f*x + e)^2 + (cos(f*x + e)^2 - 1)*sin(f*x + e) - cos(f*x + e) - 1))
)/((a*c^2 - a*c*d)*f), -1/2*(2*a*d*sqrt(-d/(a*c + a*d))*arctan(1/2*sqrt(a*s
in(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-d/(a*c + a*d))/(d*cos(f*x
+ e))) + sqrt(2)*sqrt(a)*c*log(-(cos(f*x + e)^2 - (cos(f*x + e) - 2)*sin(f
*x + e) - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e) - sin(f*x + e) +
1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(
f*x + e) - cos(f*x + e) - 2)) - sqrt(a)*(c - d)*log((a*cos(f*x + e)^3 - 7*a
*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 + (cos(f*x + e) + 3)*sin(f*x + e) - 2*c
os(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a)*sqrt(a) - 9*a*cos(f*x + e) + (a*c
os(f*x + e)^2 + 8*a*cos(f*x + e) - a)*sin(f*x + e) - a)/(cos(f*x + e)^3 + c
os(f*x + e)^2 + (cos(f*x + e)^2 - 1)*sin(f*x + e) - cos(f*x + e) - 1)))/((a
*c^2 - a*c*d)*f)]
```

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a(\sin(e + fx) + 1)} (c + d \sin(e + fx)) \sin(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sin(f\*x+e)/(c+d\*sin(f\*x+e))/(a+a\*sin(f\*x+e))^(1/2),x)

[Out] Integral(1/(sqrt(a\*(sin(e + f\*x) + 1))\*(c + d\*sin(e + f\*x))\*sin(e + f\*x)), x)

**Giac [A]**

time = 0.50, size = 195, normalized size = 1.18

$$\frac{\sqrt{2} \left( \frac{2\sqrt{2}d^2 \arctan\left(\frac{\sqrt{2}d \sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)}{\sqrt{-cd - d^2}}\right)}{(c^2 - cd)\sqrt{-cd - d^2}} + \frac{\sqrt{2} \log\left(\frac{-2\sqrt{2} + 4 \sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)}{2\sqrt{2} + 4 \sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)}\right)}{c} + \frac{\log(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) + 1)}{c-d} - \frac{\log(-\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) + 1)}{c-d} \right)}{2\sqrt{a} f \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sin(f\*x+e)/(c+d\*sin(f\*x+e))/(a+a\*sin(f\*x+e))^(1/2),x, algorithm="giac")

[Out] -1/2\*sqrt(2)\*(2\*sqrt(2)\*d^2\*arctan(sqrt(2)\*d\*sin(-1/4\*pi + 1/2\*f\*x + 1/2\*e)/sqrt(-c\*d - d^2))/((c^2 - c\*d)\*sqrt(-c\*d - d^2)) + sqrt(2)\*log(abs(-2\*sqrt(2) + 4\*sin(-1/4\*pi + 1/2\*f\*x + 1/2\*e))/abs(2\*sqrt(2) + 4\*sin(-1/4\*pi + 1/2\*f\*x + 1/2\*e)))/c + log(sin(-1/4\*pi + 1/2\*f\*x + 1/2\*e) + 1)/(c - d) - log(-sin(-1/4\*pi + 1/2\*f\*x + 1/2\*e) + 1)/(c - d)/(sqrt(a)\*f\*sgn(cos(-1/4\*pi + 1/2\*f\*x + 1/2\*e)))

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin(e + fx) \sqrt{a + a \sin(e + fx)} (c + d \sin(e + fx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sin(e + f\*x)\*(a + a\*sin(e + f\*x))^(1/2)\*(c + d\*sin(e + f\*x))),x)

[Out] int(1/(sin(e + f\*x)\*(a + a\*sin(e + f\*x))^(1/2)\*(c + d\*sin(e + f\*x))), x)

$$3.25 \quad \int \frac{\sqrt{g \sin(e + fx)} \sqrt{a + a \sin(e + fx)}}{c + d \sin(e + fx)} dx$$

**Optimal.** Leaf size=149

$$\frac{2\sqrt{a} \sqrt{g} \tan^{-1} \left( \frac{\sqrt{a} \sqrt{g} \cos(e+fx)}{\sqrt{g \sin(e+fx)} \sqrt{a + a \sin(e+fx)}} \right)}{df} + \frac{2\sqrt{a} \sqrt{c} \sqrt{g} \tan^{-1} \left( \frac{\sqrt{a} \sqrt{c} \sqrt{g}}{\sqrt{c+d} \sqrt{g \sin(e+fx)}} \right)}{d\sqrt{c+d} f}$$

[Out]  $-2*\arctan(\cos(f*x+e)*a^{(1/2)}*g^{(1/2)}/(g*\sin(f*x+e))^{(1/2)}/(a+a*\sin(f*x+e))^{(1/2)})*a^{(1/2)}*g^{(1/2)}/d/f+2*\arctan(\cos(f*x+e)*a^{(1/2)}*c^{(1/2)}*g^{(1/2)}/(c+d)^{(1/2)}/(g*\sin(f*x+e))^{(1/2)}/(a+a*\sin(f*x+e))^{(1/2)})*a^{(1/2)}*c^{(1/2)}*g^{(1/2)}/d/f/(c+d)^{(1/2)}$

**Rubi [A]**

time = 0.34, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 39,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$ , Rules used = {3007, 2854, 211, 3009}

$$\frac{2\sqrt{a} \sqrt{c} \sqrt{g} \text{ArcTan} \left( \frac{\sqrt{a} \sqrt{c} \sqrt{g} \cos(e+fx)}{\sqrt{c+d} \sqrt{a \sin(e+fx) + a} \sqrt{g \sin(e+fx)}} \right)}{df \sqrt{c+d}} - \frac{2\sqrt{a} \sqrt{g} \text{ArcTan} \left( \frac{\sqrt{a} \sqrt{g} \cos(e+fx)}{\sqrt{a \sin(e+fx) + a} \sqrt{g \sin(e+fx)}} \right)}{df}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[g\*Sin[e + f\*x]]\*Sqrt[a + a\*Sin[e + f\*x]])/(c + d\*Sin[e + f\*x]),x]

[Out]  $(-2*\text{Sqrt}[a]*\text{Sqrt}[g]*\text{ArcTan}[(\text{Sqrt}[a]*\text{Sqrt}[g]*\text{Cos}[e + f*x])/(\text{Sqrt}[g*\text{Sin}[e + f*x]]*\text{Sqrt}[a + a*\text{Sin}[e + f*x]])]/(d*f) + (2*\text{Sqrt}[a]*\text{Sqrt}[c]*\text{Sqrt}[g]*\text{ArcTan}[(\text{Sqrt}[a]*\text{Sqrt}[c]*\text{Sqrt}[g]*\text{Cos}[e + f*x])/(\text{Sqrt}[c + d]*\text{Sqrt}[g*\text{Sin}[e + f*x]]*\text{Sqrt}[a + a*\text{Sin}[e + f*x]])]/(d*\text{Sqrt}[c + d]*f))$

**Rule 211**

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

**Rule 2854**

Int[Sqrt[(a\_) + (b\_)\*sin[(e\_) + (f\_)\*(x\_)]]/Sqrt[(c\_) + (d\_)\*sin[(e\_) + (f\_)\*(x\_)]], x\_Symbol] := Dist[-2\*(b/f), Subst[Int[1/(b + d\*x^2), x], x, b\*(Cos[e + f\*x]/(Sqrt[a + b\*Sin[e + f\*x]]\*Sqrt[c + d\*Sin[e + f\*x]])]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

**Rule 3007**

Int[(Sqrt[(g\_)\*sin[(e\_) + (f\_)\*(x\_)]]\*Sqrt[(a\_) + (b\_)\*sin[(e\_) + (f\_)\*(x\_)])]/((c\_) + (d\_)\*sin[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Dist[g/d, In





$$E^{\left(\left(2I\right)\left(e+f x\right)\right)}] + \left(\left(-I\right)c + \sqrt{-c^2+d^2}\right) \operatorname{ArcTanh}\left[E^{\left(I\left(e+f x\right)\right)}\right] / \sqrt{-1+E^{\left(\left(2I\right)\left(e+f x\right)\right)}}) / d + \left(\left(I + \left(c-d\right) / \sqrt{-c^2+d^2}\right) \left(\sqrt{2}\right) \sqrt{c} \sqrt{c-I \sqrt{-c^2+d^2}} \operatorname{ArcTan}\left[\left(d + \left(Ic + \sqrt{-c^2+d^2}\right) E^{\left(I\left(e+f x\right)\right)}\right) / \left(\sqrt{2}\right) \sqrt{c} \sqrt{c-I \sqrt{-c^2+d^2}} \sqrt{-1+E^{\left(\left(2I\right)\left(e+f x\right)\right)}}\right] - \left(Ic + \sqrt{-c^2+d^2}\right) \operatorname{ArcTanh}\left[E^{\left(I\left(e+f x\right)\right)}\right] / \sqrt{-1+E^{\left(\left(2I\right)\left(e+f x\right)\right)}}\right) / d \sqrt{g \sin\left[e+f x\right]} \sqrt{a\left(1+\sin\left[e+f x\right]\right)} / \left(\sqrt{2}\right) d E^{\left(\left(\left(5I\right) / 2\right)\left(e+f x\right)\right)} \left(\left(-I\right)\left(-1+E^{\left(\left(2I\right)\left(e+f x\right)\right)}\right)\right) / E^{\left(I\left(e+f x\right)\right)} \left(5 / 2\right) f \left(\cos\left[\left(e+f x\right) / 2\right] + \sin\left[\left(e+f x\right) / 2\right]\right) \sqrt{\sin\left[e+f x\right]}$$

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 1024 vs.  $2(117) = 234$ .

time = 0.51, size = 1025, normalized size = 6.88

method	result	size
default	Expression too large to display	1025

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sin(f*x+e))^(1/2)*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f*(g*sin(f*x+e))^(1/2)*(a*(1+sin(f*x+e)))^(1/2)*(-1+cos(f*x+e))*(2*2^(1/2))*(((c-d)*(c+d))^(1/2)+d)*c^(1/2)*arctanh(c*(-(-1+cos(f*x+e))/sin(f*x+e))^(1/2)/(((c-d)*(c+d))^(1/2)-d)*c^(1/2))*(-(c-d)*(c+d))^(1/2)*c*2*2^(1/2)*(((c-d)*(c+d))^(1/2)+d)*c^(1/2)*arctanh(c*(-(-1+cos(f*x+e))/sin(f*x+e))^(1/2)/(((c-d)*(c+d))^(1/2)-d)*c^(1/2))*c^2-2*2^(1/2)*(((c-d)*(c+d))^(1/2)+d)*c^(1/2)*arctanh(c*(-(-1+cos(f*x+e))/sin(f*x+e))^(1/2)/(((c-d)*(c+d))^(1/2)-d)*c^(1/2))*c*d-2*2^(1/2)*(((c-d)*(c+d))^(1/2)-d)*c^(1/2)*arctan(c*(-(-1+cos(f*x+e))/sin(f*x+e))^(1/2)/(((c-d)*(c+d))^(1/2)+d)*c^(1/2))*(-(c-d)*(c+d))^(1/2)*c*2*2^(1/2)*(((c-d)*(c+d))^(1/2)-d)*c^(1/2)*arctan(c*(-(-1+cos(f*x+e))/sin(f*x+e))^(1/2)/(((c-d)*(c+d))^(1/2)+d)*c^(1/2))*c^2-2*2^(1/2)*(((c-d)*(c+d))^(1/2)-d)*c^(1/2)*arctan(c*(-(-1+cos(f*x+e))/sin(f*x+e))^(1/2)/(((c-d)*(c+d))^(1/2)+d)*c^(1/2))*c*d+(((c-d)*(c+d))^(1/2)+d)*c^(1/2)*(((c-d)*(c+d))^(1/2)-d)*c^(1/2)*ln(-((-(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*2^(1/2)*sin(f*x+e)+sin(f*x+e)-cos(f*x+e)+1)/((-(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*2^(1/2)*sin(f*x+e)-sin(f*x+e)+cos(f*x+e)-1))*(-(c-d)*(c+d))^(1/2)+4*(((c-d)*(c+d))^(1/2)+d)*c^(1/2)*(((c-d)*(c+d))^(1/2)-d)*c^(1/2)*arctan((-(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*2^(1/2)+1)*(-(c-d)*(c+d))^(1/2)+4*(((c-d)*(c+d))^(1/2)+d)*c^(1/2)*(((c-d)*(c+d))^(1/2)-d)*c^(1/2)*arctan((-(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*2^(1/2)-1)*(-(c-d)*(c+d))^(1/2)+(((c-d)*(c+d))^(1/2)+d)*c^(1/2)*(((c-d)*(c+d))^(1/2)-d)*c^(1/2)*ln(-((-(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*2^(1/2)*sin(f*x+e)-sin(f*x+e)+cos(f*x+e)-1)/((-(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*2^(1/2)*sin(f*x+e)+sin(f*x+e)-cos(f*x+e)+1))*(-(c-d)*(c+d))^(1/2))/sin(f*x+e)/(-1+cos(f*x+e)
```

$$\frac{-\sin(f*x+e)}{(-(-1+\cos(f*x+e))/\sin(f*x+e))^{1/2}*2^{1/2}/(-(c-d)*(c+d))^{1/2}} / \frac{(((-c-d)*(c+d))^{1/2}-d)*c^{1/2}}{(((c-d)*(c+d))^{1/2}+d)*c^{1/2}}$$

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*sin(f\*x+e))^(1/2)\*(a+a\*sin(f\*x+e))^(1/2)/(c+d\*sin(f\*x+e)),x, algorithm="maxima")

[Out] integrate(sqrt(a\*sin(f\*x + e) + a)\*sqrt(g\*sin(f\*x + e))/(d\*sin(f\*x + e) + c), x)

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 287 vs. 2(123) = 246.

time = 1.70, size = 3409, normalized size = 22.88

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*sin(f\*x+e))^(1/2)\*(a+a\*sin(f\*x+e))^(1/2)/(c+d\*sin(f\*x+e)),x, algorithm="fricas")

[Out] [1/4\*(sqrt(-a\*c\*g/(c + d))\*log(((128\*a\*c^4 + 256\*a\*c^3\*d + 160\*a\*c^2\*d^2 + 32\*a\*c\*d^3 + a\*d^4)\*g\*cos(f\*x + e)^5 - (128\*a\*c^4 + 192\*a\*c^3\*d + 64\*a\*c^2\*d^2 - 4\*a\*c\*d^3 - a\*d^4)\*g\*cos(f\*x + e)^4 - 2\*(208\*a\*c^4 + 368\*a\*c^3\*d + 195\*a\*c^2\*d^2 + 32\*a\*c\*d^3 + a\*d^4)\*g\*cos(f\*x + e)^3 + 2\*(64\*a\*c^4 + 94\*a\*c^3\*d + 29\*a\*c^2\*d^2 - 4\*a\*c\*d^3 - a\*d^4)\*g\*cos(f\*x + e)^2 + (289\*a\*c^4 + 480\*a\*c^3\*d + 230\*a\*c^2\*d^2 + 32\*a\*c\*d^3 + a\*d^4)\*g\*cos(f\*x + e) + 8\*((16\*c^4 + 40\*c^3\*d + 34\*c^2\*d^2 + 11\*c\*d^3 + d^4)\*cos(f\*x + e)^4 + 51\*c^4 + 110\*c^3\*d + 76\*c^2\*d^2 + 18\*c\*d^3 + d^4 - (24\*c^4 + 52\*c^3\*d + 35\*c^2\*d^2 + 7\*c\*d^3)\*cos(f\*x + e)^3 - (66\*c^4 + 149\*c^3\*d + 110\*c^2\*d^2 + 29\*c\*d^3 + 2\*d^4)\*cos(f\*x + e)^2 + (25\*c^4 + 53\*c^3\*d + 35\*c^2\*d^2 + 7\*c\*d^3)\*cos(f\*x + e) - (51\*c^4 + 110\*c^3\*d + 76\*c^2\*d^2 + 18\*c\*d^3 + d^4 - (16\*c^4 + 40\*c^3\*d + 34\*c^2\*d^2 + 11\*c\*d^3 + d^4)\*cos(f\*x + e)^3 - (40\*c^4 + 92\*c^3\*d + 69\*c^2\*d^2 + 18\*c\*d^3 + d^4)\*cos(f\*x + e)^2 + (26\*c^4 + 57\*c^3\*d + 41\*c^2\*d^2 + 11\*c\*d^3 + d^4)\*cos(f\*x + e))\*sin(f\*x + e))\*sqrt(-a\*c\*g/(c + d))\*sqrt(a\*sin(f\*x + e) + a)\*sqrt(g\*sin(f\*x + e)) + (a\*c^4 + 4\*a\*c^3\*d + 6\*a\*c^2\*d^2 + 4\*a\*c\*d^3 + a\*d^4)\*g + ((128\*a\*c^4 + 256\*a\*c^3\*d + 160\*a\*c^2\*d^2 + 32\*a\*c\*d^3 + a\*d^4)\*g\*cos(f\*x + e)^4 + 4\*(64\*a\*c^4 + 112\*a\*c^3\*d + 56\*a\*c^2\*d^2 + 7\*a\*c\*d^3)\*g\*cos(f\*x + e)^3 - 2\*(80\*a\*c^4 + 144\*a\*c^3\*d + 83\*a\*c^2\*d^2 + 18\*a\*c\*d^3 + a\*d^4)\*g\*cos(f\*x + e)^2 - 4\*(72\*a\*c^4 + 119\*a\*c^3\*d + 56\*a\*c^2\*d^2 + 7\*a\*c\*d^3)\*g\*cos(f\*x + e) + (a\*c^4 + 4\*a\*c^3\*d + 6\*a\*c^2\*d^2 + 4\*a\*c\*d^3 + a\*d^4)\*g)\*sin(f\*x + e))/(d^4\*cos(f\*x + e)^5 + (4\*c\*d^3 + d^4)\*cos(f\*x + e)^4 + c

$$\begin{aligned}
&^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4 - 2*(3*c^2*d^2 + d^4)*\cos(f*x + e) \\
&^3 - 2*(2*c^3*d + 3*c^2*d^2 + 4*c*d^3 + d^4)*\cos(f*x + e)^2 + (c^4 + 6*c^2* \\
&d^2 + d^4)*\cos(f*x + e) + (d^4*\cos(f*x + e)^4 - 4*c*d^3*\cos(f*x + e)^3 + c^ \\
&4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4 - 2*(3*c^2*d^2 + 2*c*d^3 + d^4)*\cos \\
&(f*x + e)^2 + 4*(c^3*d + c*d^3)*\cos(f*x + e))*\sin(f*x + e))) + \sqrt{-a*g}*l \\
&og((128*a*g*\cos(f*x + e)^5 - 128*a*g*\cos(f*x + e)^4 - 416*a*g*\cos(f*x + e)^ \\
&3 + 128*a*g*\cos(f*x + e)^2 + 289*a*g*\cos(f*x + e) - 8*(16*\cos(f*x + e)^4 - \\
&24*\cos(f*x + e)^3 - 66*\cos(f*x + e)^2 + (16*\cos(f*x + e)^3 + 40*\cos(f*x + e) \\
&)^2 - 26*\cos(f*x + e) - 51)*\sin(f*x + e) + 25*\cos(f*x + e) + 51)*\sqrt{-a*g} \\
&)*\sqrt{a*\sin(f*x + e) + a}*\sqrt{g*\sin(f*x + e)} + a*g + (128*a*g*\cos(f*x + e) \\
&)^4 + 256*a*g*\cos(f*x + e)^3 - 160*a*g*\cos(f*x + e)^2 - 288*a*g*\cos(f*x + e) \\
& + a*g)*\sin(f*x + e))/(\cos(f*x + e) + \sin(f*x + e) + 1)))/(d*f), -1/4*(2*s \\
&qrt(a*c*g/(c + d))*\arctan(1/4*((8*c^2 + 8*c*d + d^2)*\cos(f*x + e)^2 - 9*c^2 \\
&- 8*c*d - d^2 + 2*(4*c^2 + 3*c*d)*\sin(f*x + e))*\sqrt{a*c*g/(c + d)}*\sqrt{a \\
&}* \sin(f*x + e) + a)*\sqrt{g*\sin(f*x + e)})/(a*c^2*g*\cos(f*x + e)*\sin(f*x + e) \\
& + (2*a*c^2 + a*c*d)*g*\cos(f*x + e)^3 - (2*a*c^2 + a*c*d)*g*\cos(f*x + e))) - \\
&\sqrt{-a*g}*log((128*a*g*\cos(f*x + e)^5 - 128*a*g*\cos(f*x + e)^4 - 416*a*g* \\
&\cos(f*x + e)^3 + 128*a*g*\cos(f*x + e)^2 + 289*a*g*\cos(f*x + e) - 8*(16*\cos( \\
&f*x + e)^4 - 24*\cos(f*x + e)^3 - 66*\cos(f*x + e)^2 + (16*\cos(f*x + e)^3 + 4 \\
&0*\cos(f*x + e)^2 - 26*\cos(f*x + e) - 51)*\sin(f*x + e) + 25*\cos(f*x + e) + 5 \\
&1)*\sqrt{-a*g}*\sqrt{a*\sin(f*x + e) + a}*\sqrt{g*\sin(f*x + e)} + a*g + (128*a* \\
&g*\cos(f*x + e)^4 + 256*a*g*\cos(f*x + e)^3 - 160*a*g*\cos(f*x + e)^2 - 288*a* \\
&g*\cos(f*x + e) + a*g)*\sin(f*x + e))/(\cos(f*x + e) + \sin(f*x + e) + 1)))/(d* \\
&f), 1/4*(2*\sqrt{a*g}*\arctan(1/4*\sqrt{a*g}*(8*\cos(f*x + e)^2 + 8*\sin(f*x + e) \\
&) - 9)*\sqrt{a*\sin(f*x + e) + a}*\sqrt{g*\sin(f*x + e)})/(2*a*g*\cos(f*x + e)^3 \\
& + a*g*\cos(f*x + e)*\sin(f*x + e) - 2*a*g*\cos(f*x + e))) + \sqrt{-a*c*g/(c + d) \\
&))*log(((128*a*c^4 + 256*a*c^3*d + 160*a*c^2*d^2 + 32*a*c*d^3 + a*d^4)*g*co \\
&s(f*x + e)^5 - (128*a*c^4 + 192*a*c^3*d + 64*a*c^2*d^2 - 4*a*c*d^3 - a*d^4) \\
&)*g*\cos(f*x + e)^4 - 2*(208*a*c^4 + 368*a*c^3*d + 195*a*c^2*d^2 + 32*a*c*d^3 \\
& + a*d^4)*g*\cos(f*x + e)^3 + 2*(64*a*c^4 + 94*a*c^3*d + 29*a*c^2*d^2 - 4*a* \\
&c*d^3 - a*d^4)*g*\cos(f*x + e)^2 + (289*a*c^4 + 480*a*c^3*d + 230*a*c^2*d^2 \\
& + 32*a*c*d^3 + a*d^4)*g*\cos(f*x + e) + 8*((16*c^4 + 40*c^3*d + 34*c^2*d^2 + \\
&11*c*d^3 + d^4)*\cos(f*x + e)^4 + 51*c^4 + 110*c^3*d + 76*c^2*d^2 + 18*c*d^ \\
&3 + d^4 - (24*c^4 + 52*c^3*d + 35*c^2*d^2 + 7*c*d^3)*\cos(f*x + e)^3 - (66*c \\
&^4 + 149*c^3*d + 110*c^2*d^2 + 29*c*d^3 + 2*d^4)*\cos(f*x + e)^2 + (25*c^4 + \\
&53*c^3*d + 35*c^2*d^2 + 7*c*d^3)*\cos(f*x + e) - (51*c^4 + 110*c^3*d + 76*c \\
&^2*d^2 + 18*c*d^3 + d^4 - (16*c^4 + 40*c^3*d + 34*c^2*d^2 + 11*c*d^3 + d^4) \\
&)*\cos(f*x + e)^3 - (40*c^4 + 92*c^3*d + 69*c^2*d^2 + 18*c*d^3 + d^4)*\cos(f*x \\
& + e)^2 + (26*c^4 + 57*c^3*d + 41*c^2*d^2 + 11*c*d^3 + d^4)*\cos(f*x + e))*s \\
&\sin(f*x + e))*\sqrt{-a*c*g/(c + d)}*\sqrt{a*\sin(f*x + e) + a}*\sqrt{g*\sin(f*x + \\
&e)} + (a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4)*g + ((128*a*c^ \\
&4 + 256*a*c^3*d + 160*a*c^2*d^2 + 32*a*c*d^3 + a*d^4)*g*\cos(f*x + e)^4 + 4* \\
&(64*a*c^4 + 112*a*c^3*d + 56*a*c^2*d^2 + 7*a*c*d^3)*g*\cos(f*x + e)^3 - 2*(8 \\
&0*a*c^4 + 144*a*c^3*d + 83*a*c^2*d^2 + 18*a*c*d^3 + a*d^4)*g*\cos(f*x + e)^2 \\
&- 4*(72*a*c^4 + 119*a*c^3*d + 56*a*c^2*d^2 + 7*a*c*d^3)*g*\cos(f*x + e) + (
\end{aligned}$$

$a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a\dots$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a(\sin(e+fx)+1)} \sqrt{g \sin(e+fx)}}{c+d \sin(e+fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*sin(f\*x+e))\*\*(1/2)\*(a+a\*sin(f\*x+e))\*\*(1/2)/(c+d\*sin(f\*x+e)),x)

[Out] Integral(sqrt(a\*(sin(e + f\*x) + 1))\*sqrt(g\*sin(e + f\*x))/(c + d\*sin(e + f\*x)), x)

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*sin(f\*x+e))^(1/2)\*(a+a\*sin(f\*x+e))^(1/2)/(c+d\*sin(f\*x+e)),x, algorithm="giac")

[Out] Timed out

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{g \sin(e+fx)} \sqrt{a+a \sin(e+fx)}}{c+d \sin(e+fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((g\*sin(e + f\*x))^(1/2)\*(a + a\*sin(e + f\*x))^(1/2))/(c + d\*sin(e + f\*x)),x)

[Out] int(((g\*sin(e + f\*x))^(1/2)\*(a + a\*sin(e + f\*x))^(1/2))/(c + d\*sin(e + f\*x)), x)

$$3.26 \quad \int \frac{\sqrt{a + a \sin(e + fx)}}{\sqrt{g \sin(e + fx)} (c + d \sin(e + fx))} dx$$

**Optimal.** Leaf size=83

$$-\frac{2\sqrt{a} \tan^{-1}\left(\frac{\sqrt{a} \sqrt{c} \sqrt{g} \cos(e+fx)}{\sqrt{c+d} \sqrt{g \sin(e+fx)} \sqrt{a + a \sin(e+fx)}}\right)}{\sqrt{c} \sqrt{c+d} f \sqrt{g}}$$

[Out]  $-2*\arctan(\cos(f*x+e)*a^{(1/2)}*c^{(1/2)}*g^{(1/2)/(c+d)^{(1/2)/(g*\sin(f*x+e))^{(1/2)/(a+a*\sin(f*x+e))^{(1/2)}}*a^{(1/2)/f/c^{(1/2)/(c+d)^{(1/2)/g^{(1/2)}}$

**Rubi [A]**

time = 0.14, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 39,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$ , Rules used = {3009, 211}

$$-\frac{2\sqrt{a} \text{ArcTan}\left(\frac{\sqrt{a} \sqrt{c} \sqrt{g} \cos(e+fx)}{\sqrt{c+d} \sqrt{a \sin(e+fx) + a} \sqrt{g \sin(e+fx)}}\right)}{\sqrt{c} f \sqrt{g} \sqrt{c+d}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + a\*Sin[e + f\*x]]/(Sqrt[g\*Sin[e + f\*x]]\*(c + d\*Sin[e + f\*x])),x]

[Out]  $(-2*\text{Sqrt}[a]*\text{ArcTan}[(\text{Sqrt}[a]*\text{Sqrt}[c]*\text{Sqrt}[g]*\text{Cos}[e + f*x])/(\text{Sqrt}[c + d]*\text{Sqrt}[g*\text{Sin}[e + f*x]]*\text{Sqrt}[a + a*\text{Sin}[e + f*x]])]/(\text{Sqrt}[c]*\text{Sqrt}[c + d]*f*\text{Sqrt}[g])$

**Rule 211**

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

**Rule 3009**

Int[Sqrt[(a\_) + (b\_)\*sin[(e\_) + (f\_)\*(x\_)]]/(Sqrt[(g\_)\*sin[(e\_) + (f\_)\*(x\_)])\*((c\_) + (d\_)\*sin[(e\_) + (f\_)\*(x\_)])], x\_Symbol] := Dist[-2\*(b/f), Subst[Int[1/(b\*c + a\*d + c\*g\*x^2), x], x, b\*(Cos[e + f\*x]/(Sqrt[g\*Sin[e + f\*x]]\*Sqrt[a + b\*Sin[e + f\*x]])], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\int \frac{\sqrt{a + a \sin(e + fx)}}{\sqrt{g \sin(e + fx)} (c + d \sin(e + fx))} dx = \frac{(2a) \text{Subst} \left( \int \frac{1}{ac + ad + c g x^2} dx, x, \frac{a \cos(e + fx)}{\sqrt{g \sin(e + fx)} \sqrt{a + a \sin(e + fx)}} \right)}{f}$$

$$= - \frac{2\sqrt{a} \tan^{-1} \left( \frac{\sqrt{a} \sqrt{c} \sqrt{g} \cos(e + fx)}{\sqrt{c + d} \sqrt{g \sin(e + fx)} \sqrt{a + a \sin(e + fx)}} \right)}{\sqrt{c} \sqrt{c + d} f \sqrt{g}}$$

**Mathematica [C]** Result contains complex when optimal does not.  
 time = 47.76, size = 436, normalized size = 5.25

$$\frac{(1 + i) \sqrt{c + \sqrt{-c^2 + d^2}} (ic - id + \sqrt{-c^2 + d^2}) \tan^{-1} \left( \frac{a(-ic + \sqrt{-c^2 + d^2}) \cos(e + fx)}{\sqrt{2} \sqrt{c} \sqrt{c + \sqrt{-c^2 + d^2}} \sqrt{-1 + \cos(2e + fx)} + i \sin(2e + fx)} \right) + \sqrt{c - \sqrt{-c^2 + d^2}} (-ic + id + \sqrt{-c^2 + d^2}) \tan^{-1} \left( \frac{a(ic + \sqrt{-c^2 + d^2}) \cos(e + fx)}{\sqrt{2} \sqrt{c} \sqrt{c - \sqrt{-c^2 + d^2}} \sqrt{-1 + \cos(2e + fx)} + i \sin(2e + fx)} \right)}{\sqrt{2} \sqrt{c} \sqrt{-c^2 + d^2} f (\cos(\frac{1}{2}(e + fx)) + \sin(\frac{1}{2}(e + fx))) (g \sin(e + fx))^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]/(Sqrt[g*Sin[e + f*x]]*(c + d*Sin[e + f*x])),x]
```

```
[Out] ((1/4 + I/4)*g*(Sqrt[c + I*Sqrt[-c^2 + d^2]]*(I*c - I*d + Sqrt[-c^2 + d^2]))*ArcTan[(d - ((-I)*c + Sqrt[-c^2 + d^2]))*(Cos[e + f*x] + I*Sin[e + f*x])]/(Sqrt[2]*Sqrt[c]*Sqrt[c + I*Sqrt[-c^2 + d^2]]*Sqrt[-1 + Cos[2*(e + f*x)] + I*Sin[2*(e + f*x)]]) + Sqrt[c - I*Sqrt[-c^2 + d^2]]*((-I)*c + I*d + Sqrt[-c^2 + d^2])*ArcTan[(d + (I*c + Sqrt[-c^2 + d^2]))*(Cos[e + f*x] + I*Sin[e + f*x])]/(Sqrt[2]*Sqrt[c]*Sqrt[c - I*Sqrt[-c^2 + d^2]]*Sqrt[-1 + Cos[2*(e + f*x)] + I*Sin[2*(e + f*x)]])]*Sqrt[a*(1 + Sin[e + f*x])]*(Cos[(3*(e + f*x))/2] - I*Sin[(3*(e + f*x))/2])*(-1 + Cos[2*(e + f*x)] + I*Sin[2*(e + f*x)])^(3/2))/(Sqrt[2]*Sqrt[c]*d*Sqrt[-c^2 + d^2]*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(g*Sin[e + f*x])^(3/2))
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 525 vs. 2(63) = 126.  
 time = 0.31, size = 526, normalized size = 6.34

method	result
default	$2 \sqrt{-\frac{-1 + \cos(fx + e)}{\sin(fx + e)}} \sqrt{a(1 + \sin(fx + e))} \sin(fx + e) \left( \arctan \left( \frac{c \sqrt{-\frac{-1 + \cos(fx + e)}{\sin(fx + e)}}}{\sqrt{(\sqrt{-(c - d)(c + d)} + d)c}} \right) \sqrt{\dots} \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))/(g*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)`

[Out] 
$$\begin{aligned} & -2/f * (-(-1+\cos(f*x+e))/\sin(f*x+e))^{1/2} * (a*(1+\sin(f*x+e)))^{1/2} * \sin(f*x+e) \\ & * (\arctan(c*(-(-1+\cos(f*x+e))/\sin(f*x+e))^{1/2}/(((c-d)*(c+d))^{1/2}+d)*c \\ & )^{1/2}) * (((c-d)*(c+d))^{1/2}-d)*c^{1/2} * (-c-d)*(c+d)^{1/2} - \arctan(c*( \\ & -(-1+\cos(f*x+e))/\sin(f*x+e))^{1/2}/(((c-d)*(c+d))^{1/2}+d)*c)^{1/2} * (((c-d) \\ & *(c+d))^{1/2}-d)*c^{1/2} * c + \arctan(c*(-(-1+\cos(f*x+e))/\sin(f*x+e))^{1/2} \\ & /(((c-d)*(c+d))^{1/2}+d)*c)^{1/2}) * (((c-d)*(c+d))^{1/2}-d)*c^{1/2} * d \\ & - \operatorname{arctanh}(c*(-(-1+\cos(f*x+e))/\sin(f*x+e))^{1/2}/(((c-d)*(c+d))^{1/2}-d)*c \\ & )^{1/2}) * (((c-d)*(c+d))^{1/2}+d)*c^{1/2} * (-c-d)*(c+d)^{1/2} - \operatorname{arctanh}(c*( \\ & -(-1+\cos(f*x+e))/\sin(f*x+e))^{1/2}/(((c-d)*(c+d))^{1/2}-d)*c)^{1/2} * (((c-d) \\ & *(c+d))^{1/2}+d)*c^{1/2} * c + \operatorname{arctanh}(c*(-(-1+\cos(f*x+e))/\sin(f*x+e))^{1/2} \\ & /(((c-d)*(c+d))^{1/2}-d)*c)^{1/2}) * (((c-d)*(c+d))^{1/2}+d)*c^{1/2} * \\ & d) / (g*\sin(f*x+e))^{1/2} / (-1+\cos(f*x+e)-\sin(f*x+e)) / (-c-d)*(c+d)^{1/2} / (((c-d) \\ & *(c+d))^{1/2}-d)*c^{1/2} / (((c-d)*(c+d))^{1/2}+d)*c^{1/2} \end{aligned}$$

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))/(g*sin(f*x+e))^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(a*sin(f*x + e) + a)/((d*sin(f*x + e) + c)*sqrt(g*sin(f*x + e))), x)`

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 176 vs. 2(66) = 132.

time = 1.05, size = 1341, normalized size = 16.16

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))/(g*sin(f*x+e))^(1/2),x, algorithm="fricas")`

[Out] 
$$\begin{aligned} & [1/4*\sqrt{-a/((c^2 + c*d)*g)}]*\log((((128*a*c^4 + 256*a*c^3*d + 160*a*c^2*d^2 \\ & + 32*a*c*d^3 + a*d^4)*\cos(f*x + e)^5 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4 \\ & *a*c*d^3 + a*d^4 - (128*a*c^4 + 192*a*c^3*d + 64*a*c^2*d^2 - 4*a*c*d^3 - a \\ & *d^4)*\cos(f*x + e)^4 - 2*(208*a*c^4 + 368*a*c^3*d + 195*a*c^2*d^2 + 32*a*c*d \\ & ^3 + a*d^4)*\cos(f*x + e)^3 + 2*(64*a*c^4 + 94*a*c^3*d + 29*a*c^2*d^2 - 4*a \\ & *c*d^3 - a*d^4)*\cos(f*x + e)^2 - 8*(51*c^5 + 110*c^4*d + 76*c^3*d^2 + 18*c^2 \\ & *d^3 + c*d^4 + (16*c^5 + 40*c^4*d + 34*c^3*d^2 + 11*c^2*d^3 + c*d^4)*\cos(f* \end{aligned}$$

$$\begin{aligned}
& x + e)^4 - (24c^5 + 52c^4d + 35c^3d^2 + 7c^2d^3) \cos(fx + e)^3 - (66c^5 + 149c^4d + 110c^3d^2 + 29c^2d^3 + 2cd^4) \cos(fx + e)^2 + (25c^5 + 53c^4d + 35c^3d^2 + 7c^2d^3) \cos(fx + e) - (51c^5 + 110c^4d + 76c^3d^2 + 18c^2d^3 + cd^4 - (16c^5 + 40c^4d + 34c^3d^2 + 11c^2d^3 + cd^4) \cos(fx + e)^3 - (40c^5 + 92c^4d + 69c^3d^2 + 18c^2d^3 + cd^4) \cos(fx + e)^2 + (26c^5 + 57c^4d + 41c^3d^2 + 11c^2d^3 + cd^4) \cos(fx + e)) \sin(fx + e) \sqrt{a \sin(fx + e) + a} \sqrt{g \sin(fx + e)} \sqrt{-a / ((c^2 + cd)g)} + (289a^2c^4 + 480a^2c^3d + 230a^2c^2d^2 + 32a^2cd^3 + a^2d^4) \cos(fx + e) + (a^2c^4 + 4a^2c^3d + 6a^2c^2d^2 + 4a^2cd^3 + a^2d^4 + (128a^2c^4 + 256a^2c^3d + 160a^2c^2d^2 + 32a^2cd^3 + a^2d^4) \cos(fx + e)^4 + 4(64a^2c^4 + 112a^2c^3d + 56a^2c^2d^2 + 7a^2cd^3) \cos(fx + e)^3 - 2(80a^2c^4 + 144a^2c^3d + 83a^2c^2d^2 + 18a^2cd^3 + a^2d^4) \cos(fx + e)^2 - 4(72a^2c^4 + 119a^2c^3d + 56a^2c^2d^2 + 7a^2cd^3) \cos(fx + e)) \sin(fx + e) / (d^4 \cos(fx + e)^5 + (4cd^3 + d^4) \cos(fx + e)^4 + c^4 + 4c^3d + 6c^2d^2 + 4cd^3 + d^4 - 2(3c^2d^2 + d^4) \cos(fx + e)^3 - 2(2c^3d + 3c^2d^2 + 4cd^3 + d^4) \cos(fx + e)^2 + (c^4 + 6c^2d^2 + d^4) \cos(fx + e) + (d^4 \cos(fx + e)^4 - 4cd^3 \cos(fx + e)^3 + c^4 + 4c^3d + 6c^2d^2 + 4cd^3 + d^4 - 2(3c^2d^2 + 2cd^3 + d^4) \cos(fx + e)^2 + 4(c^3d + cd^3) \cos(fx + e)) \sin(fx + e)) / f, 1/2 \sqrt{a / ((c^2 + cd)g)} \arctan(1/4 * ((8c^2 + 8cd + d^2) \cos(fx + e)^2 - 9c^2 - 8cd - d^2 + 2(4c^2 + 3cd) \sin(fx + e)) \sqrt{a \sin(fx + e) + a} \sqrt{g \sin(fx + e)} \sqrt{a / ((c^2 + cd)g)} / ((2ac + ad) \cos(fx + e)^3 + ac \cos(fx + e) \sin(fx + e) - (2ac + ad) \cos(fx + e))) / f]
\end{aligned}$$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a(\sin(e + fx) + 1)}}{\sqrt{g \sin(e + fx)} (c + d \sin(e + fx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*sin(f\*x+e))\*\*(1/2)/(c+d\*sin(f\*x+e))/(g\*sin(f\*x+e))\*\*(1/2),x)

[Out] Integral(sqrt(a\*(sin(e + f\*x) + 1))/(sqrt(g\*sin(e + f\*x))\*(c + d\*sin(e + f\*x))), x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*sin(f\*x+e))^(1/2)/(c+d\*sin(f\*x+e))/(g\*sin(f\*x+e))^(1/2),x, algorithm="giac")



[Out] integrate(sqrt(a\*sin(f\*x + e) + a)/((d\*sin(f\*x + e) + c)\*sqrt(g\*sin(f\*x + e))), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a + a \sin(e + f x)}}{\sqrt{g \sin(e + f x)} (c + d \sin(e + f x))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a\*sin(e + f\*x))^(1/2)/((g\*sin(e + f\*x))^(1/2)\*(c + d\*sin(e + f\*x))), x)

[Out] int((a + a\*sin(e + f\*x))^(1/2)/((g\*sin(e + f\*x))^(1/2)\*(c + d\*sin(e + f\*x))), x)

$$3.27 \quad \int \frac{\sqrt{g \sin(e + fx)}}{\sqrt{a + a \sin(e + fx)} (c + d \sin(e + fx))} dx$$

**Optimal.** Leaf size=166

$$\frac{\sqrt{2} \sqrt{g} \tan^{-1} \left( \frac{\sqrt{a} \sqrt{g} \cos(e+fx)}{\sqrt{2} \sqrt{g \sin(e+fx)} \sqrt{a + a \sin(e+fx)}} \right)}{\sqrt{a} (c-d)f} - \frac{2\sqrt{c} \sqrt{g} \tan^{-1} \left( \frac{\sqrt{a} \sqrt{c} \sqrt{g} \cos(e+fx)}{\sqrt{c+d} \sqrt{g \sin(e+fx)} \sqrt{a + a \sin(e+fx)}} \right)}{\sqrt{a} (c-d)\sqrt{c+d} f}$$

[Out] arctan(1/2\*cos(f\*x+e)\*a^(1/2)\*g^(1/2)\*2^(1/2)/(g\*sin(f\*x+e))^(1/2)/(a+a\*sin(f\*x+e))^(1/2))\*2^(1/2)\*g^(1/2)/(c-d)/f/a^(1/2)-2\*arctan(cos(f\*x+e)\*a^(1/2)\*c^(1/2)\*g^(1/2)/(c+d)^(1/2)/(g\*sin(f\*x+e))^(1/2)/(a+a\*sin(f\*x+e))^(1/2))\*c^(1/2)\*g^(1/2)/(c-d)/f/a^(1/2)/(c+d)^(1/2)

**Rubi [A]**

time = 0.35, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 39,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$ , Rules used = {3015, 2861, 211, 3009}

$$\frac{\sqrt{2} \sqrt{g} \text{ArcTan} \left( \frac{\sqrt{a} \sqrt{g} \cos(e+fx)}{\sqrt{2} \sqrt{a \sin(e+fx) + a} \sqrt{g \sin(e+fx)}} \right)}{\sqrt{a} f(c-d)} - \frac{2\sqrt{c} \sqrt{g} \text{ArcTan} \left( \frac{\sqrt{a} \sqrt{c} \sqrt{g} \cos(e+fx)}{\sqrt{c+d} \sqrt{a \sin(e+fx) + a} \sqrt{g \sin(e+fx)}} \right)}{\sqrt{a} f(c-d)\sqrt{c+d}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[g\*Sin[e + f\*x]]/(Sqrt[a + a\*Sin[e + f\*x]]\*(c + d\*Sin[e + f\*x])),x]

[Out] (Sqrt[2]\*Sqrt[g]\*ArcTan[(Sqrt[a]\*Sqrt[g]\*Cos[e + f\*x])/(Sqrt[2]\*Sqrt[g\*Sin[e + f\*x]]\*Sqrt[a + a\*Sin[e + f\*x]])]/(Sqrt[a]\*(c - d)\*f) - (2\*Sqrt[c]\*Sqrt[g]\*ArcTan[(Sqrt[a]\*Sqrt[c]\*Sqrt[g]\*Cos[e + f\*x])/(Sqrt[c + d]\*Sqrt[g\*Sin[e + f\*x]]\*Sqrt[a + a\*Sin[e + f\*x]])]/(Sqrt[a]\*(c - d)\*Sqrt[c + d]\*f)

**Rule 211**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

**Rule 2861**

Int[1/(Sqrt[(a\_) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)]]\*Sqrt[(c\_.) + (d\_.)\*sin[(e\_.) + (f\_.)\*(x\_)]]), x\_Symbol] :> Dist[-2\*(a/f), Subst[Int[1/(2\*b^2 - (a\*c - b\*d)\*x^2), x], x, b\*(Cos[e + f\*x]/(Sqrt[a + b\*Sin[e + f\*x]]\*Sqrt[c + d\*Sin[e + f\*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

**Rule 3009**

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/(Sqrt[(g_)*sin[(e_) + (f_
)*(x_)]]*((c_) + (d_)*sin[(e_) + (f_)*(x_)])), x_Symbol] :> Dist[-2*(b/f
), Subst[Int[1/(b*c + a*d + c*g*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[g*Sin[e
+ f*x]]*Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

### Rule 3015

```
Int[Sqrt[(g_)*sin[(e_) + (f_)*(x_)]]/(Sqrt[(a_) + (b_)*sin[(e_) + (f_
)*(x_)]]*((c_) + (d_)*sin[(e_) + (f_)*(x_)])), x_Symbol] :> Dist[(-a)*(g
/(b*c - a*d)), Int[1/(Sqrt[g*Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]]), x], x
] + Dist[c*(g/(b*c - a*d)), Int[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[g*Sin[e + f*
x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[b*c - a*d, 0] && (EqQ[a^2 - b^2, 0] || EqQ[c^2 - d^2, 0])
```

### Rubi steps

$$\int \frac{\sqrt{g \sin(e + fx)}}{\sqrt{a + a \sin(e + fx)} (c + d \sin(e + fx))} dx = -\frac{g \int \frac{1}{\sqrt{g \sin(e + fx)} \sqrt{a + a \sin(e + fx)}} dx}{c - d} + \frac{(cg) \int \frac{1}{\sqrt{g \sin(e + fx)} \sqrt{a + a \sin(e + fx)}} dx}{c - d}$$

$$= \frac{(2ag) \text{Subst}\left(\int \frac{1}{2a^2 + agx^2} dx, x, \frac{a \cos(e + fx)}{\sqrt{g \sin(e + fx)} \sqrt{a + a \sin(e + fx)}}\right)}{(c - d)f}$$

$$= \frac{\sqrt{2} \sqrt{g} \tan^{-1}\left(\frac{\sqrt{a} \sqrt{g} \cos(e + fx)}{\sqrt{2} \sqrt{g \sin(e + fx)} \sqrt{a + a \sin(e + fx)}}\right)}{\sqrt{a} (c - d)f}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 2568 vs. 2(166) = 332.

time = 24.66, size = 2568, normalized size = 15.47

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[g*Sin[e + f*x]]/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x
])),x]
```

```
[Out] (((-((Sqrt[c]*(c - d + Sqrt[-c^2 + d^2])*ArcTan[(Sqrt[2]*Sqrt[-d + Sqrt[-c^2
+ d^2]]*Sqrt[Tan[(e + f*x)/4]]]/(Sqrt[c]*Sqrt[-1 + Tan[(e + f*x)/4]^2])))/
(Sqrt[-c^2 + d^2]*Sqrt[-d + Sqrt[-c^2 + d^2]])) + 2*ArcTanh[(Sqrt[2]*Sqrt[T
an[(e + f*x)/4]]/Sqrt[-1 + Tan[(e + f*x)/4]^2]] - (Sqrt[c]*(-c + d + Sqrt[
```



+ Tan[(e + f\*x)/4]^2)))) - (Sqrt[c]\*(-c + d + Sqrt[-c^2 + d^2])\*(-1/2\*(Sqrt[d + Sqrt[-c^2 + d^2]]\*Sec[(e + f\*x)/4]^2\*Tan[(e + f\*x)/4]^(3/2))/(Sqrt[2]\*Sqrt[c]\*(-1 + Tan[(e + f\*x)/4]^(3/2)) + (Sqrt[d + Sqrt[-c^2 + d^2]]\*Sec[(e + f\*x)/4]^2)/(4\*Sqrt[2]\*Sqrt[c]\*Sqrt[Tan[(e + f\*x)/4]]\*Sqrt[-1 + Tan[(e + f\*x)/4]^2])))/(Sqrt[-c^2 + d^2]\*Sqrt[d + Sqrt[-c^2 + d^2]]\*(1 - (2\*(d + Sqrt[-c^2 + d^2])\*Tan[(e + f\*x)/4])/(c\*(-1 + Tan[(e + f\*x)/4]^2))))))/(Sqrt[2]\*(c - d)\*Sqrt[Tan[(e + f\*x)/4]]\*Sqrt[-1 + Tan[(e + f\*x)/4]^2]))

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 612 vs.  $2(131) = 262$ .

time = 0.37, size = 613, normalized size = 3.69

method	result
default	$\frac{\sqrt{g \sin(fx + e)}^{-1 + \cos(fx + e) - \sin(fx + e)} \left( 2 \arctan \left( \sqrt{-\frac{-1 + \cos(fx + e)}{\sin(fx + e)}} \right) \sqrt{-(c - d)(c + d)} \sqrt{\sqrt{-(c - d)(c + d)}} \right)}{\dots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*sin(f\*x+e))^(1/2)/(c+d\*sin(f\*x+e))/(a+a\*sin(f\*x+e))^(1/2),x,method=\_RETURNVERBOSE)

[Out]  $\frac{1}{f} \frac{\sqrt{g \sin(fx + e)}^{-1 + \cos(fx + e) - \sin(fx + e)} \left( 2 \arctan \left( \sqrt{-\frac{-1 + \cos(fx + e)}{\sin(fx + e)}} \right) \sqrt{-(c - d)(c + d)} \sqrt{\sqrt{-(c - d)(c + d)}} \right)}{\sqrt{g \sin(fx + e)}^{1/2} \sqrt{c + d \sin(fx + e)} \sqrt{a + a \sin(fx + e)}} \dots$

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, a
lgorithm="maxima")
```

```
[Out] integrate(sqrt(g*sin(f*x + e))/(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) +
c)), x)
```

**Fricas [A]**

time = 1.81, size = 3170, normalized size = 19.10

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, a
lgorithm="fricas")
```

```
[Out] [-1/4*(sqrt(2)*sqrt(-g/a)*log((17*g*cos(f*x + e)^3 - 4*sqrt(2)*(3*cos(f*x +
e)^2 + (3*cos(f*x + e) + 4)*sin(f*x + e) - cos(f*x + e) - 4)*sqrt(a*sin(f*
x + e) + a)*sqrt(g*sin(f*x + e))*sqrt(-g/a) + 3*g*cos(f*x + e)^2 - 18*g*cos
(f*x + e) + (17*g*cos(f*x + e)^2 + 14*g*cos(f*x + e) - 4*g)*sin(f*x + e) -
4*g)/(cos(f*x + e)^3 + 3*cos(f*x + e)^2 + (cos(f*x + e)^2 - 2*cos(f*x + e)
- 4)*sin(f*x + e) - 2*cos(f*x + e) - 4)) + sqrt(-c*g/(a*c + a*d))*log(((128
*c^4 + 256*c^3*d + 160*c^2*d^2 + 32*c*d^3 + d^4)*g*cos(f*x + e)^5 - (128*c^
4 + 192*c^3*d + 64*c^2*d^2 - 4*c*d^3 - d^4)*g*cos(f*x + e)^4 - 2*(208*c^4 +
368*c^3*d + 195*c^2*d^2 + 32*c*d^3 + d^4)*g*cos(f*x + e)^3 + 2*(64*c^4 + 9
4*c^3*d + 29*c^2*d^2 - 4*c*d^3 - d^4)*g*cos(f*x + e)^2 + (289*c^4 + 480*c^3
*d + 230*c^2*d^2 + 32*c*d^3 + d^4)*g*cos(f*x + e) + 8*((16*c^4 + 40*c^3*d +
34*c^2*d^2 + 11*c*d^3 + d^4)*cos(f*x + e)^4 + 51*c^4 + 110*c^3*d + 76*c^2*
d^2 + 18*c*d^3 + d^4 - (24*c^4 + 52*c^3*d + 35*c^2*d^2 + 7*c*d^3)*cos(f*x +
e)^3 - (66*c^4 + 149*c^3*d + 110*c^2*d^2 + 29*c*d^3 + 2*d^4)*cos(f*x + e)^
2 + (25*c^4 + 53*c^3*d + 35*c^2*d^2 + 7*c*d^3)*cos(f*x + e) - (51*c^4 + 110
*c^3*d + 76*c^2*d^2 + 18*c*d^3 + d^4 - (16*c^4 + 40*c^3*d + 34*c^2*d^2 + 11
*c*d^3 + d^4)*cos(f*x + e)^3 - (40*c^4 + 92*c^3*d + 69*c^2*d^2 + 18*c*d^3 +
d^4)*cos(f*x + e)^2 + (26*c^4 + 57*c^3*d + 41*c^2*d^2 + 11*c*d^3 + d^4)*co
s(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(g*sin(f*x + e))*sqr
t(-c*g/(a*c + a*d)) + (c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4)*g + ((128
*c^4 + 256*c^3*d + 160*c^2*d^2 + 32*c*d^3 + d^4)*g*cos(f*x + e)^4 + 4*(64*c
^4 + 112*c^3*d + 56*c^2*d^2 + 7*c*d^3)*g*cos(f*x + e)^3 - 2*(80*c^4 + 144*c
^3*d + 83*c^2*d^2 + 18*c*d^3 + d^4)*g*cos(f*x + e)^2 - 4*(72*c^4 + 119*c^3*
d + 56*c^2*d^2 + 7*c*d^3)*g*cos(f*x + e) + (c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c
*d^3 + d^4)*g)*sin(f*x + e))/(d^4*cos(f*x + e)^5 + (4*c*d^3 + d^4)*cos(f*x
+ e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4 - 2*(3*c^2*d^2 + d^4)*co
s(f*x + e)^3 - 2*(2*c^3*d + 3*c^2*d^2 + 4*c*d^3 + d^4)*cos(f*x + e)^2 + (c^
4 + 6*c^2*d^2 + d^4)*cos(f*x + e) + (d^4*cos(f*x + e)^4 - 4*c*d^3*cos(f*x +
e)^3 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4 - 2*(3*c^2*d^2 + 2*c*d^3
+ d^4)*cos(f*x + e)^2 + 4*(c^3*d + c*d^3)*cos(f*x + e))*sin(f*x + e)))/((c
- d)*f), -1/4*(sqrt(2)*sqrt(-g/a)*log((17*g*cos(f*x + e)^3 - 4*sqrt(2)*(3*
```

```

cos(f*x + e)^2 + (3*cos(f*x + e) + 4)*sin(f*x + e) - cos(f*x + e) - 4)*sqrt
(a*sin(f*x + e) + a)*sqrt(g*sin(f*x + e))*sqrt(-g/a) + 3*g*cos(f*x + e)^2 -
18*g*cos(f*x + e) + (17*g*cos(f*x + e)^2 + 14*g*cos(f*x + e) - 4*g)*sin(f*
x + e) - 4*g)/(cos(f*x + e)^3 + 3*cos(f*x + e)^2 + (cos(f*x + e)^2 - 2*cos(
f*x + e) - 4)*sin(f*x + e) - 2*cos(f*x + e) - 4)) - 2*sqrt(c*g/(a*c + a*d))
*arctan(1/4*((8*c^2 + 8*c*d + d^2)*cos(f*x + e)^2 - 9*c^2 - 8*c*d - d^2 + 2
*(4*c^2 + 3*c*d)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(g*sin(f*x + e)
)*sqrt(c*g/(a*c + a*d))/((2*c^2 + c*d)*g*cos(f*x + e)^3 + c^2*g*cos(f*x + e)
)*sin(f*x + e) - (2*c^2 + c*d)*g*cos(f*x + e)))/((c - d)*f), -1/4*(2*sqrt(
2)*sqrt(g/a)*arctan(1/4*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(g*sin(f*x + e)
))*sqrt(g/a)*(3*sin(f*x + e) - 1)/(g*cos(f*x + e)*sin(f*x + e))) + sqrt(-c*
g/(a*c + a*d))*log(((128*c^4 + 256*c^3*d + 160*c^2*d^2 + 32*c*d^3 + d^4)*g*
cos(f*x + e)^5 - (128*c^4 + 192*c^3*d + 64*c^2*d^2 - 4*c*d^3 - d^4)*g*cos(f
*x + e)^4 - 2*(208*c^4 + 368*c^3*d + 195*c^2*d^2 + 32*c*d^3 + d^4)*g*cos(f*
x + e)^3 + 2*(64*c^4 + 94*c^3*d + 29*c^2*d^2 - 4*c*d^3 - d^4)*g*cos(f*x + e)
)^2 + (289*c^4 + 480*c^3*d + 230*c^2*d^2 + 32*c*d^3 + d^4)*g*cos(f*x + e) +
8*((16*c^4 + 40*c^3*d + 34*c^2*d^2 + 11*c*d^3 + d^4)*cos(f*x + e)^4 + 51*c
^4 + 110*c^3*d + 76*c^2*d^2 + 18*c*d^3 + d^4 - (24*c^4 + 52*c^3*d + 35*c^2*
d^2 + 7*c*d^3)*cos(f*x + e)^3 - (66*c^4 + 149*c^3*d + 110*c^2*d^2 + 29*c*d^
3 + 2*d^4)*cos(f*x + e)^2 + (25*c^4 + 53*c^3*d + 35*c^2*d^2 + 7*c*d^3)*cos(
f*x + e) - (51*c^4 + 110*c^3*d + 76*c^2*d^2 + 18*c*d^3 + d^4 - (16*c^4 + 40
*c^3*d + 34*c^2*d^2 + 11*c*d^3 + d^4)*cos(f*x + e)^3 - (40*c^4 + 92*c^3*d +
69*c^2*d^2 + 18*c*d^3 + d^4)*cos(f*x + e)^2 + (26*c^4 + 57*c^3*d + 41*c^2*
d^2 + 11*c*d^3 + d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*
sqrt(g*sin(f*x + e))*sqrt(-c*g/(a*c + a*d)) + (c^4 + 4*c^3*d + 6*c^2*d^2 +
4*c*d^3 + d^4)*g + ((128*c^4 + 256*c^3*d + 160*c^2*d^2 + 32*c*d^3 + d^4)*g*
cos(f*x + e)^4 + 4*(64*c^4 + 112*c^3*d + 56*c^2*d^2 + 7*c*d^3)*g*cos(f*x +
e)^3 - 2*(80*c^4 + 144*c^3*d + 83*c^2*d^2 + 18*c*d^3 + d^4)*g*cos(f*x + e)^
2 - 4*(72*c^4 + 119*c^3*d + 56*c^2*d^2 + 7*c*d^3)*g*cos(f*x + e) + (c^4 + 4
*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4)*g)*sin(f*x + e))/(d^4*cos(f*x + e)^5 +
(4*c*d^3 + d^4)*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4
- 2*(3*c^2*d^2 + d^4)*cos(f*x + e)^3 - 2*(2*c^3*d + 3*c^2*d^2 + 4*c*d^3 + d
^4)*cos(f*x + e)^2 + (c^4 + 6*c^2*d^2 + d^4)*cos(f*x + e) + (d^4*cos(f*x +
e)^4 - 4*c*d^3*cos(f*x + e)^3 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4 -
2*(3*c^2*d^2 + 2*c*d^3 + d^4)*cos(f*x + e)^2 + 4*(c^3*d + c*d^3)*cos(f*x +
e))*sin(f*x + e)))/((c - d)*f), -1/2*(sqrt(2)...

```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{g \sin(e + fx)}}{\sqrt{a(\sin(e + fx) + 1)} (c + d \sin(e + fx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*sin(f\*x+e))\*\*(1/2)/(c+d\*sin(f\*x+e))/(a+a\*sin(f\*x+e))\*\*(1/2),x)

[Out] Integral(sqrt(g\*sin(e + f\*x))/(sqrt(a\*(sin(e + f\*x) + 1))\*(c + d\*sin(e + f\*x))), x)

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*sin(f\*x+e))^(1/2)/(c+d\*sin(f\*x+e))/(a+a\*sin(f\*x+e))^(1/2),x, algorithm="giac")

[Out] Timed out

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{g \sin(e + f x)}}{\sqrt{a + a \sin(e + f x)} (c + d \sin(e + f x))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*sin(e + f\*x))^(1/2)/((a + a\*sin(e + f\*x))^(1/2)\*(c + d\*sin(e + f\*x))),x)

[Out] int((g\*sin(e + f\*x))^(1/2)/((a + a\*sin(e + f\*x))^(1/2)\*(c + d\*sin(e + f\*x))), x)



$$3.28 \quad \int \frac{1}{\sqrt{g \sin(e + fx)} \sqrt{a + a \sin(e + fx)} (c + d \sin(e + fx))} dx$$

**Optimal.** Leaf size=168

$$-\frac{\sqrt{2} \tan^{-1} \left( \frac{\sqrt{a} \sqrt{g} \cos(e+fx)}{\sqrt{2} \sqrt{g \sin(e+fx)} \sqrt{a + a \sin(e+fx)}} \right)}{\sqrt{a} (c-d) f \sqrt{g}} + \frac{2d \tan^{-1} \left( \frac{\sqrt{a} \sqrt{c} \sqrt{g} \cos(e+fx)}{\sqrt{c+d} \sqrt{g \sin(e+fx)} \sqrt{a + a \sin(e+fx)}} \right)}{\sqrt{a} \sqrt{c} (c-d) \sqrt{c+d} f \sqrt{g}}$$

[Out]  $-\arctan(1/2*\cos(f*x+e)*a^{(1/2)}*g^{(1/2)}*2^{(1/2)}/(g*\sin(f*x+e))^{(1/2)}/(a+a*\sin(f*x+e))^{(1/2)})*2^{(1/2)}/(c-d)/f/a^{(1/2)}/g^{(1/2)}+2*d*\arctan(\cos(f*x+e)*a^{(1/2)}/2)*c^{(1/2)}*g^{(1/2)}/(c+d)^{(1/2)}/(g*\sin(f*x+e))^{(1/2)}/(a+a*\sin(f*x+e))^{(1/2)}/(c-d)/f/a^{(1/2)}/c^{(1/2)}/(c+d)^{(1/2)}/g^{(1/2)}$

**Rubi [A]**

time = 0.35, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 39,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$ , Rules used = {3017, 2861, 211, 3009}

$$\frac{2d \text{ArcTan} \left( \frac{\sqrt{a} \sqrt{c} \sqrt{g} \cos(e+fx)}{\sqrt{c+d} \sqrt{a \sin(e+fx) + a} \sqrt{g \sin(e+fx)}} \right)}{\sqrt{a} \sqrt{c} f \sqrt{g} (c-d) \sqrt{c+d}} - \frac{\sqrt{2} \text{ArcTan} \left( \frac{\sqrt{a} \sqrt{g} \cos(e+fx)}{\sqrt{2} \sqrt{a \sin(e+fx) + a} \sqrt{g \sin(e+fx)}} \right)}{\sqrt{a} f \sqrt{g} (c-d)}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[g\*Sin[e + f\*x]]\*Sqrt[a + a\*Sin[e + f\*x]]\*(c + d\*Sin[e + f\*x])), x]

[Out]  $-\left(\frac{\text{Sqrt}[2]*\text{ArcTan}[\left(\frac{\text{Sqrt}[a]*\text{Sqrt}[g]*\text{Cos}[e + f*x]}{\text{Sqrt}[2]*\text{Sqrt}[g*\text{Sin}[e + f*x]]*\text{Sqrt}[a + a*\text{Sin}[e + f*x]]}\right)]}{\text{Sqrt}[a]*(c - d)*f*\text{Sqrt}[g]}\right) + \left(\frac{2*d*\text{ArcTan}[\left(\frac{\text{Sqrt}[a]*\text{Sqrt}[c]*\text{Sqrt}[g]*\text{Cos}[e + f*x]}{\text{Sqrt}[c + d]*\text{Sqrt}[g*\text{Sin}[e + f*x]]*\text{Sqrt}[a + a*\text{Sin}[e + f*x]]}\right)]}{\text{Sqrt}[a]*\text{Sqrt}[c]*(c - d)*\text{Sqrt}[c + d]*f*\text{Sqrt}[g]}\right)$

**Rule 211**

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]/a)\*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

**Rule 2861**

Int[1/(Sqrt[(a\_) + (b\_)\*sin[(e\_) + (f\_)\*(x\_)])\*Sqrt[(c\_) + (d\_)\*sin[(e\_) + (f\_)\*(x\_)])], x\_Symbol] := Dist[-2\*(a/f), Subst[Int[1/(2\*b^2 - (a\*c - b\*d)\*x^2), x], x, b\*(Cos[e + f\*x]/(Sqrt[a + b\*Sin[e + f\*x]]\*Sqrt[c + d\*Sin[e + f\*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

**Rule 3009**

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/(Sqrt[(g_)*sin[(e_) + (f_)*(x_)]]*((c_) + (d_)*sin[(e_) + (f_)*(x_)])), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d + c*g*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[g*Sin[e + f*x]])*Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

### Rule 3017

```
Int[1/(Sqrt[(g_)*sin[(e_) + (f_)*(x_)]]*Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((c_) + (d_)*sin[(e_) + (f_)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(Sqrt[g*Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]]), x], x] - Dist[d/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[g*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && (EqQ[a^2 - b^2, 0] || EqQ[c^2 - d^2, 0])
```

### Rubi steps

$$\int \frac{1}{\sqrt{g \sin(e + fx)} \sqrt{a + a \sin(e + fx)} (c + d \sin(e + fx))} dx = \frac{\int \frac{1}{\sqrt{g \sin(e + fx)} \sqrt{a + a \sin(e + fx)}} dx}{c - d}$$

$$= -\frac{(2a) \text{Subst}\left(\int \frac{1}{2a^2 + agx^2} dx, x, \frac{\sqrt{a} \sqrt{g \sin(e + fx)}}{\sqrt{a + a \sin(e + fx)}}\right)}{(c - d)f}$$

$$= -\frac{\sqrt{2} \tan^{-1}\left(\frac{\sqrt{a} \sqrt{g \cos(e + fx)}}{\sqrt{2} \sqrt{g \sin(e + fx)} \sqrt{a + a \sin(e + fx)}}\right)}{\sqrt{a} (c - d) f \sqrt{g}}$$

**Mathematica [B]** Leaf count is larger than twice the leaf count of optimal. 3062 vs. 2(168) = 336.

time = 24.89, size = 3062, normalized size = 18.23

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[g*Sin[e + f*x]]*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x]
```

```
[Out] -((((-(d*(c - d + Sqrt[-c^2 + d^2])*Sqrt[d + Sqrt[-c^2 + d^2]]*ArcTan[(Sqrt[2]*Sqrt[-d + Sqrt[-c^2 + d^2]]*Sqrt[Tan[(e + f*x)/4]])]/(Sqrt[c]*Sqrt[-1 + Tan[(e + f*x)/4]^2])) + 2*Sqrt[c]*Sqrt[-c^2 + d^2]*Sqrt[-d + Sqrt[-c^2 + d^2]]*Sqrt[d + Sqrt[-c^2 + d^2]]*ArcTanh[(Sqrt[2]*Sqrt[Tan[(e + f*x)/4]])]/Sqr
```





$$\frac{1}{\left(\left(-\left(c-d\right)\left(c+d\right)\right)^{\frac{1}{2}}-d\right) c^{\frac{1}{2}}\left(\left(-\left(c-d\right)\left(c+d\right)\right)^{\frac{1}{2}}+d\right) c^{\frac{1}{2}} d^{\frac{1}{2}}\left(g \sin \left(f x+e\right)\right)^{\frac{1}{2}}\left(a\left(1+\sin \left(f x+e\right)\right)\right)^{\frac{1}{2}}\left(-1+\cos \left(f x+e\right)\right)\left(c-d\right)\left(-\left(c-d\right)\left(c+d\right)\right)^{\frac{1}{2}}\left(\left(-\left(c-d\right)\left(c+d\right)\right)^{\frac{1}{2}}-d\right) c^{\frac{1}{2}}\left(\left(-\left(c-d\right)\left(c+d\right)\right)^{\frac{1}{2}}+d\right) c^{\frac{1}{2}}\right)}$$

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+d\*sin(f\*x+e))/(g\*sin(f\*x+e))^(1/2)/(a+a\*sin(f\*x+e))^(1/2),x,  
algorithm="maxima")

[Out] integrate(1/(sqrt(a\*sin(f\*x + e) + a)\*(d\*sin(f\*x + e) + c)\*sqrt(g\*sin(f\*x + e))), x)

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 323 vs. 2(139) = 278.

time = 1.88, size = 3297, normalized size = 19.62

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+d\*sin(f\*x+e))/(g\*sin(f\*x+e))^(1/2)/(a+a\*sin(f\*x+e))^(1/2),x,  
algorithm="fricas")

[Out] 
$$\begin{aligned} & \left[-\frac{1}{4} \sqrt{2} \left(a c^2 + a c d\right) g \sqrt{-1/(a g)} \log \left(\left(4 \sqrt{2}\right) \left(3 \cos (f x + e)^2 + \left(3 \cos (f x + e) + 4\right) \sin (f x + e) - \cos (f x + e) - 4\right) \sqrt{a \sin (f x + e) + a} \sqrt{g \sin (f x + e)} \sqrt{-1/(a g)} + 17 \cos (f x + e)^3 + 3 \cos (f x + e)^2 + \left(17 \cos (f x + e)^2 + 14 \cos (f x + e) - 4\right) \sin (f x + e) - 18 \cos (f x + e) - 4\right) / \left(\cos (f x + e)^3 + 3 \cos (f x + e)^2 + \left(\cos (f x + e)^2 - 2 \cos (f x + e) - 4\right) \sin (f x + e) - 2 \cos (f x + e) - 4\right) - \sqrt{-\left(a c^2 + a c d\right) g} d \log \left(\left(\left(128 a c^4 + 256 a c^3 d + 160 a c^2 d^2 + 32 a c d^3 + a d^4\right) g \cos (f x + e)^5 - \left(128 a c^4 + 192 a c^3 d + 64 a c^2 d^2 - 4 a c d^3 - a d^4\right) g \cos (f x + e)^4 - 2\left(208 a c^4 + 368 a c^3 d + 195 a c^2 d^2 + 32 a c d^3 + a d^4\right) g \cos (f x + e)^3 + 2\left(64 a c^4 + 94 a c^3 d + 29 a c^2 d^2 - 4 a c d^3 - a d^4\right) g \cos (f x + e)^2 + \left(289 a c^4 + 480 a c^3 d + 230 a c^2 d^2 + 32 a c d^3 + a d^4\right) g \cos (f x + e) + 8\left(\left(16 c^3 + 24 c^2 d + 10 c d^2 + d^3\right) \cos (f x + e)^4 - \left(24 c^3 + 28 c^2 d + 7 c d^2\right) \cos (f x + e)^3 + 51 c^3 + 59 c^2 d + 17 c d^2 + d^3 - \left(66 c^3 + 83 c^2 d + 27 c d^2 + 2 d^3\right) \cos (f x + e)^2 + \left(25 c^3 + 28 c^2 d + 7 c d^2\right) \cos (f x + e) + \left(\left(16 c^3 + 24 c^2 d + 10 c d^2 + d^3\right) \cos (f x + e)^3 - 51 c^3 - 59 c^2 d - 17 c d^2 - d^3 + \left(40 c^3 + 52 c^2 d + 17 c d^2 + d^3\right) \cos (f x + e)^2 - \left(26 c^3 + 31 c^2 d + 10 c d^2 + d^3\right) \cos (f x + e)\right) \sin (f x + e)\right) \sqrt{-\left(a c^2 + a c d\right) g} \sqrt{a \sin (f x + e) + a} \sqrt{g \sin (f x + e)} + \left(a c^4 + 4 a c^3 d + 6 a c^2 d^2 + \right. \end{aligned}$$

$$\begin{aligned}
& 2 + 4*a*c*d^3 + a*d^4)*g + ((128*a*c^4 + 256*a*c^3*d + 160*a*c^2*d^2 + 32*a*c*d^3 + a*d^4)*g*\cos(f*x + e)^4 + 4*(64*a*c^4 + 112*a*c^3*d + 56*a*c^2*d^2 + 7*a*c*d^3)*g*\cos(f*x + e)^3 - 2*(80*a*c^4 + 144*a*c^3*d + 83*a*c^2*d^2 + 18*a*c*d^3 + a*d^4)*g*\cos(f*x + e)^2 - 4*(72*a*c^4 + 119*a*c^3*d + 56*a*c^2*d^2 + 7*a*c*d^3)*g*\cos(f*x + e) + (a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4)*g*\sin(f*x + e))/(d^4*\cos(f*x + e)^5 + (4*c*d^3 + d^4)*\cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4 - 2*(3*c^2*d^2 + d^4)*\cos(f*x + e)^3 - 2*(2*c^3*d + 3*c^2*d^2 + 4*c*d^3 + d^4)*\cos(f*x + e)^2 + (c^4 + 6*c^2*d^2 + d^4)*\cos(f*x + e) + (d^4*\cos(f*x + e)^4 - 4*c*d^3*\cos(f*x + e)^3 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4 - 2*(3*c^2*d^2 + 2*c*d^3 + d^4)*\cos(f*x + e)^2 + 4*(c^3*d + c*d^3)*\cos(f*x + e))*\sin(f*x + e)))/((a*c^3 - a*c*d^2)*f*g), 1/4*(2*sqrt(2)*(a*c^2 + a*c*d)*g*sqrt(1/(a*g))*arctan(1/4*sqrt(2)*sqrt(a*\sin(f*x + e) + a)*sqrt(g*\sin(f*x + e))*sqrt(1/(a*g)))*(3*\sin(f*x + e) - 1)/(\cos(f*x + e)*\sin(f*x + e))) + sqrt(-(a*c^2 + a*c*d)*g)*d*log(((128*a*c^4 + 256*a*c^3*d + 160*a*c^2*d^2 + 32*a*c*d^3 + a*d^4)*g*\cos(f*x + e)^5 - (128*a*c^4 + 192*a*c^3*d + 64*a*c^2*d^2 - 4*a*c*d^3 - a*d^4)*g*\cos(f*x + e)^4 - 2*(208*a*c^4 + 368*a*c^3*d + 195*a*c^2*d^2 + 32*a*c*d^3 + a*d^4)*g*\cos(f*x + e)^3 + 2*(64*a*c^4 + 94*a*c^3*d + 29*a*c^2*d^2 - 4*a*c*d^3 - a*d^4)*g*\cos(f*x + e)^2 + (289*a*c^4 + 480*a*c^3*d + 230*a*c^2*d^2 + 32*a*c*d^3 + a*d^4)*g*\cos(f*x + e) + 8*((16*c^3 + 24*c^2*d + 10*c*d^2 + d^3)*\cos(f*x + e)^4 - (24*c^3 + 28*c^2*d + 7*c*d^2)*\cos(f*x + e)^3 + 51*c^3 + 59*c^2*d + 17*c*d^2 + d^3 - (66*c^3 + 83*c^2*d + 27*c*d^2 + 2*d^3)*\cos(f*x + e)^2 + (25*c^3 + 28*c^2*d + 7*c*d^2)*\cos(f*x + e) + ((16*c^3 + 24*c^2*d + 10*c*d^2 + d^3)*\cos(f*x + e)^3 - 51*c^3 - 59*c^2*d - 17*c*d^2 - d^3 + (40*c^3 + 52*c^2*d + 17*c*d^2 + d^3)*\cos(f*x + e)^2 - (26*c^3 + 31*c^2*d + 10*c*d^2 + d^3)*\cos(f*x + e))*\sin(f*x + e))*sqrt(-(a*c^2 + a*c*d)*g)*sqrt(a*\sin(f*x + e) + a)*sqrt(g*\sin(f*x + e)) + (a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4)*g + ((128*a*c^4 + 256*a*c^3*d + 160*a*c^2*d^2 + 32*a*c*d^3 + a*d^4)*g*\cos(f*x + e)^4 + 4*(64*a*c^4 + 112*a*c^3*d + 56*a*c^2*d^2 + 7*a*c*d^3)*g*\cos(f*x + e)^3 - 2*(80*a*c^4 + 144*a*c^3*d + 83*a*c^2*d^2 + 18*a*c*d^3 + a*d^4)*g*\cos(f*x + e)^2 - 4*(72*a*c^4 + 119*a*c^3*d + 56*a*c^2*d^2 + 7*a*c*d^3)*g*\cos(f*x + e) + (a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4)*g*\sin(f*x + e))/(d^4*\cos(f*x + e)^5 + (4*c*d^3 + d^4)*\cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4 - 2*(3*c^2*d^2 + d^4)*\cos(f*x + e)^3 - 2*(2*c^3*d + 3*c^2*d^2 + 4*c*d^3 + d^4)*\cos(f*x + e)^2 + (c^4 + 6*c^2*d^2 + d^4)*\cos(f*x + e) + (d^4*\cos(f*x + e)^4 - 4*c*d^3*\cos(f*x + e)^3 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4 - 2*(3*c^2*d^2 + 2*c*d^3 + d^4)*\cos(f*x + e)^2 + 4*(c^3*d + c*d^3)*\cos(f*x + e))*\sin(f*x + e)))/((a*c^3 - a*c*d^2)*f*g), -1/4*(sqrt(2)*(a*c^2 + a*c*d)*g*sqrt(-1/(a*g))*log((4*sqrt(2)*(3*\cos(f*x + e)^2 + (3*\cos(f*x + e) + 4)*\sin(f*x + e) - \cos(f*x + e) - 4)*sqrt(a*\sin(f*x + e) + a)*sqrt(g*\sin(f*x + e))*sqrt(-1/(a*g)) + 17*\cos(f*x + e)^3 + 3*\cos(f*x + e)^2 + (17*\cos(f*x + e)^2 + 14*\cos(f*x + e) - 4)*\sin(f*x + e) - 18*\cos(f*x + e) - 4)/(\cos(f*x + e)^3 + 3*\cos(f*x + e)^2 + (\cos(f*x + e)^2 - 2*\cos(f*x + e) - 4)*\sin(f*x + e) - 2*\cos(f*x + e) - 4)) + 2*sqrt((a*c^2 + a*c*d)*g)*d*arctan(1/4*((8*c^2 + 8*c*d + d^2)*\cos(f*x + e
\end{aligned}$$

)<sup>2</sup> - 9\*c<sup>2</sup> - 8\*c\*d - d<sup>2</sup> + 2\*(4\*c<sup>2</sup> + 3\*c\*d)\*sin(f\*x + e))\*sqrt((a\*c<sup>2</sup> + a\*c\*d)\*g)\*sqrt(a\*sin(f\*x + e) + a)\*sqrt(g\*sin(f\*x + e))/((2\*a\*c<sup>3</sup> + 3\*a\*c<sup>2</sup>\*d + a\*c\*d<sup>2</sup>)\*g\*cos(f\*x + e)<sup>3</sup> + (a\*c<sup>3</sup> + a\*c<sup>2</sup>\*...

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a(\sin(e+fx)+1)} \sqrt{g \sin(e+fx)} (c+d \sin(e+fx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+d\*sin(f\*x+e))/(g\*sin(f\*x+e))\*\*(1/2)/(a+a\*sin(f\*x+e))\*\*(1/2), x)

[Out] Integral(1/(sqrt(a\*(sin(e + f\*x) + 1))\*sqrt(g\*sin(e + f\*x))\*(c + d\*sin(e + f\*x))), x)

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+d\*sin(f\*x+e))/(g\*sin(f\*x+e))^(1/2)/(a+a\*sin(f\*x+e))^(1/2), x, algorithm="giac")

[Out] Timed out

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{g \sin(e+fx)} \sqrt{a+a \sin(e+fx)} (c+d \sin(e+fx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((g\*sin(e + f\*x))^(1/2)\*(a + a\*sin(e + f\*x))^(1/2)\*(c + d\*sin(e + f\*x))), x)

[Out] int(1/((g\*sin(e + f\*x))^(1/2)\*(a + a\*sin(e + f\*x))^(1/2)\*(c + d\*sin(e + f\*x))), x)

$$3.29 \quad \int \frac{\csc(e+fx) \sqrt{a+b \sin(e+fx)}}{c+c \sin(e+fx)} dx$$

**Optimal.** Leaf size=238

$$\frac{E\left(\frac{1}{2}\left(e - \frac{\pi}{2} + fx\right) \middle| \frac{2b}{a+b}\right) \sqrt{a+b \sin(e+fx)}}{cf \sqrt{\frac{a+b \sin(e+fx)}{a+b}}} - \frac{(a-b)F\left(\frac{1}{2}\left(e - \frac{\pi}{2} + fx\right) \middle| \frac{2b}{a+b}\right) \sqrt{\frac{a+b \sin(e+fx)}{a+b}}}{cf \sqrt{a+b \sin(e+fx)}} + \frac{2a\Pi\left(2; \frac{e+fx-\frac{\pi}{2}}{2} \middle| \frac{2b}{a+b}\right)}{cf \sqrt{a+b \sin(e+fx)}}$$

[Out] cos(f\*x+e)\*(a+b\*sin(f\*x+e))^(1/2)/f/(c+c\*sin(f\*x+e))-(sin(1/2\*e+1/4\*Pi+1/2\*f\*x)^2)^(1/2)/sin(1/2\*e+1/4\*Pi+1/2\*f\*x)\*EllipticE(cos(1/2\*e+1/4\*Pi+1/2\*f\*x), 2^(1/2)\*(b/(a+b))^(1/2))\*(a+b\*sin(f\*x+e))^(1/2)/c/f/((a+b\*sin(f\*x+e))/(a+b))^(1/2)+(a-b)\*(sin(1/2\*e+1/4\*Pi+1/2\*f\*x)^2)^(1/2)/sin(1/2\*e+1/4\*Pi+1/2\*f\*x)\*EllipticF(cos(1/2\*e+1/4\*Pi+1/2\*f\*x), 2^(1/2)\*(b/(a+b))^(1/2))\*((a+b\*sin(f\*x+e))/(a+b))^(1/2)/c/f/(a+b\*sin(f\*x+e))^(1/2)-2\*a\*(sin(1/2\*e+1/4\*Pi+1/2\*f\*x)^2)^(1/2)/sin(1/2\*e+1/4\*Pi+1/2\*f\*x)\*EllipticPi(cos(1/2\*e+1/4\*Pi+1/2\*f\*x), 2^(1/2)\*(b/(a+b))^(1/2))\*((a+b\*sin(f\*x+e))/(a+b))^(1/2)/c/f/(a+b\*sin(f\*x+e))^(1/2)

**Rubi [A]**

time = 0.34, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$ , Rules used = {3014, 2886, 2884, 2847, 2831, 2742, 2740, 2734, 2732}

$$\frac{\cos(e+fx)\sqrt{a+b \sin(e+fx)}}{f(c \sin(e+fx)+c)} - \frac{(a-b)\sqrt{\frac{a+b \sin(e+fx)}{a+b}} F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right) \middle| \frac{2b}{a+b}\right)}{cf \sqrt{a+b \sin(e+fx)}} + \frac{\sqrt{a+b \sin(e+fx)} E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right) \middle| \frac{2b}{a+b}\right)}{cf \sqrt{\frac{a+b \sin(e+fx)}{a+b}}} + \frac{2a\sqrt{\frac{a+b \sin(e+fx)}{a+b}} \Pi\left(2; \frac{1}{2}\left(e+fx-\frac{\pi}{2}\right) \middle| \frac{2b}{a+b}\right)}{cf \sqrt{a+b \sin(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[(Csc[e + f\*x]\*Sqrt[a + b\*Sin[e + f\*x]])/(c + c\*Sin[e + f\*x]),x]

[Out] (EllipticE[(e - Pi/2 + f\*x)/2, (2\*b)/(a + b)]\*Sqrt[a + b\*Sin[e + f\*x]])/(c\*f\*Sqrt[(a + b\*Sin[e + f\*x])/(a + b)]) - ((a - b)\*EllipticF[(e - Pi/2 + f\*x)/2, (2\*b)/(a + b)]\*Sqrt[(a + b\*Sin[e + f\*x])/(a + b)])/(c\*f\*Sqrt[a + b\*Sin[e + f\*x]]) + (2\*a\*EllipticPi[2, (e - Pi/2 + f\*x)/2, (2\*b)/(a + b)]\*Sqrt[(a + b\*Sin[e + f\*x])/(a + b)])/(c\*f\*Sqrt[a + b\*Sin[e + f\*x]]) + (Cos[e + f\*x]\*Sqrt[a + b\*Sin[e + f\*x]])/(f\*(c + c\*Sin[e + f\*x]))

Rule 2732

Int[Sqrt[(a\_) + (b\_)\*sin[(c\_) + (d\_)\*(x\_)]], x\_Symbol] := Simp[2\*(Sqrt[a + b]/d)\*EllipticE[(1/2)\*(c - Pi/2 + d\*x), 2\*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2734



```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
0] && !GtQ[a + b, 0]
```

#### Rule 2740

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

#### Rule 2742

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a
+ b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

#### Rule 2831

```
Int[((c_) + (d_)*sin[(e_) + (f_)*(x_)])/Sqrt[(a_) + (b_)*sin[(e_) + (
f_)*(x_)]], x_Symbol] := Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

#### Rule 2847

```
Int[((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_)/((a_) + (b_)*sin[(e_) +
(f_)*(x_)]), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x]))], x] + Dist[d/(a*(b*c - a*d))
, Int[(c + d*Sin[e + f*x])^n*(a^n - b*(n + 1)*Sin[e + f*x]), x], x] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c
^2 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

#### Rule 2884

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_)
+ (f_)*(x_)]]), x_Symbol] := Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

#### Rule 2886

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_)
+ (f_)*(x_)]]), x_Symbol] := Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
```



```
[In] Integrate[(Csc[e + f*x]*Sqrt[a + b*Sin[e + f*x]])/(c + c*Sin[e + f*x]),x]
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-8*Sin[(e + f*x)/2]*Sqrt[a + b*Sin[e + f*x]] + (Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((( -2*I)*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[e + f*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[e + f*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[e + f*x]]], (a + b)/(a - b)])))*Sec[e + f*x]*Sqrt[-((b*(-1 + Sin[e + f*x]))/(a + b))]*Sqrt[-((b*(1 + Sin[e + f*x]))/(a - b))])/(a*b*Sqrt[-(a + b)^(-1)]) + 4*Sqrt[a + b*Sin[e + f*x]] - (4*b*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]/Sqrt[a + b*Sin[e + f*x]] - (2*(4*a + b)*EllipticPi[2, (-2*e + Pi - 2*f*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]/Sqrt[a + b*Sin[e + f*x]])))/(4*c*f*(1 + Sin[e + f*x]))
```

**Maple [A]**

time = 33.06, size = 593, normalized size = 2.49

method	result
default	$\frac{\sqrt{-(-b \sin(fx + e) - a) (\cos^2(fx + e))}}{\left( \frac{2\left(\frac{a}{b} - 1\right) \sqrt{\frac{a + b \sin(fx + e)}{a - b}} \sqrt{\frac{(1 - \sin(fx + e))b}{a + b}} \sqrt{\frac{(-1 - \sin(fx + e))}{a - b}}}{\sqrt{-(-b \sin(fx + e) - a)}} $

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^(1/2)/sin(f*x+e)/(c+c*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-b*sin(f*x+e)-a)*cos(f*x+e)^2)^(1/2)/c*(-2*(1/b*a-1)*((a+b*sin(f*x+e))/(a-b))^(1/2)*(1/(a+b)*(1-sin(f*x+e))*b)^(1/2)*(1/(a-b)*(-1-sin(f*x+e))*b)^(1/2)/(-(-b*sin(f*x+e)-a)*cos(f*x+e)^2)^(1/2)*b*EllipticPi(((a+b*sin(f*x+e))/(a-b))^(1/2),-(-1/b*a+1)*b/a,((a-b)/(a+b))^(1/2))+(-a+b)*(-(-b*sin(f*x+e)-a)*sin(f*x+e)+b*sin(f*x+e)+a)/(a-b)/((1+sin(f*x+e))*(sin(f*x+e)-1)*(-b*sin(f*x+e)-a))^(1/2)-2*b/(2*a-2*b)*(1/b*a-1)*((a+b*sin(f*x+e))/(a-b))^(1/2)*(1/(a+b)*(1-sin(f*x+e))*b)^(1/2)*(1/(a-b)*(-1-sin(f*x+e))*b)^(1/2)/(-(-b*sin(f*x+e)-a)*cos(f*x+e)^2)^(1/2)*EllipticF(((a+b*sin(f*x+e))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))-b/(a-b)*(1/b*a-1)*((a+b*sin(f*x+e))/(a-b))^(1/2)*(1/(a+b)*(1-sin(f*x+e))*b)^(1/2)*(1/(a-b)*(-1-sin(f*x+e))*b)^(1/2)/(-(-b*sin(f*x+e)-a)*cos(f*x+e)^2)^(1/2)*((-1/b*a-1)*EllipticE(((a+b*sin(f*x+e))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))+EllipticF(((a+b*sin(f*x+e))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))))/cos(f*x+e)/(a+b*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2)/sin(f*x+e)/(c+c*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)/((c*sin(f*x + e) + c)*sin(f*x + e)), x)
```

**Fricas [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2)/sin(f*x+e)/(c+c*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: failed of mode Union(SparseUnivariatePolynomial(SimpleAlgebraicExtension(InnerPrimeField(17),SparseUnivariatePolynomial(InnerPrimeField(17)),?^2+2*?+13)),failed)
```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{a + b \sin(e + fx)}}{\sin^2(e + fx) + \sin(e + fx)} dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**(1/2)/sin(f*x+e)/(c+c*sin(f*x+e)),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x))/(sin(e + f*x)**2 + sin(e + f*x)), x)/c
```

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2)/sin(f*x+e)/(c+c*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)/((c*sin(f*x + e) + c)*sin(f*x + e)), x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a + b \sin(e + fx)}}{\sin(e + fx) (c + c \sin(e + fx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^(1/2)/(sin(e + f*x)*(c + c*sin(e + f*x))),x)
```

```
[Out] int((a + b*sin(e + f*x))^(1/2)/(sin(e + f*x)*(c + c*sin(e + f*x))), x)
```

$$3.30 \quad \int \frac{\csc(e+fx)}{\sqrt{a+b\sin(e+fx)}(c+c\sin(e+fx))} dx$$

Optimal. Leaf size=246

$$\frac{E\left(\frac{1}{2}\left(e - \frac{\pi}{2} + fx\right) \middle| \frac{2b}{a+b}\right) \sqrt{a+b\sin(e+fx)}}{(a-b)cf \sqrt{\frac{a+b\sin(e+fx)}{a+b}}} - \frac{F\left(\frac{1}{2}\left(e - \frac{\pi}{2} + fx\right) \middle| \frac{2b}{a+b}\right) \sqrt{\frac{a+b\sin(e+fx)}{a+b}}}{cf \sqrt{a+b\sin(e+fx)}} + \frac{2\Pi\left(2; \frac{1}{2}\left(e - \frac{\pi}{2}\right) \middle| \frac{2b}{a+b}\right)}{cf}$$

[Out]  $\cos(f*x+e)*(a+b*\sin(f*x+e))^{(1/2)}/(a-b)/f/(c+c*\sin(f*x+e))-(\sin(1/2*e+1/4*Pi+1/2*f*x))^{(1/2)}/\sin(1/2*e+1/4*Pi+1/2*f*x)*\text{EllipticE}(\cos(1/2*e+1/4*Pi+1/2*f*x), 2^{(1/2)}*(b/(a+b))^{(1/2)})*(a+b*\sin(f*x+e))^{(1/2)}/(a-b)/c/f/((a+b*\sin(f*x+e))/(a+b))^{(1/2)}+(\sin(1/2*e+1/4*Pi+1/2*f*x))^{(1/2)}/\sin(1/2*e+1/4*Pi+1/2*f*x)*\text{EllipticF}(\cos(1/2*e+1/4*Pi+1/2*f*x), 2^{(1/2)}*(b/(a+b))^{(1/2)})*((a+b*\sin(f*x+e))/(a+b))^{(1/2)}/c/f/(a+b*\sin(f*x+e))^{(1/2)}-2*(\sin(1/2*e+1/4*Pi+1/2*f*x))^{(1/2)}/\sin(1/2*e+1/4*Pi+1/2*f*x)*\text{EllipticPi}(\cos(1/2*e+1/4*Pi+1/2*f*x), 2, 2^{(1/2)}*(b/(a+b))^{(1/2)})*((a+b*\sin(f*x+e))/(a+b))^{(1/2)}/c/f/(a+b*\sin(f*x+e))^{(1/2)}$

Rubi [A]

time = 0.33, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$ , Rules used = {3020, 2886, 2884, 2847, 2831, 2742, 2740, 2734, 2732}

$$\frac{\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{f(a-b)(c\sin(e+fx)+c)} - \frac{\sqrt{\frac{a+b\sin(e+fx)}{a+b}} F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right) \middle| \frac{2b}{a+b}\right)}{cf\sqrt{a+b\sin(e+fx)}} + \frac{\sqrt{a+b\sin(e+fx)} E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right) \middle| \frac{2b}{a+b}\right)}{cf(a-b)\sqrt{\frac{a+b\sin(e+fx)}{a+b}}} + \frac{2\sqrt{\frac{a+b\sin(e+fx)}{a+b}} \Pi\left(2; \frac{1}{2}\left(e+fx-\frac{\pi}{2}\right) \middle| \frac{2b}{a+b}\right)}{cf\sqrt{a+b\sin(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[Csc[e + f\*x]/(Sqrt[a + b\*Sin[e + f\*x]]\*(c + c\*Sin[e + f\*x])),x]

[Out]  $(\text{EllipticE}[(e - \text{Pi}/2 + f*x)/2, (2*b)/(a + b)]*\text{Sqrt}[a + b*\text{Sin}[e + f*x]])/((a - b)*c*f*\text{Sqrt}[(a + b*\text{Sin}[e + f*x])/(a + b)]) - (\text{EllipticF}[(e - \text{Pi}/2 + f*x)/2, (2*b)/(a + b)]*\text{Sqrt}[(a + b*\text{Sin}[e + f*x])/(a + b)])/(c*f*\text{Sqrt}[a + b*\text{Sin}[e + f*x]]) + (2*\text{EllipticPi}[2, (e - \text{Pi}/2 + f*x)/2, (2*b)/(a + b)]*\text{Sqrt}[(a + b*\text{Sin}[e + f*x])/(a + b)])/(c*f*\text{Sqrt}[a + b*\text{Sin}[e + f*x]]) + (\text{Cos}[e + f*x]*\text{Sqrt}[a + b*\text{Sin}[e + f*x]])/((a - b)*f*(c + c*\text{Sin}[e + f*x]))$

Rule 2732

Int[Sqrt[(a\_) + (b\_)\*sin[(c\_) + (d\_)\*(x\_)]], x\_Symbol] :> Simp[2\*(Sqrt[a + b]/d)\*EllipticE[(1/2)\*(c - Pi/2 + d\*x), 2\*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2734

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)], x_Symbol] := Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
0] && !GtQ[a + b, 0]
```

#### Rule 2740

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)], x_Symbol] := Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

#### Rule 2742

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)], x_Symbol] := Dist[Sqrt[(a
+ b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

#### Rule 2831

```
Int[((c_) + (d_)*sin[(e_) + (f_)*(x_)])/Sqrt[(a_) + (b_)*sin[(e_) + (
f_)*(x_)], x_Symbol] := Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

#### Rule 2847

```
Int[((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_)/((a_) + (b_)*sin[(e_) +
(f_)*(x_)]), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x]))], x] + Dist[d/(a*(b*c - a*d))
, Int[(c + d*Sin[e + f*x])^n*(a^n - b*(n + 1)*Sin[e + f*x]), x], x] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c
^2 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

#### Rule 2884

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_)
+ (f_)*(x_)])], x_Symbol] := Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

#### Rule 2886

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_)
+ (f_)*(x_)])], x_Symbol] := Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
```





```
[In] Integrate[Csc[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*(c + c*Sin[e + f*x])),x]
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-8*Sin[(e + f*x)/2]*Sqrt[a + b*Sin[e + f*x]] - (Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(((2*I)*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[e + f*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[e + f*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[e + f*x]]], (a + b)/(a - b)]))*Sec[e + f*x]*Sqrt[-((b*(-1 + Sin[e + f*x]))/(a + b))]*Sqrt[-((b*(1 + Sin[e + f*x]))/(a - b))])/(a*b*Sqrt[-(a + b)^(-1)]) - 4*Sqrt[a + b*Sin[e + f*x]] + (4*b*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]/Sqrt[a + b*Sin[e + f*x]] + (2*(4*a - 3*b)*EllipticPi[2, (-2*e + Pi - 2*f*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]/Sqrt[a + b*Sin[e + f*x]])))/(4*(a - b)*c*f*(1 + Sin[e + f*x]))
```

**Maple [A]**

time = 30.58, size = 587, normalized size = 2.39

method	result
default	$\frac{\sqrt{-(-b \sin(fx + e) - a) (\cos^2(fx + e))}}{\left( \frac{2\left(\frac{a}{b} - 1\right) \sqrt{\frac{a + b \sin(fx + e)}{a - b}} \sqrt{\frac{(1 - \sin(fx + e))b}{a + b}} \sqrt{\frac{(-1 - \sin(fx + e))}{a - b}}}{\sqrt{-(-b \sin(fx + e) - a)}} $

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(f*x+e)/(c+c*sin(f*x+e))/(a+b*sin(f*x+e))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] (-(-b*sin(f*x+e)-a)*cos(f*x+e)^2)^(1/2)/c*(-2*(1/b*a-1)*((a+b*sin(f*x+e))/(a-b))^(1/2)*(1/(a+b)*(1-sin(f*x+e))*b)^(1/2)*(1/(a-b)*(-1-sin(f*x+e))*b)^(1/2)/(-(-b*sin(f*x+e)-a)*cos(f*x+e)^2)^(1/2)*b/a*EllipticPi(((a+b*sin(f*x+e))/(a-b))^(1/2),-(-1/b*a+1)*b/a,((a-b)/(a+b))^(1/2))+(-b*sin(f*x+e)^2-a*sin(f*x+e)+b*sin(f*x+e)+a)/(a-b)/((1+sin(f*x+e))*(sin(f*x+e)-1)*(-b*sin(f*x+e)-a))^(1/2)+2*b/(2*a-2*b)*(1/b*a-1)*((a+b*sin(f*x+e))/(a-b))^(1/2)*(1/(a+b)*(1-sin(f*x+e))*b)^(1/2)*(1/(a-b)*(-1-sin(f*x+e))*b)^(1/2)/(-(-b*sin(f*x+e)-a)*cos(f*x+e)^2)^(1/2)*EllipticF(((a+b*sin(f*x+e))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))+b/(a-b)*(1/b*a-1)*((a+b*sin(f*x+e))/(a-b))^(1/2)*(1/(a+b)*(1-sin(f*x+e))*b)^(1/2)*(1/(a-b)*(-1-sin(f*x+e))*b)^(1/2)/(-(-b*sin(f*x+e)-a)*cos(f*x+e)^2)^(1/2)*((-1/b*a-1)*EllipticE(((a+b*sin(f*x+e))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))+EllipticF(((a+b*sin(f*x+e))/(a-b))^(1/2),((a-b)/(a+b))^(1/2)))/cos(f*x+e)/(a+b*sin(f*x+e))^(1/2)/f
```

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(f*x+e)/(c+c*sin(f*x+e))/(a+b*sin(f*x+e))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(1/(sqrt(b*sin(f*x + e) + a)*(c*sin(f*x + e) + c)*sin(f*x + e)), x
)
```

**Fricas** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(f*x+e)/(c+c*sin(f*x+e))/(a+b*sin(f*x+e))^(1/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b \sin(e + fx)} \sin^2(e + fx) + \sqrt{a + b \sin(e + fx)} \sin(e + fx)} dx$$

c

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(f*x+e)/(c+c*sin(f*x+e))/(a+b*sin(f*x+e))^(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*sin(e + f*x))*sin(e + f*x)**2 + sqrt(a + b*sin(e + f
*x))*sin(e + f*x)), x)/c
```

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(f*x+e)/(c+c*sin(f*x+e))/(a+b*sin(f*x+e))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate(1/(sqrt(b*sin(f*x + e) + a)*(c*sin(f*x + e) + c)*sin(f*x + e)), x
)
```

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sin(e + fx) \sqrt{a + b \sin(e + fx)} (c + c \sin(e + fx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)*(a + b*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))),x)
```

```
[Out] int(1/(sin(e + f*x)*(a + b*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))), x)
```

$$3.31 \quad \int \frac{\sqrt{g \sin(e + fx)} \sqrt{a + b \sin(e + fx)}}{c + c \sin(e + fx)} dx$$

Optimal. Leaf size=267

$$\frac{2\sqrt{g} \Pi\left(\frac{b}{a+b}; \sin^{-1}\left(\frac{\sqrt{a+b} \sqrt{g \sin(e+fx)}}{\sqrt{g} \sqrt{a+b \sin(e+fx)}}\right) \mid -\frac{a-b}{a+b}\right) \sec(e+fx) \sqrt{\frac{a(1-\sin(e+fx))}{a+b \sin(e+fx)}} \sqrt{\frac{a(1+\sin(e+fx))}{a+b \sin(e+fx)}}}{\sqrt{a+b} cf}$$

```
[Out] 2*EllipticPi((a+b)^(1/2)*(g*sin(f*x+e))^(1/2)/g^(1/2)/(a+b*sin(f*x+e))^(1/2), b/(a+b), ((-a+b)/(a+b))^(1/2))*sec(f*x+e)*(a+b*sin(f*x+e))*g^(1/2)*(a*(1-sin(f*x+e))/(a+b*sin(f*x+e)))^(1/2)*(a*(1+sin(f*x+e))/(a+b*sin(f*x+e)))^(1/2)/c/f/(a+b)^(1/2)+g*EllipticE(cos(f*x+e)/(1+sin(f*x+e)), ((-a+b)/(a+b))^(1/2))*((sin(f*x+e)/(1+sin(f*x+e)))^(1/2)*(a+b*sin(f*x+e))^(1/2)/c/f/(g*sin(f*x+e))^(1/2)/((a+b*sin(f*x+e))/(a+b)/(1+sin(f*x+e)))^(1/2))
```

Rubi [A]

time = 0.34, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 39,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {3007, 2890, 3011}

$$\frac{g \sqrt{\frac{\sin(e+fx)}{\sin(e+fx)+1}} \sqrt{a+b \sin(e+fx)} E\left(\text{ArcSin}\left(\frac{\cos(e+fx)}{\sin(e+fx)+1}\right) \mid -\frac{a-b}{a+b}\right) + 2\sqrt{g} \sec(e+fx) \sqrt{\frac{a(1-\sin(e+fx))}{a+b \sin(e+fx)}} \sqrt{\frac{a(\sin(e+fx)+1)}{a+b \sin(e+fx)}} (a+b \sin(e+fx)) \Pi\left(\frac{b}{a+b}; \text{ArcSin}\left(\frac{\sqrt{a+b} \sqrt{g \sin(e+fx)}}{\sqrt{g} \sqrt{a+b \sin(e+fx)}}\right) \mid -\frac{a-b}{a+b}\right)}{cf \sqrt{g \sin(e+fx)} \sqrt{\frac{a+b \sin(e+fx)}{(a+b)(\sin(e+fx)+1)}} + \frac{cf \sqrt{a+b}}{cf \sqrt{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[g*Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]])/(c + c*Sin[e + f*x]),x]
```

```
[Out] (2*Sqrt[g]*EllipticPi[b/(a + b), ArcSin[(Sqrt[a + b]*Sqrt[g*Sin[e + f*x]])/(Sqrt[g]*Sqrt[a + b*Sin[e + f*x]])], -((a - b)/(a + b))]*Sec[e + f*x]*Sqrt[(a*(1 - Sin[e + f*x]))/(a + b*Sin[e + f*x])]*Sqrt[(a*(1 + Sin[e + f*x]))/(a + b*Sin[e + f*x])]*(a + b*Sin[e + f*x])]/(Sqrt[a + b]*c*f) + (g*EllipticE[ArcSin[Cos[e + f*x]/(1 + Sin[e + f*x])], -((a - b)/(a + b))]*Sqrt[Sin[e + f*x]/(1 + Sin[e + f*x])]*Sqrt[a + b*Sin[e + f*x]])/(c*f*Sqrt[g*Sin[e + f*x]]*Sqrt[(a + b*Sin[e + f*x])/((a + b)*(1 + Sin[e + f*x]))])
```

Rule 2890

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e + f*x])))]*Sqrt[(-b*c - a*d)*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
```

$c + d)/((a + b)*(c - d))], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{PosQ}[(a + b)/(c + d)]$

### Rule 3007

$\text{Int}[(\text{Sqrt}[(g_.)*\sin[(e_.) + (f_.)*(x_.)])*\text{Sqrt}[(a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)])]/((c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_.)]), x\_Symbol] :> \text{Dist}[g/d, \text{Int}[\text{Sqrt}[a + b*\text{Sin}[e + f*x]]/\text{Sqrt}[g*\text{Sin}[e + f*x]], x], x] - \text{Dist}[c*(g/d), \text{Int}[\text{Sqrt}[a + b*\text{Sin}[e + f*x]]/(\text{Sqrt}[g*\text{Sin}[e + f*x]]*(c + d*\text{Sin}[e + f*x])), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ (\text{EqQ}[a^2 - b^2, 0] \ || \ \text{EqQ}[c^2 - d^2, 0])$

### Rule 3011

$\text{Int}[\text{Sqrt}[(a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]]/(\text{Sqrt}[(g_.)*\sin[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_.)])), x\_Symbol] :> \text{Simp}[(-\text{Sqrt}[a + b*\text{Sin}[e + f*x]])*(\text{Sqrt}[d*(\text{Sin}[e + f*x]/(c + d*\text{Sin}[e + f*x]))]/(d*f*\text{Sqrt}[g*\text{Sin}[e + f*x]]*\text{Sqrt}[c^2*((a + b*\text{Sin}[e + f*x])/((a*c + b*d)*(c + d*\text{Sin}[e + f*x])))])))*\text{EllipticE}[\text{ArcSin}[c*(\text{Cos}[e + f*x]/(c + d*\text{Sin}[e + f*x]))], (b*c - a*d)/(b*c + a*d)], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{EqQ}[c^2 - d^2, 0]$

### Rubi steps

$$\int \frac{\sqrt{g \sin(e + fx)} \sqrt{a + b \sin(e + fx)}}{c + c \sin(e + fx)} dx = - \left( g \int \frac{\sqrt{a + b \sin(e + fx)}}{\sqrt{g \sin(e + fx)} (c + c \sin(e + fx))} dx \right) + \frac{g \int \frac{\sqrt{a + b \sin(e + fx)}}{\sqrt{g \sin(e + fx)}} dx}{c + c \sin(e + fx)}$$

$$= \frac{2\sqrt{g} \Pi\left(\frac{b}{a+b}; \sin^{-1}\left(\frac{\sqrt{a+b} \sqrt{g \sin(e+fx)}}{\sqrt{g} \sqrt{a+b \sin(e+fx)}}\right) \middle| -\frac{a-b}{a+b}\right) \sec(e)}{c + c \sin(e + fx)}$$

**Mathematica [C]** Result contains complex when optimal does not.  
time = 87.34, size = 13199, normalized size = 49.43

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(Sqrt[g\*Sin[e + f\*x]]\*Sqrt[a + b\*Sin[e + f\*x]])/(c + c\*Sin[e + f\*x]), x]

[Out] Result too large to show

**Maple [C]** Result contains complex when optimal does not.  
time = 0.96, size = 22961, normalized size = 86.00

method	result	size
default	Expression too large to display	22961

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g*sin(f*x+e))^(1/2)*(a+b*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e)),x,method=_RETURNVERBOSE)`

[Out] result too large to display

**Maxima [F]**  
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*sin(f*x+e))^(1/2)*(a+b*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e)),x, algorithm="maxima")`

[Out] `integrate(sqrt(b*sin(f*x + e) + a)*sqrt(g*sin(f*x + e))/(c*sin(f*x + e) + c), x)`

**Fricas [F]**  
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*sin(f*x+e))^(1/2)*(a+b*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e)),x, algorithm="fricas")`

[Out] `integral(sqrt(b*sin(f*x + e) + a)*sqrt(g*sin(f*x + e))/(c*sin(f*x + e) + c), x)`

**Sympy [F]**  
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{g \sin(e + fx)} \sqrt{a + b \sin(e + fx)}}{\sin(e + fx) + 1} dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*sin(f\*x+e))\*\*(1/2)\*(a+b\*sin(f\*x+e))\*\*(1/2)/(c+c\*sin(f\*x+e)),x)

[Out] Integral(sqrt(g\*sin(e + f\*x))\*sqrt(a + b\*sin(e + f\*x))/(sin(e + f\*x) + 1), x)/c

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*sin(f\*x+e))^(1/2)\*(a+b\*sin(f\*x+e))^(1/2)/(c+c\*sin(f\*x+e)),x, algorithm="giac")

[Out] integrate(sqrt(b\*sin(f\*x + e) + a)\*sqrt(g\*sin(f\*x + e))/(c\*sin(f\*x + e) + c), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{g \sin(e + f x)} \sqrt{a + b \sin(e + f x)}}{c + c \sin(e + f x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((g\*sin(e + f\*x))^(1/2)\*(a + b\*sin(e + f\*x))^(1/2))/(c + c\*sin(e + f\*x)),x)

[Out] int(((g\*sin(e + f\*x))^(1/2)\*(a + b\*sin(e + f\*x))^(1/2))/(c + c\*sin(e + f\*x)), x)

$$3.32 \quad \int \frac{\sqrt{a + b \sin(e + fx)}}{\sqrt{g \sin(e + fx)} (c + c \sin(e + fx))} dx$$

Optimal. Leaf size=116

$$\frac{E\left(\sin^{-1}\left(\frac{\cos(e+fx)}{1+\sin(e+fx)}\right) \middle| -\frac{a-b}{a+b}\right) \sqrt{\frac{\sin(e+fx)}{1+\sin(e+fx)}} \sqrt{a+b \sin(e+fx)}}{cf \sqrt{g \sin(e+fx)} \sqrt{\frac{a+b \sin(e+fx)}{(a+b)(1+\sin(e+fx))}}}$$

[Out] -EllipticE(cos(f\*x+e)/(1+sin(f\*x+e)),((-a+b)/(a+b))^(1/2))\*(sin(f\*x+e)/(1+sin(f\*x+e)))^(1/2)\*(a+b\*sin(f\*x+e))^(1/2)/c/f/(g\*sin(f\*x+e))^(1/2)/((a+b\*sin(f\*x+e))/(a+b)/(1+sin(f\*x+e)))^(1/2)

Rubi [A]

time = 0.13, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 39,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$ , Rules used = {3011}

$$\frac{\sqrt{\frac{\sin(e+fx)}{\sin(e+fx)+1}} \sqrt{a+b \sin(e+fx)} E\left(\text{ArcSin}\left(\frac{\cos(e+fx)}{\sin(e+fx)+1}\right) \middle| -\frac{a-b}{a+b}\right)}{cf \sqrt{g \sin(e+fx)} \sqrt{\frac{a+b \sin(e+fx)}{(a+b)(\sin(e+fx)+1)}}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b\*Sin[e + f\*x]]/(Sqrt[g\*Sin[e + f\*x]]\*(c + c\*Sin[e + f\*x])),x]

[Out] -((EllipticE[ArcSin[Cos[e + f\*x]/(1 + Sin[e + f\*x])], -(a - b)/(a + b)]\*Sqrt[Sin[e + f\*x]/(1 + Sin[e + f\*x])]\*Sqrt[a + b\*Sin[e + f\*x]]/(c\*f\*Sqrt[g\*Sin[e + f\*x]]\*Sqrt[(a + b\*Sin[e + f\*x])/((a + b)\*(1 + Sin[e + f\*x]))]))

Rule 3011

Int[Sqrt[(a\_) + (b\_)\*sin[(e\_) + (f\_)\*(x\_)]]/(Sqrt[(g\_)\*sin[(e\_) + (f\_)\*(x\_)]]\*(c\_) + (d\_)\*sin[(e\_) + (f\_)\*(x\_)]), x\_Symbol] :> Simp[(-Sqrt[a + b\*Sin[e + f\*x]]\*(Sqrt[d\*(Sin[e + f\*x]/(c + d\*Sin[e + f\*x]))]/(d\*f\*Sqrt[g\*Sin[e + f\*x]]\*Sqrt[c^2\*((a + b\*Sin[e + f\*x])/((a\*c + b\*d)\*(c + d\*Sin[e + f\*x]))])))\*EllipticE[ArcSin[c\*(Cos[e + f\*x]/(c + d\*Sin[e + f\*x]))], (b\*c - a\*d)/(b\*c + a\*d), x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]

Rubi steps





$$\begin{aligned}
& [\text{Sin}[e + f*x]]*\text{Sqrt}[a + b*\text{Sin}[e + f*x]] - (\text{Cos}[e + f*x]*\text{Sqrt}[a + b*\text{Sin}[e + \\
& f*x]]*(1 - \text{Cos}[e + f*x] + \text{Sin}[e + f*x] - (2*a*(\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[(-b + \\
& \text{Sqrt}[-a^2 + b^2] - a*\text{Tan}[(e + f*x)/2])/ \text{Sqrt}[-a^2 + b^2]]/\text{Sqrt}[2]], (2*\text{Sqrt} \\
& [-a^2 + b^2])/(-b + \text{Sqrt}[-a^2 + b^2]))*\text{Tan}[(e + f*x)/2] + \text{EllipticF}[\text{ArcSin}[ \\
& \text{Sqrt}[(b + \text{Sqrt}[-a^2 + b^2] + a*\text{Tan}[(e + f*x)/2])/ \text{Sqrt}[-a^2 + b^2]]/\text{Sqrt}[2]] \\
& , (2*\text{Sqrt}[-a^2 + b^2])/(b + \text{Sqrt}[-a^2 + b^2]))*\text{Sqrt}[(a*\text{Tan}[(e + f*x)/2])/(- \\
& b + \text{Sqrt}[-a^2 + b^2]))*\text{Sqrt}[-((a*\text{Tan}[(e + f*x)/2])/(b + \text{Sqrt}[-a^2 + b^2]))] \\
& ))/(\text{Sqrt}[-a^2 + b^2]*\text{Sqrt}[(a*\text{Sec}[(e + f*x)/2]^2*(a + b*\text{Sin}[e + f*x]))/(a^2 \\
& - b^2))*\text{Sqrt}[(a*\text{Tan}[(e + f*x)/2])/(-b + \text{Sqrt}[-a^2 + b^2]))]))/(2*\text{Sin}[e + f* \\
& x]^(3/2)) + (\text{Sqrt}[a + b*\text{Sin}[e + f*x]]*(\text{Cos}[e + f*x] + \text{Sin}[e + f*x] + (a^2*S \\
& ec[(e + f*x)/2]^2*(\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[(-b + \text{Sqrt}[-a^2 + b^2] - a*\text{Tan}[(e \\
& + f*x)/2])/ \text{Sqrt}[-a^2 + b^2]]/\text{Sqrt}[2]], (2*\text{Sqrt}[-a^2 + b^2])/(-b + \text{Sqrt}[-a^2 \\
& + b^2]))*\text{Tan}[(e + f*x)/2] + \text{EllipticF}[\text{ArcSin}[\text{Sqrt}[(b + \text{Sqrt}[-a^2 + b^2] + \\
& a*\text{Tan}[(e + f*x)/2])/ \text{Sqrt}[-a^2 + b^2]]/\text{Sqrt}[2]], (2*\text{Sqrt}[-a^2 + b^2])/(b + S \\
& qrt[-a^2 + b^2]))*\text{Sqrt}[(a*\text{Tan}[(e + f*x)/2])/(-b + \text{Sqrt}[-a^2 + b^2]))*\text{Sqrt}[- \\
& ((a*\text{Tan}[(e + f*x)/2])/(b + \text{Sqrt}[-a^2 + b^2]))]))/(2*\text{Sqrt}[-a^2 + b^2]*(-b + \\
& \text{Sqrt}[-a^2 + b^2])* \text{Sqrt}[(a*\text{Sec}[(e + f*x)/2]^2*(a + b*\text{Sin}[e + f*x]))/(a^2 - b \\
& ^2))*((a*\text{Tan}[(e + f*x)/2])/(-b + \text{Sqrt}[-a^2 + b^2]))^(3/2)) + (a*((a*b*\text{Cos}[e \\
& + f*x]*\text{Sec}[(e + f*x)/2]^2)/(a^2 - b^2) + (a*\text{Sec}[(e + f*x)/2]^2*(a + b*\text{Sin}[ \\
& e + f*x])* \text{Tan}[(e + f*x)/2])/(a^2 - b^2))*(\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[(-b + \text{Sqrt} \\
& -a^2 + b^2] - a*\text{Tan}[(e + f*x)/2])/ \text{Sqrt}[-a^2 + b^2]]/\text{Sqrt}[2]], (2*\text{Sqrt}[-a^2 \\
& + b^2])/(-b + \text{Sqrt}[-a^2 + b^2]))*\text{Tan}[(e + f*x)/2] + \text{EllipticF}[\text{ArcSin}[\text{Sqrt}[( \\
& b + \text{Sqrt}[-a^2 + b^2] + a*\text{Tan}[(e + f*x)/2])/ \text{Sqrt}[-a^2 + b^2]]/\text{Sqrt}[2]], (2*S \\
& qrt[-a^2 + b^2])/(b + \text{Sqrt}[-a^2 + b^2]))*\text{Sqrt}[(a*\text{Tan}[(e + f*x)/2])/(-b + S \\
& qrt[-a^2 + b^2]))*\text{Sqrt}[-((a*\text{Tan}[(e + f*x)/2])/(b + \text{Sqrt}[-a^2 + b^2])))]/(S \\
& qrt[-a^2 + b^2]*((a*\text{Sec}[(e + f*x)/2]^2*(a + b*\text{Sin}[e + f*x]))/(a^2 - b^2))^(3 \\
& /2)*\text{Sqrt}[(a*\text{Tan}[(e + f*x)/2])/(-b + \text{Sqrt}[-a^2 + b^2]))] - (2*a*((\text{EllipticE} \\
& [\text{ArcSin}[\text{Sqrt}[(-b + \text{Sqrt}[-a^2 + b^2] - a*\text{Tan}[(e + f*x)/2])/ \text{Sqrt}[-a^2 + b^2]]/ \\
& \text{Sqrt}[2]], (2*\text{Sqrt}[-a^2 + b^2])/(-b + \text{Sqrt}[-a^2 + b^2]))*\text{Sec}[(e + f*x)/2]^2) \\
& /2 - (a*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[(b + \text{Sqrt}[-a^2 + b^2] + a*\text{Tan}[(e + f*x)/2])/ \\
& \text{Sqrt}[-a^2 + b^2]]/\text{Sqrt}[2]], (2*\text{Sqrt}[-a^2 + b^2])/(b + \text{Sqrt}[-a^2 + b^2]))*\text{Sec} \\
& [(e + f*x)/2]^2*\text{Sqrt}[(a*\text{Tan}[(e + f*x)/2])/(-b + \text{Sqrt}[-a^2 + b^2]))/(4*(b + \\
& \text{Sqrt}[-a^2 + b^2])* \text{Sqrt}[-((a*\text{Tan}[(e + f*x)/2])/(b + \text{Sqrt}[-a^2 + b^2]))]) + \\
& (a*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[(b + \text{Sqrt}[-a^2 + b^2] + a*\text{Tan}[(e + f*x)/2])/ \text{Sqrt}[- \\
& a^2 + b^2]]/\text{Sqrt}[2]], (2*\text{Sqrt}[-a^2 + b^2])/(b + \text{Sqrt}[-a^2 + b^2]))*\text{Sec}[(e + \\
& f*x)/2]^2*\text{Sqrt}[-((a*\text{Tan}[(e + f*x)/2])/(b + \text{Sqrt}[-a^2 + b^2])))]/(4*(-b + S \\
& qrt[-a^2 + b^2])* \text{Sqrt}[(a*\text{Tan}[(e + f*x)/2])/(-b + \text{Sqrt}[-a^2 + b^2]))] - (a*S \\
& ec[(e + f*x)/2]^2*\text{Tan}[(e + f*x)/2]*\text{Sqrt}[1 - (-b + \text{Sqrt}[-a^2 + b^2] - a*\text{Tan} \\
& (e + f*x)/2])/(-b + \text{Sqrt}[-a^2 + b^2]))/(4*\text{Sqrt}[2]*\text{Sqrt}[-a^2 + b^2]*\text{Sqrt}[(- \\
& b + \text{Sqrt}[-a^2 + b^2] - a*\text{Tan}[(e + f*x)/2])/ \text{Sqrt}[-a^2 + b^2]]*\text{Sqrt}[1 - (-b + \\
& \text{Sqrt}[-a^2 + b^2] - a*\text{Tan}[(e + f*x)/2])/(2*\text{Sqrt}[-a^2 + b^2]))] + (a*\text{Sec}[(e \\
& + f*x)/2]^2*\text{Sqrt}[(a*\text{Tan}[(e + f*x)/2])/(-b + \text{Sqr...}
\end{aligned}$$

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 6803 vs.  $2(108) = 216$ .

time = 0.33, size = 6804, normalized size = 58.66

method	result	size
default	Expression too large to display	6804

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))/(g*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)`

[Out] result too large to display

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))/(g*sin(f*x+e))^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(b*sin(f*x + e) + a)/((c*sin(f*x + e) + c)*sqrt(g*sin(f*x + e))), x)`

**Fricas** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))/(g*sin(f*x+e))^(1/2),x, algorithm="fricas")`

[Out] `integral(-sqrt(b*sin(f*x + e) + a)*sqrt(g*sin(f*x + e))/(c*g*cos(f*x + e)^2 - c*g*sin(f*x + e) - c*g), x)`

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b \sin(e + fx)}}{\sqrt{g \sin(e + fx)} \sin(e + fx) + \sqrt{g \sin(e + fx)}} dx$$

*c*

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sin(f*x+e))**(1/2)/(c+c*sin(f*x+e))/(g*sin(f*x+e))**(1/2),x)`

[Out] Integral(sqrt(a + b\*sin(e + f\*x))/(sqrt(g\*sin(e + f\*x))\*sin(e + f\*x) + sqrt(g\*sin(e + f\*x))), x)/c

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sin(f\*x+e))^(1/2)/(c+c\*sin(f\*x+e))/(g\*sin(f\*x+e))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b\*sin(f\*x + e) + a)/((c\*sin(f\*x + e) + c)\*sqrt(g\*sin(f\*x + e))), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a + b \sin(e + f x)}}{\sqrt{g \sin(e + f x)} (c + c \sin(e + f x))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*sin(e + f\*x))^(1/2)/((g\*sin(e + f\*x))^(1/2)\*(c + c\*sin(e + f\*x))),x)

[Out] int((a + b\*sin(e + f\*x))^(1/2)/((g\*sin(e + f\*x))^(1/2)\*(c + c\*sin(e + f\*x))), x)

$$3.33 \quad \int \frac{\sqrt{g \sin(e + fx)}}{\sqrt{a + b \sin(e + fx)} (c + c \sin(e + fx))} dx$$

Optimal. Leaf size=252

$$\frac{gE\left(\sin^{-1}\left(\frac{\cos(e+fx)}{1+\sin(e+fx)}\right)\middle|-\frac{a-b}{a+b}\right)\sqrt{\frac{\sin(e+fx)}{1+\sin(e+fx)}}\sqrt{a+b\sin(e+fx)}-2\sqrt{a+b}\sqrt{g}\sqrt{\frac{a(1-\csc(e+fx))}{a+b}}}{(a-b)cf\sqrt{g\sin(e+fx)}\sqrt{\frac{a+b\sin(e+fx)}{(a+b)(1+\sin(e+fx))}}}$$

[Out] g\*EllipticE(cos(f\*x+e)/(1+sin(f\*x+e)),((-a+b)/(a+b))^(1/2))\*(sin(f\*x+e)/(1+sin(f\*x+e)))^(1/2)\*(a+b\*sin(f\*x+e))^(1/2)/(a-b)/c/f/(g\*sin(f\*x+e))^(1/2)/((a+b\*sin(f\*x+e))/(a+b)/(1+sin(f\*x+e)))^(1/2)-2\*EllipticF(g^(1/2)\*(a+b\*sin(f\*x+e))^(1/2)/(a+b)^(1/2)/(g\*sin(f\*x+e))^(1/2),((-a-b)/(a-b))^(1/2))\*(a+b)^(1/2)\*g^(1/2)\*(a\*(1-csc(f\*x+e))/(a+b))^(1/2)\*(a\*(1+csc(f\*x+e))/(a+b))^(1/2)\*tan(f\*x+e)/(a-b)/c/f

**Rubi** [A]

time = 0.33, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 39,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {3015, 2895, 3011}

$$\frac{g\sqrt{\frac{\sin(e+fx)}{\sin(e+fx)+1}}\sqrt{a+b\sin(e+fx)}E\left(\text{ArcSin}\left(\frac{\cos(e+fx)}{\sin(e+fx)+1}\right)\middle|-\frac{a-b}{a+b}\right)-2\sqrt{g}\sqrt{a+b}\tan(e+fx)\sqrt{\frac{a(1-\csc(e+fx))}{a+b}}\sqrt{\frac{a(\csc(e+fx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{g}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{g\sin(e+fx)}}\right)\middle|-\frac{a+b}{a-b}\right)}{cf(a-b)\sqrt{g\sin(e+fx)}\sqrt{\frac{a+b\sin(e+fx)}{(a+b)(\sin(e+fx)+1)}}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[g\*Sin[e + f\*x]]/(Sqrt[a + b\*Sin[e + f\*x]]\*(c + c\*Sin[e + f\*x])),x]

[Out] (g\*EllipticE[ArcSin[Cos[e + f\*x]/(1 + Sin[e + f\*x])], -((a - b)/(a + b))] \* Sqrt[Sin[e + f\*x]/(1 + Sin[e + f\*x])] \* Sqrt[a + b\*Sin[e + f\*x]] / ((a - b) \* c \* f \* Sqrt[g\*Sin[e + f\*x]] \* Sqrt[(a + b\*Sin[e + f\*x]) / ((a + b) \* (1 + Sin[e + f\*x]))]) - (2 \* Sqrt[a + b] \* Sqrt[g] \* Sqrt[(a \* (1 - Csc[e + f\*x])) / (a + b)] \* Sqrt[(a \* (1 + Csc[e + f\*x])) / (a - b)] \* EllipticF[ArcSin[(Sqrt[g] \* Sqrt[a + b\*Sin[e + f\*x]]) / (Sqrt[a + b] \* Sqrt[g\*Sin[e + f\*x]])], -((a + b)/(a - b))] \* Tan[e + f\*x]) / ((a - b) \* c \* f)

Rule 2895

Int[1/(Sqrt[(d\_)\*sin[(e\_) + (f\_)\*(x\_)]]\*Sqrt[(a\_) + (b\_)\*sin[(e\_) + (f\_)\*(x\_)])], x\_Symbol] :> Simp[-2\*(Tan[e + f\*x]/(a\*f))\*Rt[(a + b)/d, 2]\*Sqrt[a\*((1 - Csc[e + f\*x])/(a + b))]\*Sqrt[a\*((1 + Csc[e + f\*x])/(a - b))]\*EllipticF[ArcSin[Sqrt[a + b\*Sin[e + f\*x]]/Sqrt[d\*Sin[e + f\*x]]/Rt[(a + b)/d, 2]], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

&& PosQ[(a + b)/d]

### Rule 3011

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/(Sqrt[(g_)*sin[(e_) + (f_)*
(x_)]*((c_) + (d_)*sin[(e_) + (f_)*(x_)])), x_Symbol] := Simp[(-Sqrt[
a + b*Sin[e + f*x]])*(Sqrt[d*(Sin[e + f*x]/(c + d*Sin[e + f*x]))]/(d*f*Sqrt
[g*Sin[e + f*x]]*Sqrt[c^2*((a + b*Sin[e + f*x])/((a*c + b*d)*(c + d*Sin[e +
f*x]))))]*)*EllipticE[ArcSin[c*(Cos[e + f*x]/(c + d*Sin[e + f*x]))], (b*c -
a*d)/(b*c + a*d), x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]
```

### Rule 3015

```
Int[Sqrt[(g_)*sin[(e_) + (f_)*(x_)]]/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*
(x_)]*((c_) + (d_)*sin[(e_) + (f_)*(x_)])), x_Symbol] := Dist[(-a)*(g
/(b*c - a*d)), Int[1/(Sqrt[g*Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]]), x]
] + Dist[c*(g/(b*c - a*d)), Int[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[g*Sin[e + f*
x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[b*c - a*d, 0] && (EqQ[a^2 - b^2, 0] || EqQ[c^2 - d^2, 0])
```

### Rubi steps

$$\int \frac{\sqrt{g \sin(e + fx)}}{\sqrt{a + b \sin(e + fx)} (c + c \sin(e + fx))} dx = -\frac{g \int \frac{\sqrt{a + b \sin(e + fx)}}{\sqrt{g \sin(e + fx)} (c + c \sin(e + fx))} dx}{a - b} + \frac{(ag) \int \frac{\sqrt{g \sin(e + fx)}}{\sqrt{g \sin(e + fx)}} dx}{a - b}$$

$$= \frac{g E\left(\sin^{-1}\left(\frac{\cos(e + fx)}{1 + \sin(e + fx)}\right) \middle| -\frac{a - b}{a + b}\right) \sqrt{\frac{\sin(e + fx)}{1 + \sin(e + fx)}} \sqrt{a + b \sin(e + fx)}}{(a - b) c f \sqrt{g \sin(e + fx)} \sqrt{\frac{a + b \sin(e + fx)}{(a + b)(1 + \sin(e + fx))}}}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 5708 vs.

2(252) = 504.

time = 84.56, size = 5708, normalized size = 22.65

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[g*Sin[e + f*x]]/(Sqrt[a + b*Sin[e + f*x]]*(c + c*Sin[e + f*x
])),x]
```

[Out] Result too large to show

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 6814 vs.  $2(232) = 464$ .

time = 0.44, size = 6815, normalized size = 27.04

method	result	size
default	Expression too large to display	6815

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))/(a+b*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)`

[Out] result too large to display

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))/(a+b*sin(f*x+e))^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(g*sin(f*x + e))/(sqrt(b*sin(f*x + e) + a)*(c*sin(f*x + e) + c)), x)`

**Fricas [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))/(a+b*sin(f*x+e))^(1/2),x, algorithm="fricas")`

[Out] `integral(-sqrt(b*sin(f*x + e) + a)*sqrt(g*sin(f*x + e))/(b*c*cos(f*x + e)^2 - (a + b)*c*sin(f*x + e) - (a + b)*c), x)`

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{g \sin(e + fx)}}{\sqrt{a + b \sin(e + fx)} \sin(e + fx) + \sqrt{a + b \sin(e + fx)}} dx$$

*c*

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sin(f*x+e))**(1/2)/(c+c*sin(f*x+e))/(a+b*sin(f*x+e))**(1/2),x)
[Out] Integral(sqrt(g*sin(e + f*x))/(sqrt(a + b*sin(e + f*x))*sin(e + f*x) + sqrt(a + b*sin(e + f*x))), x)/c
```

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))/(a+b*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(g*sin(f*x + e))/(sqrt(b*sin(f*x + e) + a)*(c*sin(f*x + e) + c)), x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{g \sin(e + f x)}}{\sqrt{a + b \sin(e + f x)} (c + c \sin(e + f x))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sin(e + f*x))^(1/2)/((a + b*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))),x)
```

```
[Out] int((g*sin(e + f*x))^(1/2)/((a + b*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))), x)
```



$$3.34 \int \frac{1}{\sqrt{g \sin(e + fx)} \sqrt{a + b \sin(e + fx)} (c + c \sin(e + fx))} dx$$

**Optimal.** Leaf size=256

$$\frac{E\left(\sin^{-1}\left(\frac{\cos(e+fx)}{1+\sin(e+fx)}\right)\right) \left|^{-\frac{a-b}{a+b}} \sqrt{\frac{\sin(e+fx)}{1+\sin(e+fx)}} \sqrt{a+b \sin(e+fx)} \right.}{(a-b)cf \sqrt{g \sin(e+fx)} \sqrt{\frac{a+b \sin(e+fx)}{(a+b)(1+\sin(e+fx))}}} + \frac{2b\sqrt{a+b} \sqrt{\frac{a(1-\csc(e+fx))}{a+b}}}{(a-b)cf \sqrt{g \sin(e+fx)} \sqrt{\frac{a+b \sin(e+fx)}{(a+b)(1+\sin(e+fx))}}}$$

[Out] -EllipticE(cos(f\*x+e)/(1+sin(f\*x+e)),((-a+b)/(a+b))^(1/2))\*(sin(f\*x+e)/(1+sin(f\*x+e)))^(1/2)\*(a+b\*sin(f\*x+e))^(1/2)/(a-b)/c/f/(g\*sin(f\*x+e))^(1/2)/((a+b\*sin(f\*x+e))/(a+b)/(1+sin(f\*x+e)))^(1/2)+2\*b\*EllipticF(g^(1/2)\*(a+b\*sin(f\*x+e))^(1/2)/(a+b)^(1/2)/(g\*sin(f\*x+e))^(1/2),((-a-b)/(a-b))^(1/2))\*(a+b)^(1/2)\*(a\*(1-csc(f\*x+e))/(a+b))^(1/2)\*(a\*(1+csc(f\*x+e))/(a-b))^(1/2)\*tan(f\*x+e)/a/(a-b)/c/f/g^(1/2)

**Rubi [A]**

time = 0.35, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 39,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {3017, 2895, 3011}

$$\frac{2b\sqrt{a+b} \tan(e+fx) \sqrt{\frac{a(1-\csc(e+fx))}{a+b}} \sqrt{\frac{a(\csc(e+fx)+1)}{a-b}} F\left(\text{ArcSin}\left(\frac{\sqrt{g} \sqrt{a+b \sin(e+fx)}}{\sqrt{a+b} \sqrt{g \sin(e+fx)}}\right)\right) \left|^{-\frac{a-b}{a+b}}}{ac f \sqrt{g} (a-b)} - \frac{\sqrt{\frac{\sin(e+fx)}{\sin(e+fx)+1}} \sqrt{a+b \sin(e+fx)} E\left(\text{ArcSin}\left(\frac{\cos(e+fx)}{\sin(e+fx)+1}\right)\right) \left|^{-\frac{a-b}{a+b}}}{cf(a-b) \sqrt{g \sin(e+fx)} \sqrt{\frac{a+b \sin(e+fx)}{(a+b)(\sin(e+fx)+1)}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[g\*Sin[e + f\*x]]\*Sqrt[a + b\*Sin[e + f\*x]]\*(c + c\*Sin[e + f\*x])), x]

[Out] -((EllipticE[ArcSin[Cos[e + f\*x]/(1 + Sin[e + f\*x])], -((a - b)/(a + b))] \* Sqrt[Sin[e + f\*x]/(1 + Sin[e + f\*x])] \* Sqrt[a + b\*Sin[e + f\*x]])/((a - b)\*c\*f \* Sqrt[g\*Sin[e + f\*x]] \* Sqrt[(a + b\*Sin[e + f\*x])/((a + b)\*(1 + Sin[e + f\*x]))]) + (2\*b\*Sqrt[a + b]\*Sqrt[(a\*(1 - Csc[e + f\*x]))/(a + b)] \* Sqrt[(a\*(1 + Csc[e + f\*x]))/(a - b)] \* EllipticF[ArcSin[(Sqrt[g]\*Sqrt[a + b\*Sin[e + f\*x]])/(Sqrt[a + b]\*Sqrt[g\*Sin[e + f\*x])]], -((a + b)/(a - b))] \* Tan[e + f\*x])/(a\*(a - b)\*c\*f\*Sqrt[g])

**Rule 2895**

Int[1/(Sqrt[(d\_)\*sin[e\_] + (f\_)\*(x\_)]\*Sqrt[(a\_) + (b\_)\*sin[e\_] + (f\_)\*(x\_)]), x\_Symbol] := Simp[-2\*(Tan[e + f\*x]/(a\*f))\*Rt[(a + b)/d, 2]\*Sqrt[a\*((1 - Csc[e + f\*x])/(a + b))]\*Sqrt[a\*((1 + Csc[e + f\*x])/(a - b))]\*EllipticF[ArcSin[Sqrt[a + b\*Sin[e + f\*x]]/Sqrt[d\*Sin[e + f\*x]]/Rt[(a + b)/d, 2]], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

&& PosQ[(a + b)/d]

### Rule 3011

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/(Sqrt[(g_)*sin[(e_) + (f_)*
(x_)]*((c_) + (d_)*sin[(e_) + (f_)*(x_)])), x_Symbol] :> Simp[(-Sqrt[
a + b*Sin[e + f*x]])*(Sqrt[d*(Sin[e + f*x]/(c + d*Sin[e + f*x]))]/(d*f*Sqrt
[g*Sin[e + f*x]]*Sqrt[c^2*((a + b*Sin[e + f*x])/((a*c + b*d)*(c + d*Sin[e +
f*x])))))*EllipticE[ArcSin[c*(Cos[e + f*x]/(c + d*Sin[e + f*x]))], (b*c -
a*d)/(b*c + a*d)], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]
```

### Rule 3017

```
Int[1/(Sqrt[(g_)*sin[(e_) + (f_)*(x_)]]*Sqrt[(a_) + (b_)*sin[(e_) + (f_)*
(x_)]*((c_) + (d_)*sin[(e_) + (f_)*(x_)])), x_Symbol] :> Dist[b/(b*
c - a*d), Int[1/(Sqrt[g*Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]]), x], x] - D
ist[d/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[g*Sin[e + f*x]]*(c +
d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*
d, 0] && (EqQ[a^2 - b^2, 0] || EqQ[c^2 - d^2, 0])
```

### Rubi steps

$$\int \frac{1}{\sqrt{g \sin(e + fx)} \sqrt{a + b \sin(e + fx)} (c + c \sin(e + fx))} dx = -\frac{b \int \frac{1}{\sqrt{g \sin(e + fx)} \sqrt{a + b \sin(e + fx)}} dx}{(a - b)c}$$

$$= -\frac{E\left(\sin^{-1}\left(\frac{\cos(e + fx)}{1 + \sin(e + fx)}\right) \middle| -\frac{a - b}{a + b}\right) \sqrt{\frac{\sin(e + fx)}{1 + \sin(e + fx)}}}{(a - b)cf \sqrt{g \sin(e + fx)} \sqrt{\frac{a}{a + b}}}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 33.51, size = 1661, normalized size = 6.49



Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[g*Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]]*(c + c*Sin[e + f
*x])), x]
```

```
[Out] (-2*SIN[(e + f*x)/2]*(COS[(e + f*x)/2] + SIN[(e + f*x)/2])*SIN[e + f*x]*Sqr
t[a + b*SIN[e + f*x]])/((a - b)*f*Sqrt[g*SIN[e + f*x]]*(c + c*SIN[e + f*x])
) + ((COS[(e + f*x)/2] + SIN[(e + f*x)/2])^2*Sqrt[SIN[e + f*x]]*((4*a*(a -
b)*Sqrt[((a + b)*COT[(-e + Pi/2 - f*x)/2]^2)/(-a + b)]*ELLIPTICF[ARCSIN[Sqr
t[(CSC[(-e + Pi/2 - f*x)/2]^2*(a + b*SIN[e + f*x]))/a]/Sqrt[2]], (-2*a)/(-a
+ b)]*SEC[e + f*x]*SIN[(-e + Pi/2 - f*x)/2]^4*Sqrt[-(((a + b)*CSC[(-e + Pi
/2 - f*x)/2]^2*SIN[e + f*x])/a])*Sqrt[(CSC[(-e + Pi/2 - f*x)/2]^2*(a + b*Si
n[e + f*x]))/a])/((a + b)*Sqrt[SIN[e + f*x]]*Sqrt[a + b*SIN[e + f*x]]) + (2
*a*ARCtanH[(Sqrt[b]*Sqrt[SIN[e + f*x]])/Sqrt[a + b*SIN[e + f*x]])*COS[e + f
*x]^2)/(Sqrt[b]*(1 - SIN[e + f*x]^2)) + 4*a^2*((Sqrt[((a + b)*COT[(-e + Pi/
2 - f*x)/2]^2)/(-a + b)]*ELLIPTICF[ARCSIN[Sqrt[(CSC[(-e + Pi/2 - f*x)/2]^2*
(a + b*SIN[e + f*x]))/a]/Sqrt[2]], (-2*a)/(-a + b)]*SEC[e + f*x]*SIN[(-e +
Pi/2 - f*x)/2]^4*Sqrt[-(((a + b)*CSC[(-e + Pi/2 - f*x)/2]^2*SIN[e + f*x])/a
])*Sqrt[(CSC[(-e + Pi/2 - f*x)/2]^2*(a + b*SIN[e + f*x]))/a])/((a + b)*Sqrt
[SIN[e + f*x]]*Sqrt[a + b*SIN[e + f*x]]) - (Sqrt[((a + b)*COT[(-e + Pi/2 -
f*x)/2]^2)/(-a + b)]*ELLIPTICPI[-(a/b), ARCSIN[Sqrt[(CSC[(-e + Pi/2 - f*x)/
2]^2*(a + b*SIN[e + f*x]))/a]/Sqrt[2]], (-2*a)/(-a + b)]*SEC[e + f*x]*SIN[(
-e + Pi/2 - f*x)/2]^4*Sqrt[-(((a + b)*CSC[(-e + Pi/2 - f*x)/2]^2*SIN[e + f*
x])/a])*Sqrt[(CSC[(-e + Pi/2 - f*x)/2]^2*(a + b*SIN[e + f*x]))/a])/(b*Sqrt[
SIN[e + f*x]]*Sqrt[a + b*SIN[e + f*x]]) - 2*b*((COS[e + f*x]*Sqrt[a + b*Si
n[e + f*x]])/(b*Sqrt[SIN[e + f*x]]) + (I*COS[(-e + Pi/2 - f*x)/2]*CSC[e + f
*x]*ELLIPTICE[I*ARCSINH[SIN[(-e + Pi/2 - f*x)/2]/Sqrt[SIN[e + f*x]]], (-2*a
)/(-a - b)]*Sqrt[a + b*SIN[e + f*x]])/(b*Sqrt[COS[(-e + Pi/2 - f*x)/2]^2*Cs
c[e + f*x]]*Sqrt[(CSC[e + f*x]*(a + b*SIN[e + f*x]))/(a + b)]) + (2*a*((a*S
qrt[((a + b)*COT[(-e + Pi/2 - f*x)/2]^2)/(-a + b)]*ELLIPTICF[ARCSIN[Sqrt[(C
sc[(-e + Pi/2 - f*x)/2]^2*(a + b*SIN[e + f*x]))/a]/Sqrt[2]], (-2*a)/(-a + b
)]*SEC[e + f*x]*SIN[(-e + Pi/2 - f*x)/2]^4*Sqrt[-(((a + b)*CSC[(-e + Pi/2 -
f*x)/2]^2*SIN[e + f*x])/a])*Sqrt[(CSC[(-e + Pi/2 - f*x)/2]^2*(a + b*SIN[e
+ f*x]))/a])/((a + b)*Sqrt[SIN[e + f*x]]*Sqrt[a + b*SIN[e + f*x]]) - (a*Sqr
t[((a + b)*COT[(-e + Pi/2 - f*x)/2]^2)/(-a + b)]*ELLIPTICPI[-(a/b), ARCSIN[
Sqrt[(CSC[(-e + Pi/2 - f*x)/2]^2*(a + b*SIN[e + f*x]))/a]/Sqrt[2]], (-2*a)/
(-a + b)]*SEC[e + f*x]*SIN[(-e + Pi/2 - f*x)/2]^4*Sqrt[-(((a + b)*CSC[(-e +
Pi/2 - f*x)/2]^2*SIN[e + f*x])/a])*Sqrt[(CSC[(-e + Pi/2 - f*x)/2]^2*(a + b
*SIN[e + f*x]))/a])/(b*Sqrt[SIN[e + f*x]]*Sqrt[a + b*SIN[e + f*x]])))/b +
(2*b*COT[e + f*x]*((a*LOG[-(Sqrt[b]*Sqrt[SIN[e + f*x]]) + Sqrt[a + b*SIN[e
+ f*x]])]/(2*b^(3/2)) + (Sqrt[SIN[e + f*x]]*Sqrt[a + b*SIN[e + f*x]])/(2*b
)*SIN[2*(e + f*x)]/(1 - SIN[e + f*x]^2)))/(2*(a - b)*f*Sqrt[g*SIN[e + f*x]
]*(c + c*SIN[e + f*x]))
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 9042 vs.  $2(236) = 472$ .

time = 0.37, size = 9043, normalized size = 35.32

method	result	size
default	Expression too large to display	9043

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+c*sin(f*x+e))/(g*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(1/2),x,method
=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

**Maxima [F]**

```
time = 0.00, size = 0, normalized size = 0.00
```

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+c*sin(f*x+e))/(g*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(1/2),x,
algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*sin(f*x + e) + a)*(c*sin(f*x + e) + c)*sqrt(g*sin(f*x +
e))), x)
```

**Fricas [F]**

```
time = 0.00, size = 0, normalized size = 0.00
```

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+c*sin(f*x+e))/(g*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(1/2),x,
algorithm="fricas")
```

```
[Out] integral(-sqrt(b*sin(f*x + e) + a)*sqrt(g*sin(f*x + e))/((a + b)*c*g*cos(f*
x + e)^2 - (a + b)*c*g + (b*c*g*cos(f*x + e)^2 - (a + b)*c*g)*sin(f*x + e))
, x)
```

**Sympy [F]**

```
time = 0.00, size = 0, normalized size = 0.00
```

$$\frac{\int \frac{1}{\sqrt{g \sin(e + fx)} \sqrt{a + b \sin(e + fx)} \sin(e + fx) + \sqrt{g \sin(e + fx)} \sqrt{a + b \sin(e + fx)}} dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+c*sin(f*x+e))/(g*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(1/2),
x)
```

```
[Out] Integral(1/(sqrt(g*sin(e + f*x))*sqrt(a + b*sin(e + f*x))*sin(e + f*x) + sq
rt(g*sin(e + f*x))*sqrt(a + b*sin(e + f*x))), x)/c
```

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+c*sin(f*x+e))/(g*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(1/2),x,
algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*sin(f*x + e) + a)*(c*sin(f*x + e) + c)*sqrt(g*sin(f*x +
e))), x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{g \sin(e + f x)} \sqrt{a + b \sin(e + f x)} (c + c \sin(e + f x))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((g*sin(e + f*x))^(1/2)*(a + b*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x
))),x)
```

```
[Out] int(1/((g*sin(e + f*x))^(1/2)*(a + b*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x
))), x)
```

### 3.35 $\int \csc(e+fx) \sqrt{a+a\sin(e+fx)} \sqrt{c+d\sin(e+fx)}$

Optimal. Leaf size=123

$$\frac{2\sqrt{a}\sqrt{d}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{f} - \frac{2\sqrt{a}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{f}$$

[Out]  $-2*\operatorname{arctanh}(\cos(f*x+e)*a^{(1/2)}*c^{(1/2)}/(a+a*\sin(f*x+e))^{(1/2)}/(c+d*\sin(f*x+e))^{(1/2)})*a^{(1/2)}*c^{(1/2)}/f-2*\operatorname{arctan}(\cos(f*x+e)*a^{(1/2)}*d^{(1/2)}/(a+a*\sin(f*x+e))^{(1/2)}/(c+d*\sin(f*x+e))^{(1/2)})*a^{(1/2)}*d^{(1/2)}/f$

Rubi [A]

time = 0.31, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {3028, 2854, 211, 3022, 212}

$$\frac{2\sqrt{a}\sqrt{d}\operatorname{ArcTan}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{f} - \frac{2\sqrt{a}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{f}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Csc}[e+f*x]*\operatorname{Sqrt}[a+a*\operatorname{Sin}[e+f*x]]*\operatorname{Sqrt}[c+d*\operatorname{Sin}[e+f*x]],x]$

[Out]  $(-2*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[d]*\operatorname{ArcTan}[(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[d]*\operatorname{Cos}[e+f*x])/(\operatorname{Sqrt}[a+a*\operatorname{Sin}[e+f*x]]*\operatorname{Sqrt}[c+d*\operatorname{Sin}[e+f*x]])])/f - (2*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[c]*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[c]*\operatorname{Cos}[e+f*x])/(\operatorname{Sqrt}[a+a*\operatorname{Sin}[e+f*x]]*\operatorname{Sqrt}[c+d*\operatorname{Sin}[e+f*x]])])/f$

Rule 211

$\operatorname{Int}[(a_+ + (b_+)*(x_+)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[a/b, 2]/a)*\operatorname{ArcTan}[x/\operatorname{Rt}[a/b, 2]], x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{PosQ}[a/b]$

Rule 212

$\operatorname{Int}[(a_+ + (b_+)*(x_+)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]))*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ \|\ \operatorname{LtQ}[b, 0])$

Rule 2854

$\operatorname{Int}[\operatorname{Sqrt}[(a_+ + (b_+)*\sin[(e_+ + (f_+)*(x_+)])/ \operatorname{Sqrt}[(c_+ + (d_+)*\sin[(e_+ + (f_+)*(x_+)])], x\_Symbol] \rightarrow \operatorname{Dist}[-2*(b/f), \operatorname{Subst}[\operatorname{Int}[1/(b+d*x^2), x], x, b*(\operatorname{Cos}[e+f*x]/(\operatorname{Sqrt}[a+b*\operatorname{Sin}[e+f*x]]*\operatorname{Sqrt}[c+d*\operatorname{Sin}[e+f*x]])]], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \operatorname{EqQ}[a^2 - b^2, 0] \ \&\& \operatorname{NeQ}[c^2 - d^2, 0]$

Rule 3022

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/(sin[(e_) + (f_)*(x_)]*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[-2*(a/f), Subst[Int[1/(1 - a*c*x^2), x], x, Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[b*c + a*d, 0]
```

Rule 3028

```
Int[(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]/sin[(e_) + (f_)*(x_)], x_Symbol] := Dist[d, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[c, Int[Sqrt[a + b*Sin[e + f*x]]/(Sin[e + f*x]*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && (NeQ[a^2 - b^2, 0] || NeQ[c^2 - d^2, 0])
```

Rubi steps

$$\int \csc(e + fx) \sqrt{a + a \sin(e + fx)} \sqrt{c + d \sin(e + fx)} dx = c \int \frac{\csc(e + fx) \sqrt{a + a \sin(e + fx)}}{\sqrt{c + d \sin(e + fx)}} dx + d \int \frac{\csc(e + fx) \sqrt{a + a \sin(e + fx)}}{\sqrt{c + d \sin(e + fx)}} dx$$

$$= -\frac{(2ac) \operatorname{Subst}\left(\int \frac{1}{1-ax^2} dx, x, \frac{\sqrt{a + a \sin(e + fx)}}{\sqrt{c + d \sin(e + fx)}}\right)}{f}$$

$$= -\frac{2\sqrt{a} \sqrt{d} \tan^{-1}\left(\frac{\sqrt{a} \sqrt{d} \cos(e + fx)}{\sqrt{a + a \sin(e + fx)} \sqrt{c + d \sin(e + fx)}}\right)}{f}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 2.96, size = 567, normalized size = 4.61

$$\frac{\sqrt{c} \log\left(\frac{(1 + \sqrt{c}) \sqrt{a + a \sin(e + fx)} \sqrt{c + d \sin(e + fx)}}{(1 - \sqrt{c}) \sqrt{a + a \sin(e + fx)} \sqrt{c + d \sin(e + fx)}}\right) + \sqrt{c} \log\left(\frac{(1 + \sqrt{c}) \sqrt{a + a \sin(e + fx)} \sqrt{c + d \sin(e + fx)}}{(1 - \sqrt{c}) \sqrt{a + a \sin(e + fx)} \sqrt{c + d \sin(e + fx)}}\right)}{f \cos(e + fx) \sqrt{c + d \sin(e + fx)}} - \log\left(\frac{(1 + \sqrt{c}) \sqrt{a + a \sin(e + fx)} \sqrt{c + d \sin(e + fx)}}{(1 - \sqrt{c}) \sqrt{a + a \sin(e + fx)} \sqrt{c + d \sin(e + fx)}}\right) \sqrt{c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]],x]
[Out] -(((Sqrt[c]*Log[((1/2 + I/2)*(-(Sqrt[2]*c*(-1 + E^(I*(e + f*x)))))) - I*Sqrt[2]*d*(1 + E^(I*(e + f*x))) + (2*I)*Sqrt[c]*Sqrt[2*c*E^(I*(e + f*x)) - I*d*(-1 + E^((2*I)*(e + f*x)))]*f)/(c^(3/2)*E^((I/2)*e)*(1 + E^(I*(e + f*x))))] + Sqrt[c]*Log[((1/2 + I/2)*((-I)*Sqrt[2]*d*(-1 + E^(I*(e + f*x)))) + Sqrt[2]*c*(1 + E^(I*(e + f*x))) + 2*Sqrt[c]*Sqrt[2*c*E^(I*(e + f*x)) - I*d*(-1 +
```

$$E^{((2I)*(e + f*x))}] * f) / (c^{(3/2)} * E^{((I/2)*e) * (-1 + E^{(I*(e + f*x))})}] - I * \text{Sqrt}[d] * (\text{Log}[(2 * ((-1)^{(3/4)} * d + (-1)^{(1/4)} * c * E^{(I*(e + f*x))}) + I * \text{Sqrt}[d] * \text{Sqrt}[2 * c * E^{(I*(e + f*x))} - I * d * (-1 + E^{((2I)*(e + f*x))})]) * f] / (d^{(3/2)} * E^{((I/2)*(e + 2*f*x))})] - \text{Log}[(1 + I) * \text{Sqrt}[2] * (c - I * d * \text{Cos}[e + f*x] + d * \text{Sin}[e + f*x] + (1 - I) * \text{Sqrt}[d] * \text{Sqrt}[(\text{Cos}[e + f*x] + I * \text{Sin}[e + f*x]) * (c + d * \text{Sin}[e + f*x])])]) / \text{Sqrt}[d])]) * (\text{Cos}[(e + f*x)/2] + I * \text{Sin}[(e + f*x)/2]) * \text{Sqrt}[a * (1 + \text{Sin}[e + f*x])] * \text{Sqrt}[c + d * \text{Sin}[e + f*x]]) / (f * (\text{Cos}[(e + f*x)/2] + \text{Sin}[(e + f*x)/2]) * \text{Sqrt}[(\text{Cos}[e + f*x] + I * \text{Sin}[e + f*x]) * (c + d * \text{Sin}[e + f*x])])])$$

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 4545 vs.  $2(99) = 198$ .

time = 0.34, size = 4546, normalized size = 36.96

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((a+a*\sin(f*x+e))^{(1/2)}*(c+d*\sin(f*x+e))^{(1/2)}/\sin(f*x+e), x)$

[Out] 
$$\begin{aligned} & -1/f*(2*\arctan(((c+d*\sin(f*x+e))/((d^2/c^2)^{(1/2)}*c*\sin(f*x+e)+d)*d)^{(1/2)}/ \\ & ((-d^2/c^2)^{(1/2)}*c)^{(1/2)})*c^{(5/2)}*d^2*(d^2/c^2)^{(1/2)}*((c+d*\sin(f*x+e))/ \\ & (d^2/c^2)^{(1/2)}*c*\sin(f*x+e)+d)*d)^{(1/2)}*\cos(f*x+e)-2*\arctan(((c+d*\sin(f*x+ \\ & e))/((d^2/c^2)^{(1/2)}*c*\sin(f*x+e)+d)*d)^{(1/2)}/((-d^2/c^2)^{(1/2)}*c)^{(1/2)})*c \\ & ^{(3/2)}*d^3*((c+d*\sin(f*x+e))/((d^2/c^2)^{(1/2)}*c*\sin(f*x+e)+d)*d)^{(1/2)}*\cos( \\ & f*x+e)+\arctan(((c+d*\sin(f*x+e))/((d^2/c^2)^{(1/2)}*c*\sin(f*x+e)+d)*d)^{(1/2)}/ \\ & (-d^2/c^2)^{(1/2)}*c)^{(1/2)})*c^{(5/2)}*d^2*((c+d*\sin(f*x+e))/((d^2/c^2)^{(1/2)}*c \\ & *\sin(f*x+e)+d)*d)^{(1/2)}*\cos(f*x+e)+\arctan(((c+d*\sin(f*x+e))/((d^2/c^2)^{(1/2) \\ & )*c*\sin(f*x+e)+d)*d)^{(1/2)}/((-d^2/c^2)^{(1/2)}*c)^{(1/2)})*c^{(3/2)}*d^3*(d^2/c^2 \\ & )^{(1/2)}*((c+d*\sin(f*x+e))/((d^2/c^2)^{(1/2)}*c*\sin(f*x+e)+d)*d)^{(1/2)}+\arctan( \\ & ((c+d*\sin(f*x+e))/((d^2/c^2)^{(1/2)}*c*\sin(f*x+e)+d)*d)^{(1/2)}/((-d^2/c^2)^{(1/ \\ & 2)*c)^{(1/2)})*c^{(5/2)}*d^2*((c+d*\sin(f*x+e))/((d^2/c^2)^{(1/2)}*c*\sin(f*x+e)+ \\ & d)*d)^{(1/2)}*\sin(f*x+e)+\arctan(((c+d*\sin(f*x+e))/((d^2/c^2)^{(1/2)}*c*\sin(f*x+ \\ & e)+d)*d)^{(1/2)}/((-d^2/c^2)^{(1/2)}*c)^{(1/2)})*d^4*c^{(1/2)}*((c+d*\sin(f*x+e))/((d^ \\ & 2/c^2)^{(1/2)}*c*\sin(f*x+e)+d)*d)^{(1/2)}*\sin(f*x+e)+\arctan(((c+d*\sin(f*x+e))/ \\ & (d^2/c^2)^{(1/2)}*c*\sin(f*x+e)+d)*d)^{(1/2)}/((-d^2/c^2)^{(1/2)}*c)^{(1/2)})*c^{(7/2) \\ & )*d*(d^2/c^2)^{(1/2)}*((c+d*\sin(f*x+e))/((d^2/c^2)^{(1/2)}*c*\sin(f*x+e)+d)*d)^{( \\ & 1/2)}-2*\arctan(((c+d*\sin(f*x+e))/((d^2/c^2)^{(1/2)}*c*\sin(f*x+e)+d)*d)^{(1/2)}/ \\ & (-d^2/c^2)^{(1/2)}*c)^{(1/2)})*c^{(5/2)}*d^2*(d^2/c^2)^{(1/2)}*((c+d*\sin(f*x+e))/(( \\ & d^2/c^2)^{(1/2)}*c*\sin(f*x+e)+d)*d)^{(1/2)}-2*\arctan(((c+d*\sin(f*x+e))/((d^2/c^ \\ & 2)^{(1/2)}*c*\sin(f*x+e)+d)*d)^{(1/2)}/((-d^2/c^2)^{(1/2)}*c)^{(1/2)})*c^{(3/2)}*d^3*( \\ & (c+d*\sin(f*x+e))/((d^2/c^2)^{(1/2)}*c*\sin(f*x+e)+d)*d)^{(1/2)}*\sin(f*x+e)+\arcta \\ & n(((c+d*\sin(f*x+e))/((d^2/c^2)^{(1/2)}*c*\sin(f*x+e)+d)*d)^{(1/2)}/((-d^2/c^2)^{( \\ & 1/2)*c)^{(1/2)})*d^4*c^{(1/2)}*((c+d*\sin(f*x+e))/((d^2/c^2)^{(1/2)}*c*\sin(f*x+e)+ \\ & d)*d)^{(1/2)}*\cos(f*x+e)-\arctan(((c+d*\sin(f*x+e))/((d^2/c^2)^{(1/2)}*c*\sin(f*x+ \\ & e)+d)*d)^{(1/2)}/((-d^2/c^2)^{(1/2)}*c)^{(1/2)})*c^{(7/2)}*d*(d^2/c^2)^{(1/2)}*((c+d* \\ & \sin(f*x+e))/((d^2/c^2)^{(1/2)}*c*\sin(f*x+e)+d)*d)^{(1/2)}*\sin(f*x+e)-\arctan(((c \end{aligned}$$



$$\begin{aligned}
& +d*\sin(f*x+e))/((d^2/c^2)^{(1/2)}*c*\sin(f*x+e)+d)*d)^{(1/2)}/(- (d^2/c^2)^{(1/2)}* \\
& c)^{(1/2)})*c^{(5/2)}*d^2*((c+d*\sin(f*x+e))/((d^2/c^2)^{(1/2)}*c*\sin(f*x+e)+d)*d) \\
& ^{(1/2)}+2*\arctan(((c+d*\sin(f*x+e))/((d^2/c^2)^{(1/2)}*c*\sin(f*x+e)+d)*d)^{(1/2)} \\
& /(- (d^2/c^2)^{(1/2)}*c)^{(1/2)})*c^{(3/2)}*d^3*((c+d*\sin(f*x+e))/((d^2/c^2)^{(1/2)} \\
& *c*\sin(f*x+e)+d)*d)^{(1/2)}-\arctan(((c+d*\sin(f*x+e))/((d^2/c^2)^{(1/2)}*c*\sin(f \\
& *x+e)+d)*d)^{(1/2)}/(- (d^2/c^2)^{(1/2)}*c)^{(1/2)})*d^4*c^{(1/2)}*((c+d*\sin(f*x+e)) \\
& /((d^2/c^2)^{(1/2)}*c*\sin(f*x+e)+d)*d)^{(1/2)}+\arctan(((d^2/c^2)^{(1/2)}*c*\sin(f* \\
& x+e)+d*\cos(f*x+e)-d)/((c+d*\sin(f*x+e))/((d^2/c^2)^{(1/2)}*c*\sin(f*x+e)+d)*d)^{(1/2)} \\
& /((d^2/c^2)^{(1/2)}*c*\sin(f*x+e)-d*\cos(f*x+e)+d)*((d^2/c^2)^{(1/2)}*c^2-d^2)*c*(( \\
& d^2/c^2)^{(1/2)}-1)/(((d^2/c^2)^{(1/2)}*c^4+6*(d^2/c^2)^{(1/2)}*d^2*c^2+d^4* \\
& (d^2/c^2)^{(1/2)}-4*c^2*d^2-4*d^4)*c)^{(1/2)})*c^{(3/2)}*((d^2/c^2)^{(1/2)}*c^4+ \\
& 6*(d^2/c^2)^{(1/2)}*d^2*c^2+d^4*(d^2/c^2)^{(1/2)}-4*c^2*d^2-4*d^4)*c)^{(1/2)}*(d^2/c^2)^{(1/2)}* \\
& ((c+d*\sin(f*x+e))/((d^2/c^2)^{(1/2)}*c*\sin(f*x+e)+d)*d)^{(1/2)}*(- \\
& (d^2/c^2)^{(1/2)}*c)^{(1/2)}*\sin(f*x+e)-\arctan(((d^2/c^2)^{(1/2)}*c*\sin(f*x+e)+d* \\
& \cos(f*x+e)-d)/((c+d*\sin(f*x+e))/((d^2/c^2)^{(1/2)}*c*\sin(f*x+e)+d)*d)^{(1/2)}/( \\
& (d^2/c^2)^{(1/2)}*c*\sin(f*x+e)-d*\cos(f*x+e)+d)*((d^2/c^2)^{(1/2)}*c^2-d^2)*c*(( \\
& d^2/c^2)^{(1/2)}-1)/(((d^2/c^2)^{(1/2)}*c^4+6*(d^2/c^2)^{(1/2)}*d^2*c^2+d^4*(d^2/c^2)^{(1/2)}-4*c^2*d^2- \\
& 4*d^4)*c)^{(1/2)})*d*((d^2/c^2)^{(1/2)}*c^4+6*(d^2/c^2)^{(1/2)}*d^2*c^2+d^4*(d^2/c^2)^{(1/2)}-4*c^2*d^2- \\
& 4*d^4)*c)^{(1/2)}*c^{(1/2)}*((c+d*\sin(f*x+e))/((d^2/c^2)^{(1/2)}*c*\sin(f*x+e)+d)*d)^{(1/2)}*(- \\
& (d^2/c^2)^{(1/2)}*c)^{(1/2)}*\sin(f*x+e)+\ln((c^{(1/2)}*2^{(1/2)}*((c+d*\sin(f*x+e))/(1+\cos(f*x+e)))^{(1/2)}* \\
& \sin(f*x+e)-\cos(f*x+e)*c+d*\sin(f*x+e)+c)/\sin(f*x+e)/c^{(1/2)})*c^3*d^2^{(1/2)}*((c+d*\sin(f*x+e))/(1+ \\
& \cos(f*x+e)))^{(1/2)}*(-(d^2/c^2)^{(1/2)}*c)^{(1/2)}*\sin(f*x+e)-2*\ln((c^{(1/2)}*2^{(1/2)}*((c+d*\sin(f*x+e))/(1+ \\
& \cos(f*x+e)))^{(1/2)}* \\
& \sin(f*x+e)-\cos(f*x+e)*c+d*\sin(f*x+e)+c)/\sin(f*x+e)/c^{(1/2)})*c^2*d^2*2^{(1/2)}*((c+d*\sin(f*x+e))/(1+ \\
& \cos(f*x+e)))^{(1/2)}*(-(d^2/c^2)^{(1/2)}*c)^{(1/2)}*\sin(f*x+e)+\ln((c^{(1/2)}*2^{(1/2)}*((c+d*\sin(f*x+e))/(1+ \\
& \cos(f*x+e)))^{(1/2)}* \\
& \sin(f*x+e)-\cos(f*x+e)*c+d*\sin(f*x+e)+c)/\sin(f*x+e)/c^{(1/2)})*c*d^3*2^{(1/2)}*((c+d*\sin(f*x+e))/(1+ \\
& \cos(f*x+e)))^{(1/2)}*(-(d^2/c^2)^{(1/2)}*c)^{(1/2)}*\sin(f*x+e)-\ln(-2*(c^{(1/2)}*2^{(1/2)}*((c+d*\sin(f*x+e))/(1+ \\
& \cos(f*x+e)))^{(1/2)}* \\
& \sin(f*x+e)-d*\cos(f*x+e)+c*\sin(f*x+e)+d)/(-1+\cos(f*x+e))) \\
& *c^3*d^2^{(1/2)}*((c+d*\sin(f*x+e))/(1+\cos(f*x+e)))^{(1/2)}*(-(d^2/c^2)^{(1/2)}*c)^{(1/2)}*\sin(f*x+e)+2*\ln(-2*(c^{(1/2)}*2^{(1/2)}*((c+d*\sin(f*x+e))/(1+ \\
& \cos(f*x+e)))^{(1/2)}* \\
& \sin(f*x+e)-d*\cos(f*x+e)+c*\sin(f*x+e))\dots
\end{aligned}$$

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(1/2)/sin(f*x+e),x, algo
ithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/sin(f*x + e), x
)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 349 vs.  $2(105) = 210$ .

time = 1.14, size = 3671, normalized size = 29.85

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(1/2)/sin(f*x+e),x, algo
ithm="fricas")
```

```
[Out] [1/4*(sqrt(a*c)*log(((a*c^4 - 28*a*c^3*d + 70*a*c^2*d^2 - 28*a*c*d^3 + a*d^4)*cos(f*x + e)^5 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 - (31*a*c^4 - 196*a*c^3*d + 154*a*c^2*d^2 - 4*a*c*d^3 - a*d^4)*cos(f*x + e)^4 - 2*(81*a*c^4 - 252*a*c^3*d + 150*a*c^2*d^2 - 28*a*c*d^3 + a*d^4)*cos(f*x + e)^3 + 2*(79*a*c^4 - 100*a*c^3*d + 74*a*c^2*d^2 - 4*a*c*d^3 - a*d^4)*cos(f*x + e)^2 - 8*((c^3 - 7*c^2*d + 7*c*d^2 - d^3)*cos(f*x + e)^4 - 2*(5*c^3 - 14*c^2*d + 5*c*d^2)*cos(f*x + e)^3 + 51*c^3 - 59*c^2*d + 17*c*d^2 - d^3 - 2*(18*c^3 - 33*c^2*d + 12*c*d^2 - d^3)*cos(f*x + e)^2 + 2*(13*c^3 - 14*c^2*d + 5*c*d^2)*cos(f*x + e) + ((c^3 - 7*c^2*d + 7*c*d^2 - d^3)*cos(f*x + e)^3 - 51*c^3 + 59*c^2*d - 17*c*d^2 + d^3 + (11*c^3 - 35*c^2*d + 17*c*d^2 - d^3)*cos(f*x + e)^2 - (25*c^3 - 31*c^2*d + 7*c*d^2 - d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c) + (289*a*c^4 - 476*a*c^3*d + 230*a*c^2*d^2 - 28*a*c*d^3 + a*d^4)*cos(f*x + e) + (a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 + (a*c^4 - 28*a*c^3*d + 70*a*c^2*d^2 - 28*a*c*d^3 + a*d^4)*cos(f*x + e)^4 + 32*(a*c^4 - 7*a*c^3*d + 7*a*c^2*d^2 - a*c*d^3)*cos(f*x + e)^3 - 2*(65*a*c^4 - 140*a*c^3*d + 38*a*c^2*d^2 - 12*a*c*d^3 + a*d^4)*cos(f*x + e)^2 - 32*(9*a*c^4 - 15*a*c^3*d + 7*a*c^2*d^2 - a*c*d^3)*cos(f*x + e))*sin(f*x + e))/(cos(f*x + e)^5 + cos(f*x + e)^4 - 2*cos(f*x + e)^3 - 2*cos(f*x + e)^2 + (cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sin(f*x + e) + cos(f*x + e) + 1)) + sqrt(-a*d)*log((128*a*d^4*cos(f*x + e)^5 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 + 128*(2*a*c*d^3 - a*d^4)*cos(f*x + e)^4 - 32*(5*a*c^2*d^2 - 14*a*c*d^3 + 13*a*d^4)*cos(f*x + e)^3 - 32*(a*c^3*d - 2*a*c^2*d^2 + 9*a*c*d^3 - 4*a*d^4)*cos(f*x + e)^2 - 8*(16*d^3*cos(f*x + e)^4 + 24*(c*d^2 - d^3)*cos(f*x + e)^3 - c^3 + 17*c^2*d - 59*c*d^2 + 51*d^3 - 2*(5*c^2*d - 26*c*d^2 + 33*d^3)*cos(f*x + e)^2 - (c^3 - 7*c^2*d + 31*c*d^2 - 25*d^3)*cos(f*x + e) + (16*d^3*cos(f*x + e)^3 + c^3 - 17*c^2*d + 59*c*d^2 - 51*d^3 - 8*(3*c*d^2 - 5*d^3)*cos(f*x + e)^2 - 2*(5*c^2*d - 14*c*d^2 + 13*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(-a*d
```

```

)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c) + (a*c^4 - 28*a*c^3*d +
  230*a*c^2*d^2 - 476*a*c*d^3 + 289*a*d^4)*cos(f*x + e) + (128*a*d^4*cos(f*x
  + e)^4 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 - 256*(a*c*d^
  3 - a*d^4)*cos(f*x + e)^3 - 32*(5*a*c^2*d^2 - 6*a*c*d^3 + 5*a*d^4)*cos(f*x
  + e)^2 + 32*(a*c^3*d - 7*a*c^2*d^2 + 15*a*c*d^3 - 9*a*d^4)*cos(f*x + e))*si
  n(f*x + e))/(cos(f*x + e) + sin(f*x + e) + 1))/f, 1/4*(2*sqrt(-a*c)*arctan
  (-1/4*((c^2 - 6*c*d + d^2)*cos(f*x + e)^2 - 9*c^2 + 6*c*d - d^2 + 8*(c^2 -
  c*d)*sin(f*x + e))*sqrt(-a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e)
  + c)/((a*c^2*d - a*c*d^2)*cos(f*x + e)^3 - (a*c^3 - 3*a*c^2*d)*cos(f*x + e)
  *sin(f*x + e) + (2*a*c^3 - a*c^2*d + a*c*d^2)*cos(f*x + e))) + sqrt(-a*d)*l
  og((128*a*d^4*cos(f*x + e)^5 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3
  + a*d^4 + 128*(2*a*c*d^3 - a*d^4)*cos(f*x + e)^4 - 32*(5*a*c^2*d^2 - 14*a*c
  *d^3 + 13*a*d^4)*cos(f*x + e)^3 - 32*(a*c^3*d - 2*a*c^2*d^2 + 9*a*c*d^3 - 4
  *a*d^4)*cos(f*x + e)^2 - 8*(16*d^3*cos(f*x + e)^4 + 24*(c*d^2 - d^3)*cos(f*
  x + e)^3 - c^3 + 17*c^2*d - 59*c*d^2 + 51*d^3 - 2*(5*c^2*d - 26*c*d^2 + 33*
  d^3)*cos(f*x + e)^2 - (c^3 - 7*c^2*d + 31*c*d^2 - 25*d^3)*cos(f*x + e) + (1
  6*d^3*cos(f*x + e)^3 + c^3 - 17*c^2*d + 59*c*d^2 - 51*d^3 - 8*(3*c*d^2 - 5*
  d^3)*cos(f*x + e)^2 - 2*(5*c^2*d - 14*c*d^2 + 13*d^3)*cos(f*x + e))*sin(f*x
  + e))*sqrt(-a*d)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c) + (a*c^
  4 - 28*a*c^3*d + 230*a*c^2*d^2 - 476*a*c*d^3 + 289*a*d^4)*cos(f*x + e) + (1
  28*a*d^4*cos(f*x + e)^4 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d
  ^4 - 256*(a*c*d^3 - a*d^4)*cos(f*x + e)^3 - 32*(5*a*c^2*d^2 - 6*a*c*d^3 + 5
  *a*d^4)*cos(f*x + e)^2 + 32*(a*c^3*d - 7*a*c^2*d^2 + 15*a*c*d^3 - 9*a*d^4)*
  cos(f*x + e))*sin(f*x + e))/(cos(f*x + e) + sin(f*x + e) + 1))/f, 1/4*(2*s
  qrt(a*d)*arctan(1/4*(8*d^2*cos(f*x + e)^2 - c^2 + 6*c*d - 9*d^2 - 8*(c*d -
  d^2)*sin(f*x + e))*sqrt(a*d)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) +
  c)/(2*a*d^3*cos(f*x + e)^3 - (3*a*c*d^2 - a*d^3)*cos(f*x + e)*sin(f*x + e)
  - (a*c^2*d - a*c*d^2 + 2*a*d^3)*cos(f*x + e))) + sqrt(a*c)*log(((a*c^4 - 2
  8*a*c^3*d + 70*a*c^2*d^2 - 28*a*c*d^3 + a*d^4)*cos(f*x + e)^5 + a*c^4 + 4*a
  *c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 - (31*a*c^4 - 196*a*c^3*d + 154*a*
  c^2*d^2 - 4*a*c*d^3 - a*d^4)*cos(f*x + e)^4 - 2*(81*a*c^4 - 252*a*c^3*d + 1
  50*a*c^2*d^2 - 28*a*c*d^3 + a*d^4)*cos(f*x + e)^3 + 2*(79*a*c^4 - 100*a*c^3
  *d + 74*a*c^2*d^2 - 4*a*c*d^3 - a*d^4)*cos(f*x + e)^2 - 8*((c^3 - 7*c^2*d +
  7*c*d^2 - d^3)*cos(f*x + e)^4 - 2*(5*c^3 - 14*c^2*d + 5*c*d^2)*cos(f*x + e)
  )^3 + 51*c^3 - 59*c^2*d + 17*c*d^2 - d^3 - 2*(18*c^3 - 33*c^2*d + 12*c*d^2
  - d^3)*cos(f*x + e)^2 + 2*(13*c^3 - 14*c^2*d + 5*c*d^2)*cos(f*x + e) + ((c^
  3 - 7*c^2*d + 7*c*d^2 - d^3)*cos(f*x + e)^3 - 51*c^3 + 59*c^2*d - 17*c*d^2
  + d^3 + (11*c^3 - 35*c^2*d + 17*c*d^2 - d^3)*cos(f*x + e)^2 - (25*c^3 - 31*
  c^2*d + 7*c*d^2 - d^3)*cos(f*x + e))*sin(f*x + ...

```

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a(\sin(e+fx)+1)} \sqrt{c+d\sin(e+fx)}}{\sin(e+fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)*(c+d*sin(f*x+e))**(1/2)/sin(f*x+e),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*sqrt(c + d*sin(e + f*x))/sin(e + f*x),
x)
```

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(1/2)/sin(f*x+e),x, algo
rithm="giac")
```

```
[Out] Timed out
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a + a \sin(e + f x)} \sqrt{c + d \sin(e + f x)}}{\sin(e + f x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^(1/2))/sin(e + f*x),x)
```

```
[Out] int(((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^(1/2))/sin(e + f*x), x
)
```

$$3.36 \quad \int \frac{\csc(e+fx) \sqrt{a + a \sin(e + fx)}}{\sqrt{c + d \sin(e + fx)}} dx$$

Optimal. Leaf size=61

$$-\frac{2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a} \sqrt{c} \cos(e+fx)}{\sqrt{a + a \sin(e + fx)} \sqrt{c + d \sin(e + fx)}}\right)}{\sqrt{c} f}$$

[Out]  $-2*\operatorname{arctanh}(\cos(f*x+e)*a^{(1/2)}*c^{(1/2)/(a+a*\sin(f*x+e))^{(1/2)/(c+d*\sin(f*x+e))^{(1/2)}})*a^{(1/2)}/f/c^{(1/2)}$

Rubi [A]

time = 0.13, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$ , Rules used = {3022, 212}

$$-\frac{2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a} \sqrt{c} \cos(e+fx)}{\sqrt{a \sin(e + fx) + a} \sqrt{c + d \sin(e + fx)}}\right)}{\sqrt{c} f}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(\operatorname{Csc}[e + f*x]*\operatorname{Sqrt}[a + a*\operatorname{Sin}[e + f*x]])/\operatorname{Sqrt}[c + d*\operatorname{Sin}[e + f*x]],x]$

[Out]  $(-2*\operatorname{Sqrt}[a]*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[c]*\operatorname{Cos}[e + f*x])/(\operatorname{Sqrt}[a + a*\operatorname{Sin}[e + f*x]]*\operatorname{Sqrt}[c + d*\operatorname{Sin}[e + f*x]])])/(\operatorname{Sqrt}[c]*f)$

Rule 212

$\operatorname{Int}[(a_ + (b_)*(x_)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]))*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[a, 2])], x] /;$   $\operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$

Rule 3022

$\operatorname{Int}[\operatorname{Sqrt}[(a_ + (b_)*\sin[(e_ + (f_)*(x_))])]/(\sin[(e_ + (f_)*(x_))]*\operatorname{Sqrt}[(c_ + (d_)*\sin[(e_ + (f_)*(x_))])]), x\_Symbol] \rightarrow \operatorname{Dist}[-2*(a/f), \operatorname{Subst}[\operatorname{Int}[1/(1 - a*c*x^2), x], x, \operatorname{Cos}[e + f*x]/(\operatorname{Sqrt}[a + b*\operatorname{Sin}[e + f*x]]*\operatorname{Sqrt}[c + d*\operatorname{Sin}[e + f*x]])], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \operatorname{EqQ}[a^2 - b^2, 0] \ \&\& \operatorname{NeQ}[b*c + a*d, 0]$

Rubi steps

$$\int \frac{\csc(e+fx) \sqrt{a+a \sin(e+fx)}}{\sqrt{c+d \sin(e+fx)}} dx = -\frac{(2a) \text{Subst} \left( \int \frac{1}{1-ax^2} dx, x, \frac{\cos(e+fx)}{\sqrt{a+a \sin(e+fx)} \sqrt{c+d \sin(e+fx)}} \right)}{f}$$

$$= -\frac{2\sqrt{a} \tanh^{-1} \left( \frac{\sqrt{a} \sqrt{c} \cos(e+fx)}{\sqrt{a+a \sin(e+fx)} \sqrt{c+d \sin(e+fx)}} \right)}{\sqrt{c} f}$$

**Mathematica [C]** Result contains complex when optimal does not.  
time = 2.94, size = 367, normalized size = 6.02

$$\frac{\left( \log \left( \frac{(1+i)^{\frac{1}{2}} \left( \sqrt{2} e^{-i(e+fx)} + \sqrt{2} e^{i(e+fx)} - 2\sqrt{c} \sqrt{2a^2 \cos^2(e+fx) - id(-1+e^{2i(e+fx)})} \right)^{\frac{1}{2}}}{\sqrt{c} (1+e^{2i(e+fx)})} \right) + \log \left( \frac{(1+i)^{\frac{1}{2}} \left( -\sqrt{2} e^{-i(e+fx)} + \sqrt{2} e^{i(e+fx)} + 2\sqrt{c} \sqrt{2a^2 \cos^2(e+fx) - id(-1+e^{2i(e+fx)})} \right)^{\frac{1}{2}}}{\sqrt{c} (-1+e^{2i(e+fx)})} \right) \right)}{\sqrt{c} f (\cos(\frac{1}{2}(e+fx)) + \sin(\frac{1}{2}(e+fx))) \sqrt{c+d \sin(e+fx)}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(Csc[e + f\*x]\*Sqrt[a + a\*Sin[e + f\*x]])/Sqrt[c + d\*Sin[e + f\*x]], x]

[Out] -(((Log[(-1 - I)\*E^((I/2)\*e)\*(Sqrt[2]\*c\*(-1 + E^(I\*(e + f\*x)))) + I\*Sqrt[2]\*d\*(1 + E^(I\*(e + f\*x)))] - (2\*I)\*Sqrt[c]\*Sqrt[2\*c\*E^(I\*(e + f\*x)) - I\*d\*(-1 + E^((2\*I)\*(e + f\*x)))])\*f)/(Sqrt[c]\*(1 + E^(I\*(e + f\*x))))] + Log[((1 + I)\*E^((I/2)\*e)\*((-I)\*Sqrt[2]\*d\*(-1 + E^(I\*(e + f\*x)))) + Sqrt[2]\*c\*(1 + E^(I\*(e + f\*x))) + 2\*Sqrt[c]\*Sqrt[2\*c\*E^(I\*(e + f\*x)) - I\*d\*(-1 + E^((2\*I)\*(e + f\*x)))])\*f)/(Sqrt[c]\*(-1 + E^(I\*(e + f\*x))))]\*((Cos[(e + f\*x)/2] - I\*Sin[(e + f\*x)/2])\*Sqrt[a\*(1 + Sin[e + f\*x])]\*Sqrt[(Cos[e + f\*x] + I\*Sin[e + f\*x])\*(c + d\*Sin[e + f\*x])])/(Sqrt[c]\*f\*(Cos[(e + f\*x)/2] + Sin[(e + f\*x)/2])\*Sqrt[c + d\*Sin[e + f\*x]])

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 237 vs. 2(49) = 98.

time = 20.06, size = 238, normalized size = 3.90

method	result
default	$\frac{\sqrt{a(1 + \sin(fx + e))} \sqrt{c + d \sin(fx + e)} \sqrt{2} \left( \ln \left( -\frac{-\sqrt{c} \sqrt{2} \sqrt{\frac{c+d \sin(fx+e)}{1+\cos(fx+e)}} \sin(fx+e) + \cos(fx+e)c - d \sin(fx+e)}{\sin(fx+e) \sqrt{c}} \right) \right)}{f \sin(fx+e)(-1+\cos(fx+e)-\sin(fx+e)) \sqrt{c}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a\*sin(f\*x+e))^(1/2)/sin(f\*x+e)/(c+d\*sin(f\*x+e))^(1/2),x,method=\_RETURNVERBOSE)

```
[Out] 1/f*(a*(1+sin(f*x+e)))^(1/2)*(c+d*sin(f*x+e))^(1/2)*2^(1/2)*(ln(-(-c^(1/2)*
2^(1/2)*((c+d*sin(f*x+e))/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)*c-d*s
in(f*x+e)-c)/sin(f*x+e)/c^(1/2))-ln(2*(-c^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(
1+cos(f*x+e)))^(1/2)*sin(f*x+e)+d*cos(f*x+e)-c*sin(f*x+e)-d)/(-1+cos(f*x+e)
)))*(-1+cos(f*x+e))/sin(f*x+e)/(-1+cos(f*x+e)-sin(f*x+e))/((c+d*sin(f*x+e))
/(1+cos(f*x+e)))^(1/2)/c^(1/2)
```

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/sin(f*x+e)/(c+d*sin(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)/(sqrt(d*sin(f*x + e) + c)*sin(f*x + e)),
x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 179 vs. 2(52) = 104.

time = 0.90, size = 1080, normalized size = 17.70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/sin(f*x+e)/(c+d*sin(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/4*sqrt(a/c)*log(((a*c^4 - 28*a*c^3*d + 70*a*c^2*d^2 - 28*a*c*d^3 + a*d^4
)*cos(f*x + e)^5 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 - (3
1*a*c^4 - 196*a*c^3*d + 154*a*c^2*d^2 - 4*a*c*d^3 - a*d^4)*cos(f*x + e)^4 -
2*(81*a*c^4 - 252*a*c^3*d + 150*a*c^2*d^2 - 28*a*c*d^3 + a*d^4)*cos(f*x +
e)^3 + 2*(79*a*c^4 - 100*a*c^3*d + 74*a*c^2*d^2 - 4*a*c*d^3 - a*d^4)*cos(f*
x + e)^2 - 8*((c^4 - 7*c^3*d + 7*c^2*d^2 - c*d^3)*cos(f*x + e)^4 + 51*c^4 -
59*c^3*d + 17*c^2*d^2 - c*d^3 - 2*(5*c^4 - 14*c^3*d + 5*c^2*d^2)*cos(f*x +
e)^3 - 2*(18*c^4 - 33*c^3*d + 12*c^2*d^2 - c*d^3)*cos(f*x + e)^2 + 2*(13*c
^4 - 14*c^3*d + 5*c^2*d^2)*cos(f*x + e) - (51*c^4 - 59*c^3*d + 17*c^2*d^2 -
c*d^3 - (c^4 - 7*c^3*d + 7*c^2*d^2 - c*d^3)*cos(f*x + e)^3 - (11*c^4 - 35*
c^3*d + 17*c^2*d^2 - c*d^3)*cos(f*x + e)^2 + (25*c^4 - 31*c^3*d + 7*c^2*d^2
- c*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f
*x + e) + c)*sqrt(a/c) + (289*a*c^4 - 476*a*c^3*d + 230*a*c^2*d^2 - 28*a*c*
d^3 + a*d^4)*cos(f*x + e) + (a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 +
a*d^4 + (a*c^4 - 28*a*c^3*d + 70*a*c^2*d^2 - 28*a*c*d^3 + a*d^4)*cos(f*x +
e)^4 + 32*(a*c^4 - 7*a*c^3*d + 7*a*c^2*d^2 - a*c*d^3)*cos(f*x + e)^3 - 2*(6
5*a*c^4 - 140*a*c^3*d + 38*a*c^2*d^2 - 12*a*c*d^3 + a*d^4)*cos(f*x + e)^2 -
```

$$\frac{32(9ac^4 - 15a^2c^3d + 7a^2c^2d^2 - a^2cd^3)\cos(fx + e)\sin(fx + e)}{(\cos(fx + e)^5 + \cos(fx + e)^4 - 2\cos(fx + e)^3 - 2\cos(fx + e)^2 + (\cos(fx + e)^4 - 2\cos(fx + e)^2 + 1)\sin(fx + e) + \cos(fx + e) + 1)}$$

$$\int \frac{1}{f} \frac{1}{2} \sqrt{-a/c} \arctan\left(-\frac{1}{4}((c^2 - 6cd + d^2)\cos(fx + e)^2 - 9c^2 + 6cd - d^2 + 8(c^2 - cd)\sin(fx + e))\sqrt{a\sin(fx + e) + a}\sqrt{d\sin(fx + e) + c}\sqrt{-a/c}\right)}{((acd - ad^2)\cos(fx + e)^3 - (a^2c^2 - 3a^2cd)\cos(fx + e)\sin(fx + e) + (2a^2c^2 - acd + ad^2)\cos(fx + e))}$$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a(\sin(e + fx) + 1)}}{\sqrt{c + d\sin(e + fx)} \sin(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*sin(f\*x+e))\*\*(1/2)/sin(f\*x+e)/(c+d\*sin(f\*x+e))\*\*(1/2),x)

[Out] Integral(sqrt(a\*(sin(e + f\*x) + 1))/(sqrt(c + d\*sin(e + f\*x))\*sin(e + f\*x)), x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*sin(f\*x+e))^(1/2)/sin(f\*x+e)/(c+d\*sin(f\*x+e))^(1/2),x, algorith="giac")

[Out] integrate(sqrt(a\*sin(f\*x + e) + a)/(sqrt(d\*sin(f\*x + e) + c)\*sin(f\*x + e)), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a + a \sin(e + fx)}}{\sin(e + fx) \sqrt{c + d \sin(e + fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a\*sin(e + f\*x))^(1/2)/(sin(e + f\*x)\*(c + d\*sin(e + f\*x))^(1/2)),x)

[Out] int((a + a\*sin(e + f\*x))^(1/2)/(sin(e + f\*x)\*(c + d\*sin(e + f\*x))^(1/2)), x)



$$3.37 \int \frac{\csc(e+fx) \sqrt{c+d \sin(e+fx)}}{\sqrt{a+a \sin(e+fx)}} dx$$

**Optimal.** Leaf size=140

$$\frac{2\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c} \cos(e+fx)}{\sqrt{a+a \sin(e+fx)} \sqrt{c+d \sin(e+fx)}}\right)}{\sqrt{a} f} + \frac{\sqrt{2} \sqrt{c-d} \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}}{\sqrt{2} \sqrt{a+a \sin(e+fx)}}\right)}{\sqrt{a} f}$$

[Out]  $-2*\operatorname{arctanh}(\cos(f*x+e)*a^{(1/2)}*c^{(1/2)}/(a+a*\sin(f*x+e))^{(1/2)}/(c+d*\sin(f*x+e))^{(1/2)})*c^{(1/2)}/f/a^{(1/2)}+\operatorname{arctanh}(1/2*\cos(f*x+e)*a^{(1/2)}*(c-d)^{(1/2)}*2^{(1/2)}/(a+a*\sin(f*x+e))^{(1/2)}/(c+d*\sin(f*x+e))^{(1/2)})*2^{(1/2)}*(c-d)^{(1/2)}/f/a^{(1/2)}$

**Rubi [A]**

time = 0.34, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {3023, 2861, 214, 3022, 212}

$$\frac{\sqrt{2} \sqrt{c-d} \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d} \cos(e+fx)}{\sqrt{2} \sqrt{a \sin(e+fx)+a} \sqrt{c+d \sin(e+fx)}}\right)}{\sqrt{a} f} - \frac{2\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c} \cos(e+fx)}{\sqrt{a \sin(e+fx)+a} \sqrt{c+d \sin(e+fx)}}\right)}{\sqrt{a} f}$$

Antiderivative was successfully verified.

[In] `Int[(Csc[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[a + a*Sin[e + f*x]],x]`

[Out] `(-2*Sqrt[c]*ArcTanh[(Sqrt[a]*Sqrt[c]*Cos[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])]/(Sqrt[a]*f) + (Sqrt[2]*Sqrt[c - d]*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])]/(Sqrt[a]*f))`

Rule 212

`Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

Rule 214

`Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

Rule 2861

`Int[1/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c`

```
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

### Rule 3022

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/(sin[(e_) + (f_)*(x_)]*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[-2*(a/f), Subst[Int[1/(1 - a*c*x^2), x], x, Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[b*c + a*d, 0]
```

### Rule 3023

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/(sin[(e_) + (f_)*(x_)]*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[(b*c - a*d)/c, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] + Dist[a/c, Int[Sqrt[c + d*Sin[e + f*x]]/(Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]
```

### Rubi steps

$$\int \frac{\csc(e+fx) \sqrt{c+d \sin(e+fx)}}{\sqrt{a+a \sin(e+fx)}} dx = \frac{c \int \frac{\csc(e+fx) \sqrt{a+a \sin(e+fx)}}{\sqrt{c+d \sin(e+fx)}} dx}{a} + (-c+d) \int \frac{1}{\sqrt{a+a \sin(e+fx)}} dx$$

$$= \frac{(2c) \text{Subst}\left(\int \frac{1}{1-ax^2} dx, x, \frac{\cos(e+fx)}{\sqrt{a+a \sin(e+fx)} \sqrt{c+d \sin(e+fx)}}\right)}{f}$$

$$= \frac{2\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{a} \sqrt{c} \cos(e+fx)}{\sqrt{a+a \sin(e+fx)} \sqrt{c+d \sin(e+fx)}}\right)}{\sqrt{a} f} + \frac{\sqrt{2}}{\sqrt{a}}$$

**Mathematica [B]** Leaf count is larger than twice the leaf count of optimal. 2506 vs. 2(140) = 280.

time = 27.18, size = 2506, normalized size = 17.90

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Csc[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[a + a*Sin[e + f*x]], x]
```

```

[Out] (Csc[e + f*x]*(2*Sqrt[c]*ArcTanh[(2*Sqrt[c]*Tan[(e + f*x)/4])/(Sqrt[c] + Sqrt[Sec[(e + f*x)/4]^4*(c + d*Sin[e + f*x])]) - Sqrt[c]*Tan[(e + f*x)/4]^2)] - 2*Sqrt[2]*Sqrt[c - d]*ArcTanh[(2*Sqrt[2]*Sqrt[c - d]*Tan[(e + f*x)/4])/(Sqrt[c] + Sqrt[Sec[(e + f*x)/4]^4*(c + d*Sin[e + f*x])]) + 2*Sqrt[c]*Tan[(e + f*x)/4] - Sqrt[c]*Tan[(e + f*x)/4]^2)] + Sqrt[c]*(Log[Tan[(e + f*x)/4]] - Log[c + Sqrt[c]*Sqrt[Sec[(e + f*x)/4]^4*(c + d*Sin[e + f*x])]) + 2*d*Tan[(e + f*x)/4] - c*Tan[(e + f*x)/4]^2))*Sec[(e + f*x)/4]^2*(c + d*Sin[e + f*x]))/(f*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[Sec[(e + f*x)/4]^4*(c + d*Sin[e + f*x])])*((d*Cos[e + f*x]*(2*Sqrt[c]*ArcTanh[(2*Sqrt[c]*Tan[(e + f*x)/4])/(Sqrt[c] + Sqrt[Sec[(e + f*x)/4]^4*(c + d*Sin[e + f*x])]) - Sqrt[c]*Tan[(e + f*x)/4]^2)] - 2*Sqrt[2]*Sqrt[c - d]*ArcTanh[(2*Sqrt[2]*Sqrt[c - d]*Tan[(e + f*x)/4])/(Sqrt[c] + Sqrt[Sec[(e + f*x)/4]^4*(c + d*Sin[e + f*x])]) + 2*Sqrt[c]*Tan[(e + f*x)/4] - Sqrt[c]*Tan[(e + f*x)/4]^2)] + Sqrt[c]*(Log[Tan[(e + f*x)/4]] - Log[c + Sqrt[c]*Sqrt[Sec[(e + f*x)/4]^4*(c + d*Sin[e + f*x])]) + 2*d*Tan[(e + f*x)/4] - c*Tan[(e + f*x)/4]^2))*Sec[(e + f*x)/4]^2)/(2*Sqrt[c + d*Sin[e + f*x]]*Sqrt[Sec[(e + f*x)/4]^4*(c + d*Sin[e + f*x])]) + ((2*Sqrt[c]*ArcTanh[(2*Sqrt[c]*Tan[(e + f*x)/4])/(Sqrt[c] + Sqrt[Sec[(e + f*x)/4]^4*(c + d*Sin[e + f*x])]) - Sqrt[c]*Tan[(e + f*x)/4]^2)] - 2*Sqrt[2]*Sqrt[c - d]*ArcTanh[(2*Sqrt[2]*Sqrt[c - d]*Tan[(e + f*x)/4])/(Sqrt[c] + Sqrt[Sec[(e + f*x)/4]^4*(c + d*Sin[e + f*x])]) + 2*Sqrt[c]*Tan[(e + f*x)/4] - Sqrt[c]*Tan[(e + f*x)/4]^2)] + Sqrt[c]*(Log[Tan[(e + f*x)/4]] - Log[c + Sqrt[c]*Sqrt[Sec[(e + f*x)/4]^4*(c + d*Sin[e + f*x])]) + 2*d*Tan[(e + f*x)/4] - c*Tan[(e + f*x)/4]^2))*Sec[(e + f*x)/4]^2*Sqrt[c + d*Sin[e + f*x]]*Tan[(e + f*x)/4])/(2*Sqrt[Sec[(e + f*x)/4]^4*(c + d*Sin[e + f*x])]) - ((2*Sqrt[c]*ArcTanh[(2*Sqrt[c]*Tan[(e + f*x)/4])/(Sqrt[c] + Sqrt[Sec[(e + f*x)/4]^4*(c + d*Sin[e + f*x])]) - Sqrt[c]*Tan[(e + f*x)/4]^2)] - 2*Sqrt[2]*Sqrt[c - d]*ArcTanh[(2*Sqrt[2]*Sqrt[c - d]*Tan[(e + f*x)/4])/(Sqrt[c] + Sqrt[Sec[(e + f*x)/4]^4*(c + d*Sin[e + f*x])]) + 2*Sqrt[c]*Tan[(e + f*x)/4] - Sqrt[c]*Tan[(e + f*x)/4]^2)] + Sqrt[c]*(Log[Tan[(e + f*x)/4]] - Log[c + Sqrt[c]*Sqrt[Sec[(e + f*x)/4]^4*(c + d*Sin[e + f*x])]) + 2*d*Tan[(e + f*x)/4] - c*Tan[(e + f*x)/4]^2))*Sec[(e + f*x)/4]^2*Sqrt[c + d*Sin[e + f*x]]*(d*Cos[e + f*x]*Sec[(e + f*x)/4]^4 + Sec[(e + f*x)/4]^4*(c + d*Sin[e + f*x])*Tan[(e + f*x)/4]))/(2*(Sec[(e + f*x)/4]^4*(c + d*Sin[e + f*x])^(3/2)) + (Sec[(e + f*x)/4]^2*Sqrt[c + d*Sin[e + f*x]]*((2*Sqrt[c]*((Sqrt[c]*Sec[(e + f*x)/4]^2)/(2*(Sqrt[c] + Sqrt[Sec[(e + f*x)/4]^4*(c + d*Sin[e + f*x])]) - Sqrt[c]*Tan[(e + f*x)/4]^2)) - (2*Sqrt[c]*Tan[(e + f*x)/4]*(-1/2*(Sqrt[c]*Sec[(e + f*x)/4]^2*Tan[(e + f*x)/4]) + (d*Cos[e + f*x]*Sec[(e + f*x)/4]^4 + Sec[(e + f*x)/4]^4*(c + d*Sin[e + f*x])*Tan[(e + f*x)/4]))/(2*Sqrt[Sec[(e + f*x)/4]^4*(c + d*Sin[e + f*x])])))/(Sqrt[c] + Sqrt[Sec[(e + f*x)/4]^4*(c + d*Sin[e + f*x])]) - Sqrt[c]*Tan[(e + f*x)/4]^2)^2)/(1 - (4*c*Tan[(e + f*x)/4]^2)/(Sqrt[c] + Sqrt[Sec[(e + f*x)/4]^4*(c + d*Sin[e + f*x])]) - Sqrt[c]*Tan[(e + f*x)/4]^2)^2) - (2*Sqrt[2]*Sqrt[c - d]*((Sqrt[c - d]*Sec[(e + f*x)/4]^2)/(Sqrt[2]*(Sqrt[c] + Sqrt[Sec[(e + f*x)/4]^4*(c + d*Sin[e + f*x])]) + 2*Sqrt[c]*Tan[(e + f*x)/4] - Sqrt[c]*Tan[(e + f*x)/4]^2)) - (2*Sqrt[2]*Sqrt[c - d]*Tan[(e + f*x)/4]*((Sqrt[c]*Sec[(e + f*x)/4]^2)/2 - (Sqrt[c]*Sec[(e + f*x)/4]^2*Tan[(e + f*x)/4])/2 + (

```

$$\frac{d \cos[e + f x] \sec[(e + f x)/4]^4 + \sec[(e + f x)/4]^4 (c + d \sin[e + f x]) \tan[(e + f x)/4]}{(2 \sqrt{\sec[(e + f x)/4]^4 (c + d \sin[e + f x])})} \left( \sqrt{c} + \sqrt{\sec[(e + f x)/4]^4 (c + d \sin[e + f x])} + 2 \sqrt{c} \tan[(e + f x)/4] - \sqrt{c} \tan[(e + f x)/4]^2 \right) / (1 - (8(c - d) \tan[(e + f x)/4]^2)) / (\sqrt{c} + \sqrt{\sec[(e + f x)/4]^4 (c + d \sin[e + f x])} + 2 \sqrt{c} \tan[(e + f x)/4] - \sqrt{c} \tan[(e + f x)/4]^2) + \sqrt{c} \left( \csc[(e + f x)/4] \sec[(e + f x)/4] / 4 - ((d \sec[(e + f x)/4]^2) / 2 - (c \sec[(e + f x)/4]^2 \tan[(e + f x)/4]) / 2 + (\sqrt{c} (d \cos[e + f x] \sec[(e + f x)/4]^4 + \sec[(e + f x)/4]^4 (c + d \sin[e + f x]) \tan[(e + f x)/4])) / (2 \sqrt{\sec[(e + f x)/4]^4 (c + d \sin[e + f x])})} \right) / (c + \sqrt{c} \sqrt{\sec[(e + f x)/4]^4 (c + d \sin[e + f x])} + 2 d \tan[(e + f x)/4] - c \tan[(e + f x)/4]^2) / \sqrt{\sec[(e + f x)/4]^4 (c + d \sin[e + f x])}$$

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 346 vs. 2(113) = 226.

time = 0.28, size = 347, normalized size = 2.48

method	result
default	$\frac{(-1 + \cos(fx + e) - \sin(fx + e)) \sqrt{c + d \sin(fx + e)}}{\sqrt{2c - 2d}} \ln \left( - \frac{\sqrt{2c - 2d} \sqrt{2} \sqrt{\frac{c + d \sin(fx + e)}{1 + \cos(fx + e)}} \sin(fx + e)}{-1 + \cos(fx + e)} \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+d\*sin(f\*x+e))^(1/2)/sin(f\*x+e)/(a+a\*sin(f\*x+e))^(1/2),x,method=\_RETURNVERBOSE)

[Out] 
$$-1/2/f * (-1 + \cos(fx + e) - \sin(fx + e)) * (c + d \sin(fx + e))^{1/2} * ((2c - 2d)^{1/2}) * \ln(-2 * ((2c - 2d)^{1/2}) * 2^{1/2} * ((c + d \sin(fx + e)) / (1 + \cos(fx + e)))^{1/2} * \sin(fx + e) + c \sin(fx + e) - d \sin(fx + e) + \cos(fx + e) * c - d \cos(fx + e) - c + d) / (-1 + \cos(fx + e) - \sin(fx + e)) * c^{1/2} - c * \ln(2 * (-c^{1/2}) * 2^{1/2} * ((c + d \sin(fx + e)) / (1 + \cos(fx + e)))^{1/2} * \sin(fx + e) + d \cos(fx + e) - c \sin(fx + e) - d) / (-1 + \cos(fx + e)) + \ln(-(-c^{1/2}) * 2^{1/2} * ((c + d \sin(fx + e)) / (1 + \cos(fx + e)))^{1/2} * \sin(fx + e) + \cos(fx + e) * c - d \sin(fx + e) - c) / \sin(fx + e) / c^{1/2} * c) / (a * (1 + \sin(fx + e)))^{1/2} / \sin(fx + e) * 2^{1/2} / ((c + d \sin(fx + e)) / (1 + \cos(fx + e)))^{1/2} / c^{1/2}$$

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*sin(f\*x+e))^(1/2)/sin(f\*x+e)/(a+a\*sin(f\*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(d\*sin(f\*x + e) + c)/(sqrt(a\*sin(f\*x + e) + a)\*sin(f\*x + e)), x)

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 302 vs. 2(119) = 238.

time = 0.75, size = 2911, normalized size = 20.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*sin(f\*x+e))^(1/2)/sin(f\*x+e)/(a+a\*sin(f\*x+e))^(1/2),x, algorithm="fricas")

[Out] [1/4\*(sqrt(2)\*sqrt((c - d)/a)\*log(((c^2 - 14\*c\*d + 17\*d^2)\*cos(f\*x + e))^3 + 4\*sqrt(2)\*((c - 3\*d)\*cos(f\*x + e)^2 - (3\*c - d)\*cos(f\*x + e) + ((c - 3\*d)\*cos(f\*x + e) + 4\*c - 4\*d)\*sin(f\*x + e) - 4\*c + 4\*d)\*sqrt(a\*sin(f\*x + e) + a)\*sqrt(d\*sin(f\*x + e) + c)\*sqrt((c - d)/a) - (13\*c^2 - 22\*c\*d - 3\*d^2)\*cos(f\*x + e)^2 - 4\*c^2 - 8\*c\*d - 4\*d^2 - 2\*(9\*c^2 - 14\*c\*d + 9\*d^2)\*cos(f\*x + e) + ((c^2 - 14\*c\*d + 17\*d^2)\*cos(f\*x + e)^2 - 4\*c^2 - 8\*c\*d - 4\*d^2 + 2\*(7\*c^2 - 18\*c\*d + 7\*d^2)\*cos(f\*x + e))\*sin(f\*x + e))/(cos(f\*x + e)^3 + 3\*cos(f\*x + e)^2 + (cos(f\*x + e)^2 - 2\*cos(f\*x + e) - 4)\*sin(f\*x + e) - 2\*cos(f\*x + e) - 4)) + sqrt(c/a)\*log(((c^4 - 28\*c^3\*d + 70\*c^2\*d^2 - 28\*c\*d^3 + d^4)\*cos(f\*x + e)^5 - (31\*c^4 - 196\*c^3\*d + 154\*c^2\*d^2 - 4\*c\*d^3 - d^4)\*cos(f\*x + e)^4 + c^4 + 4\*c^3\*d + 6\*c^2\*d^2 + 4\*c\*d^3 + d^4 - 2\*(81\*c^4 - 252\*c^3\*d + 150\*c^2\*d^2 - 28\*c\*d^3 + d^4)\*cos(f\*x + e)^3 + 2\*(79\*c^4 - 100\*c^3\*d + 74\*c^2\*d^2 - 4\*c\*d^3 - d^4)\*cos(f\*x + e)^2 - 8\*((c^3 - 7\*c^2\*d + 7\*c\*d^2 - d^3)\*cos(f\*x + e)^4 - 2\*(5\*c^3 - 14\*c^2\*d + 5\*c\*d^2)\*cos(f\*x + e)^3 + 51\*c^3 - 59\*c^2\*d + 17\*c\*d^2 - d^3 - 2\*(18\*c^3 - 33\*c^2\*d + 12\*c\*d^2 - d^3)\*cos(f\*x + e)^2 + 2\*(13\*c^3 - 14\*c^2\*d + 5\*c\*d^2)\*cos(f\*x + e) + ((c^3 - 7\*c^2\*d + 7\*c\*d^2 - d^3)\*cos(f\*x + e)^3 - 51\*c^3 + 59\*c^2\*d - 17\*c\*d^2 + d^3 + (11\*c^3 - 35\*c^2\*d + 17\*c\*d^2 - d^3)\*cos(f\*x + e)^2 - (25\*c^3 - 31\*c^2\*d + 7\*c\*d^2 - d^3)\*cos(f\*x + e))\*sin(f\*x + e))\*sqrt(a\*sin(f\*x + e) + a)\*sqrt(d\*sin(f\*x + e) + c)\*sqrt(c/a) + (289\*c^4 - 476\*c^3\*d + 230\*c^2\*d^2 - 28\*c\*d^3 + d^4)\*cos(f\*x + e) + ((c^4 - 28\*c^3\*d + 70\*c^2\*d^2 - 28\*c\*d^3 + d^4)\*cos(f\*x + e)^4 + c^4 + 4\*c^3\*d + 6\*c^2\*d^2 + 4\*c\*d^3 + d^4 + 32\*(c^4 - 7\*c^3\*d + 7\*c^2\*d^2 - c\*d^3)\*cos(f\*x + e)^3 - 2\*(65\*c^4 - 140\*c^3\*d + 38\*c^2\*d^2 - 12\*c\*d^3 + d^4)\*cos(f\*x + e)^2 - 32\*(9\*c^4 - 15\*c^3\*d + 7\*c^2\*d^2 - c\*d^3)\*cos(f\*x + e))\*sin(f\*x + e))/(cos(f\*x + e)^5 + cos(f\*x + e)^4 - 2\*cos(f\*x + e)^3 - 2\*cos(f\*x + e)^2 + (cos(f\*x + e)^4 - 2\*cos(f\*x + e)^2 + 1)\*sin(f\*x + e) + cos(f\*x + e) + 1))/f, 1/4\*(sqrt(2)\*sqrt((c - d)/a)\*log(((c^2 - 14\*c\*d + 17\*d^2)\*cos(f\*x + e))^3 + 4\*sqrt(2)\*((c - 3\*d)\*cos(f\*x + e)^2 - (3\*c - d)\*cos(f\*x + e) + ((c - 3\*d)\*cos(f\*x + e) + 4\*c - 4\*d)\*sin(f\*x + e) - 4\*c + 4\*d)\*sqrt(a\*sin(f\*x + e) + a)\*sqrt(d\*sin(f\*x + e) + c)\*sqrt((c - d)/a) - (13\*c^2 - 22\*c\*d - 3\*d^2)\*cos(f\*x + e)^2 - 4\*c^2 - 8\*c\*d - 4\*d^2 - 2\*(9\*c^2 - 14\*c\*d + 9\*d^2)\*cos(f\*x + e) + ((c^2 - 14\*c\*d + 17\*d^2)\*cos(f\*x + e)^2 - 4\*c^2

```

- 8*c*d - 4*d^2 + 2*(7*c^2 - 18*c*d + 7*d^2)*cos(f*x + e))*sin(f*x + e))/(
cos(f*x + e)^3 + 3*cos(f*x + e)^2 + (cos(f*x + e)^2 - 2*cos(f*x + e) - 4)*s
in(f*x + e) - 2*cos(f*x + e) - 4)) + 2*sqrt(-c/a)*arctan(-1/4*((c^2 - 6*c*d
+ d^2)*cos(f*x + e)^2 - 9*c^2 + 6*c*d - d^2 + 8*(c^2 - c*d)*sin(f*x + e))*
sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(-c/a)/((c^2*d - c*d^
2)*cos(f*x + e)^3 - (c^3 - 3*c^2*d)*cos(f*x + e)*sin(f*x + e) + (2*c^3 - c^
2*d + c*d^2)*cos(f*x + e))))/f, 1/4*(2*sqrt(2)*sqrt(-(c - d)/a)*arctan(1/4*
sqrt(2)*sqrt(a*sin(f*x + e) + a)*((c - 3*d)*sin(f*x + e) - 3*c + d)*sqrt(d*
sin(f*x + e) + c)*sqrt(-(c - d)/a)/((c*d - d^2)*cos(f*x + e)*sin(f*x + e) +
(c^2 - c*d)*cos(f*x + e))) + sqrt(c/a)*log(((c^4 - 28*c^3*d + 70*c^2*d^2 -
28*c*d^3 + d^4)*cos(f*x + e)^5 - (31*c^4 - 196*c^3*d + 154*c^2*d^2 - 4*c*d
^3 - d^4)*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4 - 2*(8
1*c^4 - 252*c^3*d + 150*c^2*d^2 - 28*c*d^3 + d^4)*cos(f*x + e)^3 + 2*(79*c^
4 - 100*c^3*d + 74*c^2*d^2 - 4*c*d^3 - d^4)*cos(f*x + e)^2 - 8*((c^3 - 7*c^
2*d + 7*c*d^2 - d^3)*cos(f*x + e)^4 - 2*(5*c^3 - 14*c^2*d + 5*c*d^2)*cos(f*
x + e)^3 + 51*c^3 - 59*c^2*d + 17*c*d^2 - d^3 - 2*(18*c^3 - 33*c^2*d + 12*c
*d^2 - d^3)*cos(f*x + e)^2 + 2*(13*c^3 - 14*c^2*d + 5*c*d^2)*cos(f*x + e) +
((c^3 - 7*c^2*d + 7*c*d^2 - d^3)*cos(f*x + e)^3 - 51*c^3 + 59*c^2*d - 17*c
*d^2 + d^3 + (11*c^3 - 35*c^2*d + 17*c*d^2 - d^3)*cos(f*x + e)^2 - (25*c^3
- 31*c^2*d + 7*c*d^2 - d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e)
+ a)*sqrt(d*sin(f*x + e) + c)*sqrt(c/a) + (289*c^4 - 476*c^3*d + 230*c^2*d
^2 - 28*c*d^3 + d^4)*cos(f*x + e) + ((c^4 - 28*c^3*d + 70*c^2*d^2 - 28*c*d^
3 + d^4)*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4 + 32*(c
^4 - 7*c^3*d + 7*c^2*d^2 - c*d^3)*cos(f*x + e)^3 - 2*(65*c^4 - 140*c^3*d +
38*c^2*d^2 - 12*c*d^3 + d^4)*cos(f*x + e)^2 - 32*(9*c^4 - 15*c^3*d + 7*c^2*
d^2 - c*d^3)*cos(f*x + e))*sin(f*x + e))/(cos(f*x + e)^5 + cos(f*x + e)^4 -
2*cos(f*x + e)^3 - 2*cos(f*x + e)^2 + (cos(f*x + e)^4 - 2*cos(f*x + e)^2 +
1)*sin(f*x + e) + cos(f*x + e) + 1))/f, 1/2*(sqrt(2)*sqrt(-(c - d)/a)*arc
tan(1/4*sqrt(2)*sqrt(a*sin(f*x + e) + a)*((c - 3*d)*sin(f*x + e) - 3*c + d)
*sqrt(d*sin(f*x + e) + c)*sqrt(-(c - d)/a)/((c*d - d^2)*cos(f*x + e)*sin(f*
x + e) + (c^2 - c*d)*cos(f*x + e))) + sqrt(-c/a)*arctan(-1/4*((c^2 - 6*c*d
+ d^2)*cos(f*x + e)^2 - 9*c^2 + 6*c*d - d^2 + 8*(c^2 - c*d)*sin(f*x + e))*
sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(-c/a)/((c^2*d - c*d^2
)*cos(f*x + e)^3 - (c^3 - 3*c^2*d)*cos(f*x + e)*sin(f*x + e) + (2*c^3 - c^2
*d + c*d^2)*cos(f*x + e))))/f]

```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + d \sin(e + fx)}}{\sqrt{a(\sin(e + fx) + 1)} \sin(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*sin(f\*x+e))\*\*(1/2)/sin(f\*x+e)/(a+a\*sin(f\*x+e))\*\*(1/2),x)

[Out] Integral(sqrt(c + d\*sin(e + f\*x))/(sqrt(a\*(sin(e + f\*x) + 1))\*sin(e + f\*x)), x)

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*sin(f\*x+e))^(1/2)/sin(f\*x+e)/(a+a\*sin(f\*x+e))^(1/2),x, algorithm="giac")

[Out] Timed out

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c + d \sin(e + f x)}}{\sin(e + f x) \sqrt{a + a \sin(e + f x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*sin(e + f\*x))^(1/2)/(sin(e + f\*x)\*(a + a\*sin(e + f\*x))^(1/2)),x)

[Out] int((c + d\*sin(e + f\*x))^(1/2)/(sin(e + f\*x)\*(a + a\*sin(e + f\*x))^(1/2)), x)

$$3.38 \quad \int \frac{\csc(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}} dx$$

Optimal. Leaf size=140

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{a}\sqrt{c}f} + \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{a}\sqrt{c-d}f}$$

[Out]  $-2*\operatorname{arctanh}(\cos(f*x+e)*a^{(1/2)}*c^{(1/2)}/(a+a*\sin(f*x+e))^{(1/2)}/(c+d*\sin(f*x+e))^{(1/2)})/f/a^{(1/2)}/c^{(1/2)}+\operatorname{arctanh}(1/2*\cos(f*x+e)*a^{(1/2)}*(c-d)^{(1/2)}*2^{(1/2)}/(a+a*\sin(f*x+e))^{(1/2)}/(c+d*\sin(f*x+e))^{(1/2)})*2^{(1/2)}/f/a^{(1/2)}/(c-d)^{(1/2)}$

Rubi [A]

time = 0.32, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {3026, 2861, 214, 3022, 212}

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{a}f\sqrt{c-d}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{a}\sqrt{c}f}$$

Antiderivative was successfully verified.

[In] Int[Csc[e + f\*x]/(Sqrt[a + a\*Sin[e + f\*x]]\*Sqrt[c + d\*Sin[e + f\*x]]),x]

[Out]  $(-2*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[c]*\operatorname{Cos}[e+f*x])/(\operatorname{Sqrt}[a+a*\sin[e+f*x]]*\operatorname{Sqrt}[c+d*\sin[e+f*x]])]/(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[c]*f) + (\operatorname{Sqrt}[2]*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[c-d]*\operatorname{Cos}[e+f*x])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a+a*\sin[e+f*x]]*\operatorname{Sqrt}[c+d*\sin[e+f*x]])]/(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[c-d]*f))$

Rule 212

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1/(Rt[a, 2]\*Rt[-b, 2]))\*ArcTanh[Rt[-b, 2]\*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 214

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-a/b, 2]/a)\*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 2861

Int[1/(Sqrt[(a\_) + (b\_)\*sin[(e\_) + (f\_)\*(x\_)]]\*Sqrt[(c\_) + (d\_)\*sin[(e\_) + (f\_)\*(x\_)])], x\_Symbol] := Dist[-2\*(a/f), Subst[Int[1/(2\*b^2 - (a\*c - b\*d)\*x^2), x], x, b\*(Cos[e + f\*x]/(Sqrt[a + b\*Sin[e + f\*x]]\*Sqrt[c + d\*Si



$n[e + f*x]))], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0]$

### Rule 3022

$\text{Int}[\text{Sqrt}[(a_) + (b_)*\sin[(e_) + (f_)*(x_)]]/(\sin[(e_) + (f_)*(x_)]*\text{Sqrt}[(c_) + (d_)*\sin[(e_) + (f_)*(x_)]]), x\_Symbol] \text{:>} \text{Dist}[-2*(a/f), \text{Subst}[\text{Int}[1/(1 - a*c*x^2), x], x, \text{Cos}[e + f*x]/(\text{Sqrt}[a + b*\text{Sin}[e + f*x]]*\text{Sqrt}[c + d*\text{Sin}[e + f*x]])], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{NeQ}[b*c + a*d, 0]$

### Rule 3026

$\text{Int}[1/(\sin[(e_) + (f_)*(x_)]*\text{Sqrt}[(a_) + (b_)*\sin[(e_) + (f_)*(x_)]]*\text{Sqrt}[(c_) + (d_)*\sin[(e_) + (f_)*(x_)]]), x\_Symbol] \text{:>} \text{Dist}[-b/a, \text{Int}[1/(\text{Sqrt}[a + b*\text{Sin}[e + f*x]]*\text{Sqrt}[c + d*\text{Sin}[e + f*x]]), x], x] + \text{Dist}[1/a, \text{Int}[\text{Sqrt}[a + b*\text{Sin}[e + f*x]]/(\text{Sin}[e + f*x]*\text{Sqrt}[c + d*\text{Sin}[e + f*x]]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& (\text{NeQ}[a^2 - b^2, 0] \parallel \text{NeQ}[c^2 - d^2, 0])$

### Rubi steps

$$\begin{aligned} \int \frac{\csc(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}} dx &= \frac{\int \frac{\csc(e+fx)\sqrt{a+a\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}} dx}{a} - \int \frac{1}{\sqrt{a+a\sin(e+fx)}} dx \\ &= -\frac{2\text{Subst}\left(\int \frac{1}{1-acx^2} dx, x, \frac{\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{f} \\ &= -\frac{2 \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{a}\sqrt{c}f} \end{aligned}$$

**Mathematica [C]** Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 42.86, size = 309693, normalized size = 2212.09

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[Csc[e + f\*x]/(Sqrt[a + a\*Sin[e + f\*x]]\*Sqrt[c + d\*Sin[e + f\*x]]), x]

[Out] Result too large to show

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 363 vs.  $2(113) = 226$ .

time = 0.24, size = 364, normalized size = 2.60

method	result
default	$\frac{(-1 + \cos(fx + e) - \sin(fx + e)) \sqrt{c + d \sin(fx + e)}}{\ln \left( \frac{-\sqrt{c} \sqrt{2} \sqrt{\frac{c + d \sin(fx + e)}{1 + \cos(fx + e)}} \sin(fx + e) + \cos(fx + e) c - d \sin(fx + e)}{\sin(fx + e) \sqrt{c}} \right)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/sin(f*x+e)/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2),x,method=_RE  
TURNVERBOSE)`

[Out] 
$$-1/2/f*(-1+\cos(f*x+e)-\sin(f*x+e))*(c+d*\sin(f*x+e))^(1/2)*(\ln(-(-c^(1/2)*2^(1/2)*((c+d*\sin(f*x+e))/(1+\cos(f*x+e)))^(1/2)*\sin(f*x+e)+\cos(f*x+e)*c-d*\sin(f*x+e)-c)/\sin(f*x+e)/c^(1/2))*(2*c-2*d)^(1/2)+2*\ln(-2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*\sin(f*x+e))/(1+\cos(f*x+e)))^(1/2)*\sin(f*x+e)+c*\sin(f*x+e)-d*\sin(f*x+e)+\cos(f*x+e)*c-d*\cos(f*x+e)-c+d)/(-1+\cos(f*x+e)-\sin(f*x+e)))*c^(1/2)-\ln(2*(-c^(1/2)*2^(1/2)*((c+d*\sin(f*x+e))/(1+\cos(f*x+e)))^(1/2)*\sin(f*x+e)+d*\cos(f*x+e)-c*\sin(f*x+e)-d)/(-1+\cos(f*x+e)))*(2*c-2*d)^(1/2))/(a*(1+\sin(f*x+e)))^(1/2)/\sin(f*x+e)*2^(1/2)/((c+d*\sin(f*x+e))/(1+\cos(f*x+e)))^(1/2)/c^(1/2))/(2*c-2*d)^(1/2)$$

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sin(f*x+e)/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/(sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sin(f*x + e)), x)`

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 304 vs.  $2(119) = 238$ .

time = 0.83, size = 3125, normalized size = 22.32

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sin(f\*x+e)/(a+a\*sin(f\*x+e))^(1/2)/(c+d\*sin(f\*x+e))^(1/2),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & [1/4*(\sqrt{2})*a*c*\log(((c^2 - 14*c*d + 17*d^2)*\cos(f*x + e)^3 - (13*c^2 - 2 \\ & 2*c*d - 3*d^2)*\cos(f*x + e)^2 + 4*\sqrt{2})*((c^2 - 4*c*d + 3*d^2)*\cos(f*x + \\ & e)^2 - 4*c^2 + 8*c*d - 4*d^2 - (3*c^2 - 4*c*d + d^2)*\cos(f*x + e) + (4*c^2 \\ & - 8*c*d + 4*d^2 + (c^2 - 4*c*d + 3*d^2)*\cos(f*x + e))*\sin(f*x + e))*\sqrt{a* \\ & \sin(f*x + e) + a}*\sqrt{d*\sin(f*x + e) + c}/\sqrt{a*c - a*d} - 4*c^2 - 8*c*d \\ & - 4*d^2 - 2*(9*c^2 - 14*c*d + 9*d^2)*\cos(f*x + e) + ((c^2 - 14*c*d + 17*d^2 \\ & )*\cos(f*x + e)^2 - 4*c^2 - 8*c*d - 4*d^2 + 2*(7*c^2 - 18*c*d + 7*d^2)*\cos(f \\ & *x + e))*\sin(f*x + e))/(\cos(f*x + e)^3 + 3*\cos(f*x + e)^2 + (\cos(f*x + e)^2 \\ & - 2*\cos(f*x + e) - 4)*\sin(f*x + e) - 2*\cos(f*x + e) - 4))/\sqrt{a*c - a*d} \\ & + \sqrt{a*c}*\log(((a*c^4 - 28*a*c^3*d + 70*a*c^2*d^2 - 28*a*c*d^3 + a*d^4)*c \\ & \cos(f*x + e)^5 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 - (31*a \\ & *c^4 - 196*a*c^3*d + 154*a*c^2*d^2 - 4*a*c*d^3 - a*d^4)*\cos(f*x + e)^4 - 2* \\ & (81*a*c^4 - 252*a*c^3*d + 150*a*c^2*d^2 - 28*a*c*d^3 + a*d^4)*\cos(f*x + e)^ \\ & 3 + 2*(79*a*c^4 - 100*a*c^3*d + 74*a*c^2*d^2 - 4*a*c*d^3 - a*d^4)*\cos(f*x + \\ & e)^2 - 8*((c^3 - 7*c^2*d + 7*c*d^2 - d^3)*\cos(f*x + e)^4 - 2*(5*c^3 - 14*c \\ & ^2*d + 5*c*d^2)*\cos(f*x + e)^3 + 51*c^3 - 59*c^2*d + 17*c*d^2 - d^3 - 2*(18 \\ & *c^3 - 33*c^2*d + 12*c*d^2 - d^3)*\cos(f*x + e)^2 + 2*(13*c^3 - 14*c^2*d + 5 \\ & *c*d^2)*\cos(f*x + e) + ((c^3 - 7*c^2*d + 7*c*d^2 - d^3)*\cos(f*x + e)^3 - 51 \\ & *c^3 + 59*c^2*d - 17*c*d^2 + d^3 + (11*c^3 - 35*c^2*d + 17*c*d^2 - d^3)*\cos \\ & (f*x + e)^2 - (25*c^3 - 31*c^2*d + 7*c*d^2 - d^3)*\cos(f*x + e))*\sin(f*x + e \\ & ))*\sqrt{a*c}*\sqrt{a*\sin(f*x + e) + a}*\sqrt{d*\sin(f*x + e) + c} + (289*a*c^4 \\ & - 476*a*c^3*d + 230*a*c^2*d^2 - 28*a*c*d^3 + a*d^4)*\cos(f*x + e) + (a*c^4 \\ & + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 + (a*c^4 - 28*a*c^3*d + 70*a* \\ & c^2*d^2 - 28*a*c*d^3 + a*d^4)*\cos(f*x + e)^4 + 32*(a*c^4 - 7*a*c^3*d + 7*a* \\ & c^2*d^2 - a*c*d^3)*\cos(f*x + e)^3 - 2*(65*a*c^4 - 140*a*c^3*d + 38*a*c^2*d^ \\ & 2 - 12*a*c*d^3 + a*d^4)*\cos(f*x + e)^2 - 32*(9*a*c^4 - 15*a*c^3*d + 7*a*c^2 \\ & *d^2 - a*c*d^3)*\cos(f*x + e))*\sin(f*x + e))/(\cos(f*x + e)^5 + \cos(f*x + e)^ \\ & 4 - 2*\cos(f*x + e)^3 - 2*\cos(f*x + e)^2 + (\cos(f*x + e)^4 - 2*\cos(f*x + e)^ \\ & 2 + 1)*\sin(f*x + e) + \cos(f*x + e) + 1))/ (a*c*f), 1/4*(\sqrt{2})*a*c*\log(((c \\ & ^2 - 14*c*d + 17*d^2)*\cos(f*x + e)^3 - (13*c^2 - 22*c*d - 3*d^2)*\cos(f*x + \\ & e)^2 + 4*\sqrt{2})*((c^2 - 4*c*d + 3*d^2)*\cos(f*x + e)^2 - 4*c^2 + 8*c*d - 4* \\ & d^2 - (3*c^2 - 4*c*d + d^2)*\cos(f*x + e) + (4*c^2 - 8*c*d + 4*d^2 + (c^2 - \\ & 4*c*d + 3*d^2)*\cos(f*x + e))*\sin(f*x + e))*\sqrt{a*\sin(f*x + e) + a}*\sqrt{d* \\ & \sin(f*x + e) + c}/\sqrt{a*c - a*d} - 4*c^2 - 8*c*d - 4*d^2 - 2*(9*c^2 - 14*c \\ & *d + 9*d^2)*\cos(f*x + e) + ((c^2 - 14*c*d + 17*d^2)*\cos(f*x + e)^2 - 4*c^2 \\ & - 8*c*d - 4*d^2 + 2*(7*c^2 - 18*c*d + 7*d^2)*\cos(f*x + e))*\sin(f*x + e))/ (c \\ & \cos(f*x + e)^3 + 3*\cos(f*x + e)^2 + (\cos(f*x + e)^2 - 2*\cos(f*x + e) - 4)*\si \\ & n(f*x + e) - 2*\cos(f*x + e) - 4))/\sqrt{a*c - a*d} + 2*\sqrt{-a*c}*\arctan(-1/ \\ & 4*((c^2 - 6*c*d + d^2)*\cos(f*x + e)^2 - 9*c^2 + 6*c*d - d^2 + 8*(c^2 - c*d) \\ & *\sin(f*x + e))*\sqrt{-a*c}*\sqrt{a*\sin(f*x + e) + a}*\sqrt{d*\sin(f*x + e) + c} \\ & /((a*c^2*d - a*c*d^2)*\cos(f*x + e)^3 - (a*c^3 - 3*a*c^2*d)*\cos(f*x + e)*\sin \\ & (f*x + e) + (2*a*c^3 - a*c^2*d + a*c*d^2)*\cos(f*x + e)))/ (a*c*f), -1/4*(2* \end{aligned}$$

```

sqrt(2)*a*c*sqrt(-1/(a*c - a*d))*arctan(-1/4*sqrt(2)*sqrt(a*sin(f*x + e) +
a)*((c - 3*d)*sin(f*x + e) - 3*c + d)*sqrt(d*sin(f*x + e) + c)*sqrt(-1/(a*c
- a*d)))/(d*cos(f*x + e)*sin(f*x + e) + c*cos(f*x + e))) - sqrt(a*c)*log(((
a*c^4 - 28*a*c^3*d + 70*a*c^2*d^2 - 28*a*c*d^3 + a*d^4)*cos(f*x + e)^5 + a*
c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 - (31*a*c^4 - 196*a*c^3*d
+ 154*a*c^2*d^2 - 4*a*c*d^3 - a*d^4)*cos(f*x + e)^4 - 2*(81*a*c^4 - 252*a*
c^3*d + 150*a*c^2*d^2 - 28*a*c*d^3 + a*d^4)*cos(f*x + e)^3 + 2*(79*a*c^4 -
100*a*c^3*d + 74*a*c^2*d^2 - 4*a*c*d^3 - a*d^4)*cos(f*x + e)^2 - 8*((c^3 -
7*c^2*d + 7*c*d^2 - d^3)*cos(f*x + e)^4 - 2*(5*c^3 - 14*c^2*d + 5*c*d^2)*co
s(f*x + e)^3 + 51*c^3 - 59*c^2*d + 17*c*d^2 - d^3 - 2*(18*c^3 - 33*c^2*d +
12*c*d^2 - d^3)*cos(f*x + e)^2 + 2*(13*c^3 - 14*c^2*d + 5*c*d^2)*cos(f*x +
e) + ((c^3 - 7*c^2*d + 7*c*d^2 - d^3)*cos(f*x + e)^3 - 51*c^3 + 59*c^2*d -
17*c*d^2 + d^3 + (11*c^3 - 35*c^2*d + 17*c*d^2 - d^3)*cos(f*x + e)^2 - (25*
c^3 - 31*c^2*d + 7*c*d^2 - d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*c)*sqrt(
a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c) + (289*a*c^4 - 476*a*c^3*d + 2
30*a*c^2*d^2 - 28*a*c*d^3 + a*d^4)*cos(f*x + e) + (a*c^4 + 4*a*c^3*d + 6*a*
c^2*d^2 + 4*a*c*d^3 + a*d^4 + (a*c^4 - 28*a*c^3*d + 70*a*c^2*d^2 - 28*a*c*d
^3 + a*d^4)*cos(f*x + e)^4 + 32*(a*c^4 - 7*a*c^3*d + 7*a*c^2*d^2 - a*c*d^3)
*cos(f*x + e)^3 - 2*(65*a*c^4 - 140*a*c^3*d + 38*a*c^2*d^2 - 12*a*c*d^3 + a
*d^4)*cos(f*x + e)^2 - 32*(9*a*c^4 - 15*a*c^3*d + 7*a*c^2*d^2 - a*c*d^3)*co
s(f*x + e))*sin(f*x + e))/(cos(f*x + e)^5 + cos(f*x + e)^4 - 2*cos(f*x + e)
^3 - 2*cos(f*x + e)^2 + (cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sin(f*x + e
) + cos(f*x + e) + 1))/(a*c*f), -1/2*(sqrt(2)*a*c*sqrt(-1/(a*c - a*d))*arc
tan(-1/4*sqrt(2)*sqrt(a*sin(f*x + e) + a)*((c - 3*d)*sin(f*x + e) - 3*c + d
)*sqrt(d*sin(f*x + e) + c)*sqrt(-1/(a*c - a*d)))...

```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a(\sin(e+fx)+1)} \sqrt{c+d\sin(e+fx)} \sin(e+fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sin(f\*x+e)/(a+a\*sin(f\*x+e))^(1/2)/(c+d\*sin(f\*x+e))^(1/2),x)

[Out] Integral(1/(sqrt(a\*(sin(e + f\*x) + 1))\*sqrt(c + d\*sin(e + f\*x))\*sin(e + f\*x)), x)

**Giac [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sin(f\*x+e)/(a+a\*sin(f\*x+e))^(1/2)/(c+d\*sin(f\*x+e))^(1/2),x, algorithm="giac")

[Out] Timed out

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin(e + fx) \sqrt{a + a \sin(e + fx)} \sqrt{c + d \sin(e + fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sin(e + f\*x)\*(a + a\*sin(e + f\*x))^(1/2)\*(c + d\*sin(e + f\*x))^(1/2)),  
x)

[Out] int(1/(sin(e + f\*x)\*(a + a\*sin(e + f\*x))^(1/2)\*(c + d\*sin(e + f\*x))^(1/2)),  
x)

$$3.39 \quad \int \frac{\sin^2(e+fx)}{(a+b\sin(e+fx))^2(c+d\sin(e+fx))} dx$$

**Optimal.** Leaf size=181

$$-\frac{2a(a^2c - 2b^2c + abd) \tan^{-1}\left(\frac{b+a \tan(\frac{1}{2}(e+fx))}{\sqrt{a^2 - b^2}}\right)}{(a^2 - b^2)^{3/2} (bc - ad)^2 f} + \frac{2c^2 \tan^{-1}\left(\frac{d+c \tan(\frac{1}{2}(e+fx))}{\sqrt{c^2 - d^2}}\right)}{(bc - ad)^2 \sqrt{c^2 - d^2} f} + \frac{a^2 \cos(e + fx)}{(a^2 - b^2) (bc - ad) f (a + b \sin(e + fx))}$$

[Out]  $-2*a*(a^2*c+a*b*d-2*b^2*c)*\arctan((b+a*\tan(1/2*f*x+1/2*e))/(a^2-b^2)^{(1/2)})/(a^2-b^2)^{(3/2)}/(-a*d+b*c)^2/f+a^2*\cos(f*x+e)/(a^2-b^2)/(-a*d+b*c)/f/(a+b*\sin(f*x+e))+2*c^2*\arctan((d+c*\tan(1/2*f*x+1/2*e))/(c^2-d^2)^{(1/2)})/(-a*d+b*c)^2/f/(c^2-d^2)^{(1/2)}$

**Rubi [A]**

time = 0.35, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$ , Rules used = {3135, 3080, 2739, 632, 210}

$$-\frac{2a(a^2c + abd - 2b^2c) \text{ArcTan}\left(\frac{a \tan(\frac{1}{2}(e+fx)) + b}{\sqrt{a^2 - b^2}}\right)}{f (a^2 - b^2)^{3/2} (bc - ad)^2} + \frac{a^2 \cos(e + fx)}{f (a^2 - b^2) (bc - ad) (a + b \sin(e + fx))} + \frac{2c^2 \text{ArcTan}\left(\frac{c \tan(\frac{1}{2}(e+fx)) + d}{\sqrt{c^2 - d^2}}\right)}{f \sqrt{c^2 - d^2} (bc - ad)^2}$$

Antiderivative was successfully verified.

[In] `Int[Sin[e + f*x]^2/((a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])),x]`

[Out]  $(-2*a*(a^2*c - 2*b^2*c + a*b*d)*\text{ArcTan}[(b + a*\text{Tan}[(e + f*x)/2])/ \text{Sqrt}[a^2 - b^2]])/((a^2 - b^2)^{(3/2)}*(b*c - a*d)^2*f) + (2*c^2*\text{ArcTan}[(d + c*\text{Tan}[(e + f*x)/2])/ \text{Sqrt}[c^2 - d^2]])/((b*c - a*d)^2*\text{Sqrt}[c^2 - d^2]*f) + (a^2*\text{Cos}[e + f*x])/((a^2 - b^2)*(b*c - a*d)*f*(a + b*\text{Sin}[e + f*x]))$

**Rule 210**

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] & & (LtQ[a, 0] || LtQ[b, 0])`

**Rule 632**

`Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]`

**Rule 2739**

`Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a`

$e^{2x^2}$ ,  $x$ ,  $\tan[(c + dx)/2]/e$ ,  $x$ ] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

### Rule 3080

Int[((A\_.) + (B\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])/(((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])), x\_Symbol] := Dist[(A\*B - a\*B)/(b\*c - a\*d), Int[1/(a + b\*Sin[e + f\*x]), x], x] + Dist[(B\*c - A\*d)/(b\*c - a\*d), Int[1/(c + d\*Sin[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

### Rule 3135

Int[((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_)\*((A\_.) + (C\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^2, x\_Symbol] := Simp[(-(A\*b^2 + a^2\*C)\*Cos[e + f\*x]\*(a + b\*Sin[e + f\*x])^(m + 1)\*((c + d\*Sin[e + f\*x])^(n + 1)/(f\*(m + 1)\*(b\*c - a\*d)\*(a^2 - b^2))), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(a^2 - b^2)), Int[(a + b\*Sin[e + f\*x])^(m + 1)\*(c + d\*Sin[e + f\*x])^n\*Simp[a\*(m + 1)\*(b\*c - a\*d)\*(A + C) + d\*(A\*b^2 + a^2\*C)\*(m + n + 2) - (c\*(A\*b^2 + a^2\*C) + b\*(m + 1)\*(b\*c - a\*d)\*(A + C))\*Sin[e + f\*x] - d\*(A\*b^2 + a^2\*C)\*(m + n + 3)\*Sin[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2\*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))

### Rubi steps

$$\begin{aligned} \int \frac{\sin^2(e + fx)}{(a + b \sin(e + fx))^2(c + d \sin(e + fx))} dx &= \frac{a^2 \cos(e + fx)}{(a^2 - b^2)(bc - ad)f(a + b \sin(e + fx))} - \frac{\int \frac{-abc - (a^2c - b^2c + ad^2)}{(a + b \sin(e + fx))(c + d \sin(e + fx))} dx}{(a^2 - b^2)(bc - ad)} \\ &= \frac{a^2 \cos(e + fx)}{(a^2 - b^2)(bc - ad)f(a + b \sin(e + fx))} + \frac{c^2 \int \frac{1}{c + d \sin(e + fx)} dx}{(bc - ad)^2} \\ &= \frac{a^2 \cos(e + fx)}{(a^2 - b^2)(bc - ad)f(a + b \sin(e + fx))} + \frac{(2c^2) \operatorname{Subst}\left(\int \frac{1}{c + d \sin(x)} dx\right)}{(bc - ad)^2} \\ &= \frac{a^2 \cos(e + fx)}{(a^2 - b^2)(bc - ad)f(a + b \sin(e + fx))} - \frac{(4c^2) \operatorname{Subst}\left(\int \frac{1}{c + d \sin(x)} dx\right)}{(bc - ad)^2} \\ &= -\frac{2a(a^2c - 2b^2c + abd) \tan^{-1}\left(\frac{b + a \tan\left(\frac{1}{2}(e + fx)\right)}{\sqrt{a^2 - b^2}}\right)}{(a^2 - b^2)^{3/2}(bc - ad)^2 f} + \frac{2c^2 \tan^{-1}\left(\frac{c + d \tan\left(\frac{1}{2}(e + fx)\right)}{\sqrt{c^2 - d^2}}\right)}{(c^2 - d^2)^{3/2}(bc - ad)^2} \end{aligned}$$

**Mathematica [A]**

time = 0.78, size = 178, normalized size = 0.98

$$\frac{2a(a^2c-2b^2c+abd) \tan^{-1}\left(\frac{b+a \tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}(bc-ad)^2} + \frac{2c^2 \tan^{-1}\left(\frac{d+c \tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(bc-ad)^2 \sqrt{c^2-d^2}} - \frac{a^2 \cos(e+fx)}{(a-b)(a+b)(-bc+ad)(a+b \sin(e+fx))}$$


---


$$f$$

Antiderivative was successfully verified.

**[In]** Integrate[Sin[e + f\*x]^2/((a + b\*Sin[e + f\*x])^2\*(c + d\*Sin[e + f\*x])),x]

**[Out]** ((-2\*a\*(a^2\*c - 2\*b^2\*c + a\*b\*d)\*ArcTan[(b + a\*Tan[(e + f\*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(3/2)\*(b\*c - a\*d)^2) + (2\*c^2\*ArcTan[(d + c\*Tan[(e + f\*x)/2])/Sqrt[c^2 - d^2]])/((b\*c - a\*d)^2\*Sqrt[c^2 - d^2]) - (a^2\*Cos[e + f\*x])/((a - b)\*(a + b)\*(-b\*c) + a\*d\*(a + b\*Sin[e + f\*x]))/f

**Maple [A]**

time = 1.97, size = 244, normalized size = 1.35

method	result
derivativedivides	$\frac{8c^2 \arctan\left(\frac{2c \tan\left(\frac{fx}{2} + \frac{e}{2}\right) + 2d}{2\sqrt{c^2 - d^2}}\right)}{(4a^2d^2 - 8abcd + 4b^2c^2)\sqrt{c^2 - d^2}} - \frac{2a \left( \frac{b(ad-bc) \tan\left(\frac{fx}{2} + \frac{e}{2}\right) + a(ad-bc)}{a^2 - b^2} + \frac{(a^2c + abd - 2b^2c) \arctan\left(\frac{2a \tan\left(\frac{fx}{2} + \frac{e}{2}\right) + 2b}{2\sqrt{a^2 - b^2}}\right)}{(a^2 - b^2)^{3/2}} \right)}{a^2d^2 - 2abcd + b^2c^2}$
default	$\frac{8c^2 \arctan\left(\frac{2c \tan\left(\frac{fx}{2} + \frac{e}{2}\right) + 2d}{2\sqrt{c^2 - d^2}}\right)}{(4a^2d^2 - 8abcd + 4b^2c^2)\sqrt{c^2 - d^2}} - \frac{2a \left( \frac{b(ad-bc) \tan\left(\frac{fx}{2} + \frac{e}{2}\right) + a(ad-bc)}{a^2 - b^2} + \frac{(a^2c + abd - 2b^2c) \arctan\left(\frac{2a \tan\left(\frac{fx}{2} + \frac{e}{2}\right) + 2b}{2\sqrt{a^2 - b^2}}\right)}{(a^2 - b^2)^{3/2}} \right)}{a^2d^2 - 2abcd + b^2c^2}$
risch	$\frac{2ia^2(ib+a e^{i(fx+e)})}{b(a^2-b^2)(ad-bc)f(-ie^{2i(fx+e)}b+ib+2a e^{i(fx+e)})} + \frac{c^2 \ln\left(e^{i(fx+e)} + \frac{ic\sqrt{-c^2+d^2}+c^2-d^2}{\sqrt{-c^2+d^2}}\right)}{\sqrt{-c^2+d^2}(ad-bc)^2f} - \frac{c^2 \ln\left(e^{i(fx+e)} + \frac{ic\sqrt{-c^2+d^2}+c^2-d^2}{\sqrt{-c^2+d^2}}\right)}{\sqrt{-c^2+d^2}}$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** int(sin(f\*x+e)^2/(a+b\*sin(f\*x+e))^2/(c+d\*sin(f\*x+e)),x,method=\_RETURNVERBOSE)

**[Out]** 1/f\*(8\*c^2/(4\*a^2\*d^2-8\*a\*b\*c\*d+4\*b^2\*c^2)/(c^2-d^2)^(1/2)\*arctan(1/2\*(2\*c\*tan(1/2\*f\*x+1/2\*e)+2\*d)/(c^2-d^2)^(1/2))-2\*a/(a^2\*d^2-2\*a\*b\*c\*d+b^2\*c^2)\*((b\*(a\*d-b\*c)/(a^2-b^2)\*tan(1/2\*f\*x+1/2\*e)+a\*(a\*d-b\*c)/(a^2-b^2))/(a\*tan(1/2\*f\*x+1/2\*e)^2+2\*b\*tan(1/2\*f\*x+1/2\*e)+a)+(a^2\*c+a\*b\*d-2\*b^2\*c)/(a^2-b^2)^(3/2)\*arctan(1/2\*(2\*a\*tan(1/2\*f\*x+1/2\*e)+2\*b)/(a^2-b^2)^(1/2))))



**Maxima [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(f\*x+e)^2/(a+b\*sin(f\*x+e))^2/(c+d\*sin(f\*x+e)),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*d^2-4\*c^2>0)', see 'assume?' for more details)

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 633 vs. 2(175) = 350.

time = 65.60, size = 2889, normalized size = 15.96

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(f\*x+e)^2/(a+b\*sin(f\*x+e))^2/(c+d\*sin(f\*x+e)),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & [1/2*((a^3*b*c^2*d - a^3*b*d^3 + (a^4 - 2*a^2*b^2)*c^3 - (a^4 - 2*a^2*b^2)* \\ & c*d^2 + (a^2*b^2*c^2*d - a^2*b^2*d^3 + (a^3*b - 2*a*b^3)*c^3 - (a^3*b - 2*a \\ & *b^3)*c*d^2)*\sin(f*x + e))*\sqrt{-a^2 + b^2}*\log(((2*a^2 - b^2)*\cos(f*x + e) \\ & ^2 - 2*a*b*\sin(f*x + e) - a^2 - b^2 + 2*(a*\cos(f*x + e)*\sin(f*x + e) + b*\cos \\ & (f*x + e))*\sqrt{-a^2 + b^2}))/((b^2*\cos(f*x + e)^2 - 2*a*b*\sin(f*x + e) - a^2 \\ & - b^2)) - ((a^4*b - 2*a^2*b^3 + b^5)*c^2*\sin(f*x + e) + (a^5 - 2*a^3*b^2 \\ & + a*b^4)*c^2)*\sqrt{-c^2 + d^2}*\log(((2*c^2 - d^2)*\cos(f*x + e)^2 - 2*c*d*\sin \\ & (f*x + e) - c^2 - d^2 + 2*(c*\cos(f*x + e)*\sin(f*x + e) + d*\cos(f*x + e))*\sqrt{-c^2 + d^2}))/ \\ & ((d^2*\cos(f*x + e)^2 - 2*c*d*\sin(f*x + e) - c^2 - d^2)) + 2 \\ & *((a^4*b - a^2*b^3)*c^3 - (a^5 - a^3*b^2)*c^2*d - (a^4*b - a^2*b^3)*c*d^2 + \\ & (a^5 - a^3*b^2)*d^3)*\cos(f*x + e))/(((a^4*b^3 - 2*a^2*b^5 + b^7)*c^4 - 2*( \\ & a^5*b^2 - 2*a^3*b^4 + a*b^6)*c^3*d + (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)* \\ & c^2*d^2 + 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*c*d^3 - (a^6*b - 2*a^4*b^3 + a^2* \\ & b^5)*d^4)*f*\sin(f*x + e) + ((a^5*b^2 - 2*a^3*b^4 + a*b^6)*c^4 - 2*(a^6*b - \\ & 2*a^4*b^3 + a^2*b^5)*c^3*d + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*c^2*d^2 \\ & + 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c*d^3 - (a^7 - 2*a^5*b^2 + a^3*b^4)*d^4)* \\ & f, -1/2*(2*((a^4*b - 2*a^2*b^3 + b^5)*c^2*\sin(f*x + e) + (a^5 - 2*a^3*b^2 \\ & + a*b^4)*c^2)*\sqrt{c^2 - d^2}*\arctan(-(c*\sin(f*x + e) + d)/(\sqrt{c^2 - d^2} \\ & *\cos(f*x + e))) - (a^3*b*c^2*d - a^3*b*d^3 + (a^4 - 2*a^2*b^2)*c^3 - (a^4 - \\ & 2*a^2*b^2)*c*d^2 + (a^2*b^2*c^2*d - a^2*b^2*d^3 + (a^3*b - 2*a*b^3)*c^3 - \\ & (a^3*b - 2*a*b^3)*c*d^2)*\sin(f*x + e))*\sqrt{-a^2 + b^2}*\log(((2*a^2 - b^2)* \end{aligned}$$

$$\begin{aligned} & \cos(f*x + e)^2 - 2*a*b*\sin(f*x + e) - a^2 - b^2 + 2*(a*\cos(f*x + e)*\sin(f*x \\ & + e) + b*\cos(f*x + e))*\sqrt{-a^2 + b^2})/(b^2*\cos(f*x + e)^2 - 2*a*b*\sin(f \\ & *x + e) - a^2 - b^2)) - 2*((a^4*b - a^2*b^3)*c^3 - (a^5 - a^3*b^2)*c^2*d - \\ & (a^4*b - a^2*b^3)*c*d^2 + (a^5 - a^3*b^2)*d^3)*\cos(f*x + e))/(((a^4*b^3 - 2 \\ & *a^2*b^5 + b^7)*c^4 - 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*c^3*d + (a^6*b - 3*a^ \\ & 4*b^3 + 3*a^2*b^5 - b^7)*c^2*d^2 + 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*c*d^3 - \\ & (a^6*b - 2*a^4*b^3 + a^2*b^5)*d^4)*f*\sin(f*x + e) + ((a^5*b^2 - 2*a^3*b^4 + \\ & a*b^6)*c^4 - 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^3*d + (a^7 - 3*a^5*b^2 + 3* \\ & a^3*b^4 - a*b^6)*c^2*d^2 + 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c*d^3 - (a^7 - 2 \\ & *a^5*b^2 + a^3*b^4)*d^4)*f), 1/2*(2*(a^3*b*c^2*d - a^3*b*d^3 + (a^4 - 2*a^2 \\ & *b^2)*c^3 - (a^4 - 2*a^2*b^2)*c*d^2 + (a^2*b^2*c^2*d - a^2*b^2*d^3 + (a^3*b \\ & - 2*a*b^3)*c^3 - (a^3*b - 2*a*b^3)*c*d^2)*\sin(f*x + e))*\sqrt{a^2 - b^2}*ar \\ & ctan(-(a*\sin(f*x + e) + b)/(\sqrt{a^2 - b^2}*\cos(f*x + e))) - ((a^4*b - 2*a^ \\ & 2*b^3 + b^5)*c^2*\sin(f*x + e) + (a^5 - 2*a^3*b^2 + a*b^4)*c^2)*\sqrt{-c^2 + \\ & d^2}*\log(((2*c^2 - d^2)*\cos(f*x + e)^2 - 2*c*d*\sin(f*x + e) - c^2 - d^2 + 2 \\ & *(c*\cos(f*x + e)*\sin(f*x + e) + d*\cos(f*x + e))*\sqrt{-c^2 + d^2}))/((d^2*\cos( \\ & f*x + e)^2 - 2*c*d*\sin(f*x + e) - c^2 - d^2)) + 2*((a^4*b - a^2*b^3)*c^3 - \\ & (a^5 - a^3*b^2)*c^2*d - (a^4*b - a^2*b^3)*c*d^2 + (a^5 - a^3*b^2)*d^3)*\cos( \\ & f*x + e))/(((a^4*b^3 - 2*a^2*b^5 + b^7)*c^4 - 2*(a^5*b^2 - 2*a^3*b^4 + a*b^ \\ & 6)*c^3*d + (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*c^2*d^2 + 2*(a^5*b^2 - 2*a \\ & ^3*b^4 + a*b^6)*c*d^3 - (a^6*b - 2*a^4*b^3 + a^2*b^5)*d^4)*f*\sin(f*x + e) + \\ & ((a^5*b^2 - 2*a^3*b^4 + a*b^6)*c^4 - 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^3*d \\ & + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*c^2*d^2 + 2*(a^6*b - 2*a^4*b^3 + a \\ & ^2*b^5)*c*d^3 - (a^7 - 2*a^5*b^2 + a^3*b^4)*d^4)*f), ((a^3*b*c^2*d - a^3*b* \\ & d^3 + (a^4 - 2*a^2*b^2)*c^3 - (a^4 - 2*a^2*b^2)*c*d^2 + (a^2*b^2*c^2*d - a^ \\ & 2*b^2*d^3 + (a^3*b - 2*a*b^3)*c^3 - (a^3*b - 2*a*b^3)*c*d^2)*\sin(f*x + e))* \\ & \sqrt{a^2 - b^2}*\arctan(-(a*\sin(f*x + e) + b)/(\sqrt{a^2 - b^2}*\cos(f*x + e)) \\ & ) - ((a^4*b - 2*a^2*b^3 + b^5)*c^2*\sin(f*x + e) + (a^5 - 2*a^3*b^2 + a*b^4) \\ & *c^2)*\sqrt{c^2 - d^2}*\arctan(-(c*\sin(f*x + e) + d)/(\sqrt{c^2 - d^2}*\cos(f*x \\ & + e))) + ((a^4*b - a^2*b^3)*c^3 - (a^5 - a^3*b^2)*c^2*d - (a^4*b - a^2*b^3) \\ & )*c*d^2 + (a^5 - a^3*b^2)*d^3)*\cos(f*x + e))/(((a^4*b^3 - 2*a^2*b^5 + b^7)* \\ & c^4 - 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*c^3*d + (a^6*b - 3*a^4*b^3 + 3*a^2*b^ \\ & 5 - b^7)*c^2*d^2 + 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*c*d^3 - (a^6*b - 2*a^4*b \\ & ^3 + a^2*b^5)*d^4)*f*\sin(f*x + e) + ((a^5*b^2 - 2*a^3*b^4 + a*b^6)*c^4 - 2* \\ & (a^6*b - 2*a^4*b^3 + a^2*b^5)*c^3*d + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6) \\ & *c^2*d^2 + 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c*d^3 - (a^7 - 2*a^5*b^2 + a^3*b \\ & ^4)*d^4)*f)] \end{aligned}$$

Sympy [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(f\*x+e)\*\*2/(a+b\*sin(f\*x+e))\*\*2/(c+d\*sin(f\*x+e)),x)

[Out] Timed out

**Giac** [A]

time = 0.56, size = 299, normalized size = 1.65

$$2 \left( \frac{\left( \pi \left\lfloor \frac{fx+e}{2\pi} + \frac{1}{2} \right\rfloor \operatorname{sgn}(c) + \arctan\left(\frac{c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + d}{\sqrt{c^2 - d^2}}\right) \right) c^2}{(b^2c^2 - 2abcd + a^2d^2)\sqrt{c^2 - d^2}} - \frac{(a^3c - 2ab^2c + a^2bd) \left( \pi \left\lfloor \frac{fx+e}{2\pi} + \frac{1}{2} \right\rfloor \operatorname{sgn}(a) + \arctan\left(\frac{a \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + b}{\sqrt{a^2 - b^2}}\right) \right)}{(a^2b^2c^2 - b^4c^2 - 2a^3bcd + 2ab^3cd + a^4d^2 - a^2b^2d^2)\sqrt{a^2 - b^2}} + \frac{ab \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + a^2}{(a^2bc - b^3c - a^3d + ab^2d) \left( a \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 2b \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + a \right)} \right) / f$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(f\*x+e)^2/(a+b\*sin(f\*x+e))^2/(c+d\*sin(f\*x+e)),x, algorithm="giac")

[Out] 2\*((pi\*floor(1/2\*(f\*x + e)/pi + 1/2)\*sgn(c) + arctan((c\*tan(1/2\*f\*x + 1/2\*e) + d)/sqrt(c^2 - d^2)))\*c^2/((b^2\*c^2 - 2\*a\*b\*c\*d + a^2\*d^2)\*sqrt(c^2 - d^2)) - (a^3\*c - 2\*a\*b^2\*c + a^2\*b\*d)\*(pi\*floor(1/2\*(f\*x + e)/pi + 1/2)\*sgn(a) + arctan((a\*tan(1/2\*f\*x + 1/2\*e) + b)/sqrt(a^2 - b^2)))/((a^2\*b^2\*c^2 - b^4\*c^2 - 2\*a^3\*b\*c\*d + 2\*a\*b^3\*c\*d + a^4\*d^2 - a^2\*b^2\*d^2)\*sqrt(a^2 - b^2)) + (a\*b\*tan(1/2\*f\*x + 1/2\*e) + a^2)/((a^2\*b\*c - b^3\*c - a^3\*d + a\*b^2\*d)\*(a\*tan(1/2\*f\*x + 1/2\*e)^2 + 2\*b\*tan(1/2\*f\*x + 1/2\*e) + a)))/f

**Mupad** [B]

time = 27.41, size = 2500, normalized size = 13.81

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(e + f\*x)^2/((a + b\*sin(e + f\*x))^2\*(c + d\*sin(e + f\*x))),x)

[Out] - ((2\*a^2)/((a^2 - b^2)\*(a\*d - b\*c)) + (2\*a\*b\*tan(e/2 + (f\*x)/2))/((a^2 - b^2)\*(a\*d - b\*c)))/(f\*(a + 2\*b\*tan(e/2 + (f\*x)/2) + a\*tan(e/2 + (f\*x)/2)^2) - (c^2\*atan(((c^2\*(d^2 - c^2)^(1/2))\*((32\*(2\*a^4\*b^4\*c^6 - a^2\*b^6\*c^6 - a^6\*b^2\*c^6 + a^8\*c^4\*d^2 - 3\*a^3\*b^5\*c^5\*d + 2\*a^5\*b^3\*c^5\*d + 2\*a^7\*b\*c^3\*d^3 + 8\*a^4\*b^4\*c^4\*d^2 - 5\*a^5\*b^3\*c^3\*d^3 + a^6\*b^2\*c^2\*d^4 - 6\*a^6\*b^2\*c^4\*d^2)))/(a^7\*d^3 - b^7\*c^3 + 2\*a^2\*b^5\*c^3 - a^4\*b^3\*c^3 + a^3\*b^4\*d^3 - 2\*a^5\*b^2\*d^3 - 3\*a^2\*b^5\*c\*d^2 - 6\*a^3\*b^4\*c^2\*d + 6\*a^4\*b^3\*c\*d^2 + 3\*a^5\*b^2\*c^2\*d + 3\*a\*b^6\*c^2\*d - 3\*a^6\*b\*c\*d^2) + (32\*tan(e/2 + (f\*x)/2)\*(2\*a^7\*b\*c^6 - 2\*a\*b^7\*c^6 - 2\*a^8\*c^5\*d + 9\*a^3\*b^5\*c^6 - 8\*a^5\*b^3\*c^6 + 2\*a^8\*c^3\*d^3 + 2\*a^2\*b^6\*c^5\*d - 13\*a^4\*b^4\*c^5\*d + 2\*a^6\*b^2\*c\*d^5 + 10\*a^6\*b^2\*c^5\*d + 4\*a^7\*b\*c^2\*d^4 - 4\*a^7\*b\*c^4\*d^2 - 8\*a^3\*b^5\*c^4\*d^2 + 16\*a^4\*b^4\*c^3\*d^3 - 10\*a^5\*b^3\*c^2\*d^4 + 13\*a^5\*b^3\*c^4\*d^2 - 13\*a^6\*b^2\*c^3\*d^3)))/(a^7\*d^3 - b^7\*c^3 + 2\*a^2\*b^5\*c^3 - a^4\*b^3\*c^3 + a^3\*b^4\*d^3 - 2\*a^5\*b^2\*d^3 - 3\*a^2\*b^5\*c\*d^2 - 6\*a^3\*b^4\*c^2\*d + 6\*a^4\*b^3\*c\*d^2 + 3\*a^5\*b^2\*c^2\*d + 3\*a\*b^6\*c^2\*d - 3\*a^6\*b\*c\*d^2) + (c^2\*(d^2 - c^2)^(1/2))\*((32\*(a\*b^9\*c^7 - a^3\*b^7\*c^7 + a^10\*c^2\*d^5 - 3\*a^2\*b^8\*c^6\*d + 2\*a^4\*b^6\*c^6\*d + a^6\*b^4\*c^6\*d - a^7\*b^3\*c\*d^6 - 4\*a^9\*b\*c^3\*d^4 + a^3\*b^7\*c^5\*d^2 + 6\*a^4\*b^6\*c^4\*d^3

$$\begin{aligned}
& - 9a^5b^5c^3d^4 + 3a^5b^5c^5d^2 + 5a^6b^4c^2d^5 - 12a^6b^4c^4d^3 + 13a^7b^3c^3d^4 - 4a^7b^3c^5d^2 - 6a^8b^2c^2d^5 + 6a^8b^2c^4d^3 + a^9b^3c^6d^2) / (a^7d^3 - b^7c^3 + 2a^2b^5c^3 - a^4b^3c^3 + a^3b^4d^3 - 2a^5b^2d^3 - 3a^2b^5c^2d^2 - 6a^3b^4c^2d + 6a^4b^3c^2d^2 + 3a^5b^2c^2d + 3a^6b^2c^2d - 3a^6b^2c^2d^2) + (32 \tan(e/2 + (f*x)/2) * (4a^2b^8c^7 - 6a^4b^6c^7 + 2a^6b^4c^7 + 2a^10c^3d^4 - 20a^3b^7c^6d + 26a^5b^5c^6d - 2a^6b^4c^6d - 8a^7b^3c^6d + 2a^8b^2c^6d + 2a^9b^2c^2d^5 - 8a^9b^2c^4d^3 - 10a^2b^8c^5d^2 + 20a^3b^7c^4d^3 - 20a^4b^6c^3d^4 + 42a^4b^6c^5d^2 + 10a^5b^5c^2d^5 - 48a^5b^5c^4d^3 + 32a^6b^4c^3d^4 - 44a^6b^4c^5d^2 - 12a^7b^3c^2d^5 + 36a^7b^3c^4d^3 - 14a^8b^2c^3d^4 + 12a^8b^2c^5d^2 + 2a^9b^2c^6d) / (a^7d^3 - b^7c^3 + 2a^2b^5c^3 - a^4b^3c^3 + a^3b^4d^3 - 2a^5b^2d^3 - 3a^2b^5c^2d^2 - 6a^3b^4c^2d + 6a^4b^3c^2d^2 + 3a^5b^2c^2d + 3a^6b^2c^2d - 3a^6b^2c^2d^2) + (c^2(d^2 - c^2))^{1/2} * ((32 * (2a^4b^8c^8 - a^2b^10c^8 - a^6b^6c^8 + a^12c^2d^6 + 2a^3b^9c^7d - 7a^5b^7c^7d - a^7b^5c^7d + 4a^7b^5c^7d + 2a^9b^3c^7d - 4a^11b^3c^3d^5 - 4a^2b^10c^6d^2 + 5a^3b^9c^5d^3 + 3a^4b^8c^6d^2 - 5a^5b^7c^3d^5 - 10a^5b^7c^5d^3 + 4a^6b^6c^2d^6 + 5a^6b^6c^4d^4 + 6a^6b^6c^6d^2 + 6a^7b^5c^3d^5 + 5a^7b^5c^5d^3 - 7a^8b^4c^2d^6 - 10a^8b^4c^4d^4 - 5a^8b^4c^6d^2 + 3a^9b^3c^3d^5 + 2a^10b^2c^2d^6 + 5a^10b^2c^4d^4 + a^11b^2c^7d - a^11b^2c^7d) / (a^7d^3 - b^7c^3 + 2a^2b^5c^3 - a^4b^3c^3 + a^3b^4d^3 - 2a^5b^2d^3 - 3a^2b^5c^2d^2 - 6a^3b^4c^2d + 6a^4b^3c^2d^2 + 3a^5b^2c^2d + 3a^6b^2c^2d - 3a^6b^2c^2d^2) - (32 \tan(e/2 + (f*x)/2) * (3a^8b^11c^8 - 3a^12c^7d - 8a^3b^9c^8 + 7a^5b^7c^8 - 2a^7b^5c^8 + 2a^12c^3d^5 - 4a^8b^11c^6d^2 - 15a^2b^10c^7d + 40a^4b^8c^7d + 4a^6b^6c^7d - 35a^6b^6c^7d - 11a^8b^4c^7d + 10a^8b^4c^7d + 10a^10b^2c^7d + 15a^11b^2c^2d^6 - 10a^11b^2c^4d^4 + 20a^2b^10c^5d^3 - 40a^3b^9c^4d^4 + 41a^3b^9c^6d^2 + 40a^4b^8c^3d^5 - 85a^4b^8c^5d^3 - 20a^5b^7c^2d^6 + 125a^5b^7c^4d^4 - 90a^5b^7c^6d^2 - 113a^6b^6c^3d^5 + 130a^6b^6c^5d^3 + 55a^7b^5c^2d^6 - 140a^7b^5c^4d^4 + 73a^7b^5c^6d^2 + 108a^8b^4c^3d^5 - 85a^8b^4c^5d^3 - 50a^9b^3c^2d^6 + 65a^9b^3c^4d^4 - 20a^9b^3c^6d^2 - 37a^10b^2c^3d^5 + 20a^10b^2c^5d^3) / (a^7d^3 - b^7c^3 + 2a^2b^5c^3 - a^4b^3c^3 + a^3b^4d^3 - 2a^5b^2d^3 - 3a^2b^5c^2d^2 - 6a^3b^4c^2d + 6a^4b^3c^2d^2 + 3a^5b^2c^2d + 3a^6b^2c^2d - 3a^6b^2c^2d^2) / (a^2d^4 - b^2c^4 - a^2c^2d^2 + b^2c^2d^2 - 2a^2b^2c^2d^3 + 2a^2b^2c^3d) * i) / (a^2d^4 - b^2c^4 - a^2c^2d^2 + b^2c^2d^2 - 2a^2b^2c^2d^3 + 2a^2b^2c^3d) + (c^2(d^2 - c^2))^{1/2} * ((32 * (2a^4b^4c^6 - a^2b^6c^6 - a^6b^2c^6 + a^8c^4d^2 - 3a^3b^5c^5d + 2a^5b^3c^5d + 2a^7b^3c^3d^3 + 8a^4b^4c^4d^2 - 5a^5b^3c^3d^3 + a^6b^2c^2d^4 - 6a^6b^2c^4d^2) / (a^7d^3 - b^7c^3 + 2a^2b^5c^3 - a^4b^3c^3 + a^3b^4d^3 - 2a^5b^2d^3 - 3a^2b^5c^2d^2 - 6a^3b^4c^2d + 6a^4b^3c^2d^2 + 3a^5b^2c^2d + 3a^6b^2c^2d - 3a^6b^2c^2d^2) + (32 \tan(e/2 + (f*x)/2) * (2a^7b^2c^6
\end{aligned}$$

$$\begin{aligned}
& - 2*a*b^7*c^6 - 2*a^8*c^5*d + 9*a^3*b^5*c^6 - 8*a^5*b^3*c^6 + 2*a^8*c^3*d^3 \\
& + 2*a^2*b^6*c^5*d - 13*a^4*b^4*c^5*d + 2*a^6*b^2*c*d^5 + 10*a^6*b^2*c^5*d \\
& + 4*a^7*b*c^2*d^4 - 4*a^7*b*c^4*d^2 - 8*a^3*b^5*c^4*d^2 + 16*a^4*b^4*c^3*d^3 \\
& - 10*a^5*b^3*c^2*d^4 + 13*a^5*b^3*c^4*d^2 - 13*a^6*b^2*c^3*d^3) / (a^7*d^3 \\
& - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3\dots
\end{aligned}$$

$$3.40 \quad \int \frac{\csc(e+fx) \sqrt{c+d \sin(e+fx)}}{a+b \sin(e+fx)} dx$$

**Optimal.** Leaf size=154

$$\frac{2c \Pi\left(2; \frac{1}{2}\left(e - \frac{\pi}{2} + fx\right) \middle| \frac{2d}{c+d}\right) \sqrt{\frac{c+d \sin(e+fx)}{c+d}}}{af \sqrt{c+d \sin(e+fx)}} - \frac{2(bc-ad) \Pi\left(\frac{2b}{a+b}; \frac{1}{2}\left(e - \frac{\pi}{2} + fx\right) \middle| \frac{2d}{c+d}\right) \sqrt{\frac{c+d \sin(e+fx)}{c+d}}}{a(a+b)f \sqrt{c+d \sin(e+fx)}}$$

[Out]  $-2*c*(\sin(1/2*e+1/4*Pi+1/2*f*x)^2)^{(1/2)}/\sin(1/2*e+1/4*Pi+1/2*f*x)*\text{EllipticPi}(\cos(1/2*e+1/4*Pi+1/2*f*x), 2, 2^{(1/2)}*(d/(c+d))^{(1/2)})*((c+d*\sin(f*x+e))/(c+d))^{(1/2)}/a/f/(c+d*\sin(f*x+e))^{(1/2)}+2*(-a*d+b*c)*(\sin(1/2*e+1/4*Pi+1/2*f*x)^2)^{(1/2)}/\sin(1/2*e+1/4*Pi+1/2*f*x)*\text{EllipticPi}(\cos(1/2*e+1/4*Pi+1/2*f*x), 2*b/(a+b), 2^{(1/2)}*(d/(c+d))^{(1/2)})*((c+d*\sin(f*x+e))/(c+d))^{(1/2)}/a/(a+b)/f/(c+d*\sin(f*x+e))^{(1/2)}$

**Rubi [A]**

time = 0.34, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {3014, 2886, 2884}

$$\frac{2c \sqrt{\frac{c+d \sin(e+fx)}{c+d}} \Pi\left(2; \frac{1}{2}\left(e+fx - \frac{\pi}{2}\right) \middle| \frac{2d}{c+d}\right)}{af \sqrt{c+d \sin(e+fx)}} - \frac{2(bc-ad) \sqrt{\frac{c+d \sin(e+fx)}{c+d}} \Pi\left(\frac{2b}{a+b}; \frac{1}{2}\left(e+fx - \frac{\pi}{2}\right) \middle| \frac{2d}{c+d}\right)}{af(a+b) \sqrt{c+d \sin(e+fx)}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(\text{Csc}[e+f*x]*\text{Sqrt}[c+d*\text{Sin}[e+f*x]])/(a+b*\text{Sin}[e+f*x]),x]$

[Out]  $(2*c*\text{EllipticPi}[2, (e - \text{Pi}/2 + f*x)/2, (2*d)/(c + d)]*\text{Sqrt}[(c + d*\text{Sin}[e + f*x])/(c + d)])/(a*f*\text{Sqrt}[c + d*\text{Sin}[e + f*x]]) - (2*(b*c - a*d)*\text{EllipticPi}[(2*b)/(a + b), (e - \text{Pi}/2 + f*x)/2, (2*d)/(c + d)]*\text{Sqrt}[(c + d*\text{Sin}[e + f*x])/(c + d)])/(a*(a + b)*f*\text{Sqrt}[c + d*\text{Sin}[e + f*x]])$

**Rule 2884**

$\text{Int}[1/(((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)])*\text{Sqrt}[(c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_.)]))], x\_Symbol] :> \text{Simp}[(2/(f*(a + b)*\text{Sqrt}[c + d]))*\text{EllipticPi}[2*(b/(a + b)), (1/2)*(e - \text{Pi}/2 + f*x), 2*(d/(c + d))], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{GtQ}[c + d, 0]$

**Rule 2886**

$\text{Int}[1/(((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)])*\text{Sqrt}[(c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_.)]))], x\_Symbol] :> \text{Dist}[\text{Sqrt}[(c + d*\text{Sin}[e + f*x])/(c + d)]/\text{Sqrt}[c + d*\text{Sin}[e + f*x]], \text{Int}[1/((a + b*\text{Sin}[e + f*x])*\text{Sqrt}[c/(c + d) + (d/(c +$

d))\*Sin[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]

### Rule 3014

Int[Sqrt[(a\_) + (b\_)\*sin[(e\_) + (f\_)\*(x\_)]]/(sin[(e\_) + (f\_)\*(x\_)]\*((c\_) + (d\_)\*sin[(e\_) + (f\_)\*(x\_)])), x\_Symbol] := Dist[a/c, Int[1/(Sin[e + f\*x]\*Sqrt[a + b\*Sin[e + f\*x]]), x], x] + Dist[(b\*c - a\*d)/c, Int[1/(Sqrt[a + b\*Sin[e + f\*x]]\*(c + d\*Sin[e + f\*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 - b^2, 0]

### Rubi steps

$$\int \frac{\csc(e+fx)\sqrt{c+d\sin(e+fx)}}{a+b\sin(e+fx)} dx = \frac{c \int \frac{\csc(e+fx)}{\sqrt{c+d\sin(e+fx)}} dx}{a} + \frac{(-bc+ad) \int \frac{1}{(a+b\sin(e+fx))\sqrt{c+d\sin(e+fx)}} dx}{a}$$

$$= \frac{\left(c\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\right) \int \frac{\csc(e+fx)}{\sqrt{\frac{c}{c+d} + \frac{d\sin(e+fx)}{c+d}}} dx}{a\sqrt{c+d\sin(e+fx)}} + \frac{(-bc+ad) \int \frac{1}{(a+b\sin(e+fx))\sqrt{c+d\sin(e+fx)}} dx}{a}$$

$$= \frac{2i\Pi\left(2; \frac{1}{2}\left(e - \frac{\pi}{2} + fx\right) \middle| \frac{2d}{c+d}\right) \sqrt{\frac{c+d\sin(e+fx)}{c+d}}}{af\sqrt{c+d\sin(e+fx)}} - \frac{2(bc-ad)\Pi\left(2; \frac{1}{2}\left(e - \frac{\pi}{2} + fx\right) \middle| \frac{2d}{c+d}\right)}{af\sqrt{c+d\sin(e+fx)}}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 23.28, size = 179, normalized size = 1.16

$$\frac{2i\left(\Pi\left(\frac{c+d}{c}, i \sinh^{-1}\left(\sqrt{-\frac{1}{c+d}}\sqrt{c+d\sin(e+fx)}\right) \middle| \frac{c+d}{c-d}\right) - \Pi\left(\frac{b(c+d)}{bc-ad}, i \sinh^{-1}\left(\sqrt{-\frac{1}{c+d}}\sqrt{c+d\sin(e+fx)}\right) \middle| \frac{c+d}{c-d}\right)\right) \sec(e+fx) \sqrt{-\frac{d(-1+\sin(e+fx))}{c+d}} \sqrt{\frac{d(1+\sin(e+fx))}{-c+d}}}{a\sqrt{-\frac{1}{c+d}}f}$$

Antiderivative was successfully verified.

[In] Integrate[(Csc[e + f\*x]\*Sqrt[c + d\*Sin[e + f\*x]])/(a + b\*Sin[e + f\*x]),x]

[Out] ((2\*I)\*(EllipticPi[(c + d)/c, I\*ArcSinh[Sqrt[-(c + d)^(-1)]\*Sqrt[c + d\*Sin[e + f\*x]]], (c + d)/(c - d)] - EllipticPi[(b\*(c + d))/(b\*c - a\*d), I\*ArcSinh[Sqrt[-(c + d)^(-1)]\*Sqrt[c + d\*Sin[e + f\*x]]], (c + d)/(c - d)]\*Sec[e + f\*x]\*Sqrt[-((d\*(-1 + Sin[e + f\*x]))/(c + d))]\*Sqrt[(d\*(1 + Sin[e + f\*x]))/(-c + d)])/(a\*Sqrt[-(c + d)^(-1)]\*f)

### Maple [A]

time = 11.44, size = 190, normalized size = 1.23

method	result
default	$\frac{2 \left( \text{EllipticPi} \left( \sqrt{\frac{c+d \sin(fx+e)}{c-d}}, -\frac{(c-d)b}{ad-bc}, \sqrt{\frac{c-d}{c+d}} \right) - \text{EllipticPi} \left( \sqrt{\frac{c+d \sin(fx+e)}{c-d}}, \frac{c-d}{c}, \sqrt{\frac{c-d}{c+d}} \right) \right) \sqrt{-\frac{d(1+\sin(fx+e))}{c-d}}}{a \cos(fx+e) \sqrt{c+d \sin(fx+e)}} f$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(1/2)/sin(f*x+e)/(a+b*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(EllipticPi(((c+d*sin(f*x+e))/(c-d))^(1/2),-(c-d)*b/(a*d-b*c),((c-d)/(c+d))^(1/2))-EllipticPi(((c+d*sin(f*x+e))/(c-d))^(1/2),(c-d)/c,((c-d)/(c+d))^(1/2)))*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(c-d)/a/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/sin(f*x+e)/(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)/((b*sin(f*x + e) + a)*sin(f*x + e)), x)
```

**Fricas** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/sin(f*x+e)/(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] Timed out
```

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+d \sin(e+fx)}}{(a+b \sin(e+fx)) \sin(e+fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(1/2)/sin(f*x+e)/(a+b*sin(f*x+e)),x)
```



[Out] Integral(sqrt(c + d\*sin(e + f\*x))/((a + b\*sin(e + f\*x))\*sin(e + f\*x)), x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*sin(f\*x+e))^(1/2)/sin(f\*x+e)/(a+b\*sin(f\*x+e)),x, algorithm="giac")

[Out] integrate(sqrt(d\*sin(f\*x + e) + c)/((b\*sin(f\*x + e) + a)\*sin(f\*x + e)), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c + d \sin(e + f x)}}{\sin(e + f x) (a + b \sin(e + f x))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*sin(e + f\*x))^(1/2)/(sin(e + f\*x)\*(a + b\*sin(e + f\*x))),x)

[Out] int((c + d\*sin(e + f\*x))^(1/2)/(sin(e + f\*x)\*(a + b\*sin(e + f\*x))), x)

$$3.41 \quad \int \frac{\csc(e+fx)}{(a+b\sin(e+fx))\sqrt{c+d\sin(e+fx)}} dx$$

Optimal. Leaf size=146

$$\frac{2\Pi\left(2; \frac{1}{2}\left(e - \frac{\pi}{2} + fx\right) \middle| \frac{2d}{c+d}\right) \sqrt{\frac{c+d\sin(e+fx)}{c+d}}}{af\sqrt{c+d\sin(e+fx)}} - \frac{2b\Pi\left(\frac{2b}{a+b}; \frac{1}{2}\left(e - \frac{\pi}{2} + fx\right) \middle| \frac{2d}{c+d}\right) \sqrt{\frac{c+d\sin(e+fx)}{c+d}}}{a(a+b)f\sqrt{c+d\sin(e+fx)}}$$

[Out]  $-2*(\sin(1/2*e+1/4*Pi+1/2*f*x))^2)^{(1/2)}/\sin(1/2*e+1/4*Pi+1/2*f*x)*\text{EllipticPi}(\cos(1/2*e+1/4*Pi+1/2*f*x), 2, 2^{(1/2)}*(d/(c+d))^{(1/2)})*((c+d*\sin(f*x+e))/(c+d))^{(1/2)}/a/f/(c+d*\sin(f*x+e))^{(1/2)}+2*b*(\sin(1/2*e+1/4*Pi+1/2*f*x))^2)^{(1/2)}/\sin(1/2*e+1/4*Pi+1/2*f*x)*\text{EllipticPi}(\cos(1/2*e+1/4*Pi+1/2*f*x), 2*b/(a+b), 2^{(1/2)}*(d/(c+d))^{(1/2)})*((c+d*\sin(f*x+e))/(c+d))^{(1/2)}/a/(a+b)/f/(c+d*\sin(f*x+e))^{(1/2)}$

Rubi [A]

time = 0.32, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {3020, 2886, 2884}

$$2\sqrt{\frac{c+d\sin(e+fx)}{c+d}} \Pi\left(2; \frac{1}{2}\left(e+fx - \frac{\pi}{2}\right) \middle| \frac{2d}{c+d}\right) - \frac{2b\sqrt{\frac{c+d\sin(e+fx)}{c+d}} \Pi\left(\frac{2b}{a+b}; \frac{1}{2}\left(e+fx - \frac{\pi}{2}\right) \middle| \frac{2d}{c+d}\right)}{af(a+b)\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

[In] `Int[Csc[e + f*x]/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]), x]`

[Out]  $(2*\text{EllipticPi}[2, (e - \text{Pi}/2 + f*x)/2, (2*d)/(c + d)]*\text{Sqrt}[(c + d*\text{Sin}[e + f*x])/ (c + d)])/(a*f*\text{Sqrt}[c + d*\text{Sin}[e + f*x]]) - (2*b*\text{EllipticPi}[(2*b)/(a + b), (e - \text{Pi}/2 + f*x)/2, (2*d)/(c + d)]*\text{Sqrt}[(c + d*\text{Sin}[e + f*x])/ (c + d)])/(a*(a + b)*f*\text{Sqrt}[c + d*\text{Sin}[e + f*x]])$

Rule 2884

`Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)])), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]`

Rule 2886

`Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)])), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +`

d))\*Sin[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]

### Rule 3020

Int[1/(sin[(e\_) + (f\_)\*(x\_)]\*Sqrt[(a\_) + (b\_)\*sin[(e\_) + (f\_)\*(x\_)])\*(c\_) + (d\_)\*sin[(e\_) + (f\_)\*(x\_)]), x\_Symbol] := Dist[1/c, Int[1/(Sin[e + f\*x]\*Sqrt[a + b\*Sin[e + f\*x])], x], x] - Dist[d/c, Int[1/(Sqrt[a + b\*Sin[e + f\*x]]\*(c + d\*Sin[e + f\*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 - b^2, 0]

### Rubi steps

$$\int \frac{\csc(e + fx)}{(a + b \sin(e + fx)) \sqrt{c + d \sin(e + fx)}} dx = \frac{\int \frac{\csc(e + fx)}{\sqrt{c + d \sin(e + fx)}} dx}{a} - \frac{b \int \frac{1}{(a + b \sin(e + fx)) \sqrt{c + d \sin(e + fx)}} dx}{a}$$

$$= \frac{\sqrt{\frac{c + d \sin(e + fx)}{c + d}} \int \frac{\csc(e + fx)}{\sqrt{\frac{c}{c + d} + \frac{d \sin(e + fx)}{c + d}}} dx}{a \sqrt{c + d \sin(e + fx)}} - \frac{b \int \frac{1}{(a + b \sin(e + fx)) \sqrt{c + d \sin(e + fx)}} dx}{a}$$

$$= \frac{2i \left( (-bc + ad) \Pi \left( \frac{c+d}{c}; i \sinh^{-1} \left( \sqrt{\frac{1}{c+d}} \sqrt{c + d \sin(e + fx)} \right) \middle| \frac{c+d}{c-d} \right) + bc \Pi \left( \frac{b(c+d)}{bc-ad}; i \sinh^{-1} \left( \sqrt{\frac{1}{c+d}} \sqrt{c + d \sin(e + fx)} \right) \middle| \frac{c+d}{c-d} \right) \right) \sec(e + fx) \sqrt{\frac{d(-1 + \sin(e + fx))}{c+d}} \sqrt{\frac{d(1 + \sin(e + fx))}{c-d}}}{ac \sqrt{\frac{1}{c+d}} (bc - ad) f} - \frac{2b \Pi \left( \frac{2}{a} \right)}{af \sqrt{c + d \sin(e + fx)}}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 23.40, size = 203, normalized size = 1.39

$$\frac{2i \left( (-bc + ad) \Pi \left( \frac{c+d}{c}; i \sinh^{-1} \left( \sqrt{\frac{1}{c+d}} \sqrt{c + d \sin(e + fx)} \right) \middle| \frac{c+d}{c-d} \right) + bc \Pi \left( \frac{b(c+d)}{bc-ad}; i \sinh^{-1} \left( \sqrt{\frac{1}{c+d}} \sqrt{c + d \sin(e + fx)} \right) \middle| \frac{c+d}{c-d} \right) \right) \sec(e + fx) \sqrt{\frac{d(-1 + \sin(e + fx))}{c+d}} \sqrt{\frac{d(1 + \sin(e + fx))}{c-d}}}{ac \sqrt{\frac{1}{c+d}} (bc - ad) f}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[e + f\*x]/((a + b\*Sin[e + f\*x])\*Sqrt[c + d\*Sin[e + f\*x]]),x]

[Out] ((-2\*I)\*((-b\*c) + a\*d)\*EllipticPi[(c + d)/c, I\*ArcSinh[Sqrt[-(c + d)^(-1)]\*Sqrt[c + d\*Sin[e + f\*x]]], (c + d)/(c - d)] + b\*c\*EllipticPi[(b\*(c + d))/(b\*c - a\*d), I\*ArcSinh[Sqrt[-(c + d)^(-1)]\*Sqrt[c + d\*Sin[e + f\*x]]], (c + d)/(c - d)]\*Sec[e + f\*x]\*Sqrt[-((d\*(-1 + Sin[e + f\*x]))/(c + d))]\*Sqrt[-((d\*(1 + Sin[e + f\*x]))/(c - d))]/(a\*c\*Sqrt[-(c + d)^(-1)]\*(b\*c - a\*d)\*f)

### Maple [A]

time = 10.96, size = 254, normalized size = 1.74

method	result
default	$\frac{2(c-d) \sqrt{\frac{c+d \sin(fx+e)}{c-d}} \sqrt{-\frac{(\sin(fx+e)-1)d}{c+d}} \sqrt{-\frac{d(1+\sin(fx+e))}{c-d}} \left( b \operatorname{EllipticPi} \left( \sqrt{\frac{c+d \sin(fx+e)}{c-d}}, -\frac{(c-d)b}{ad-bc}, \sqrt{\frac{c-d}{c+d}} \right) \right)}{a(ad-bc)c \cos(fx+e) \sqrt{c+d \sin(fx+e)}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/sin(f*x+e)/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x,method=_RETURNVE  
RBOSE)`

[Out] `-2*(c-d)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d  
*(1+sin(f*x+e))/(c-d))^(1/2)*(b*EllipticPi(((c+d*sin(f*x+e))/(c-d))^(1/2),-  
(c-d)*b/(a*d-b*c),((c-d)/(c+d))^(1/2))*c+EllipticPi(((c+d*sin(f*x+e))/(c-d)  
)^(1/2),(c-d)/c,((c-d)/(c+d))^(1/2))*a*d-EllipticPi(((c+d*sin(f*x+e))/(c-d)  
)^(1/2),(c-d)/c,((c-d)/(c+d))^(1/2))*b*c)/a/(a*d-b*c)/c/cos(f*x+e)/(c+d*sin  
(f*x+e))^(1/2)/f`

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sin(f*x+e)/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, algorithm  
="maxima")`

[Out] `integrate(1/((b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sin(f*x + e)), x  
)`

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sin(f*x+e)/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, algorithm  
="fricas")`

[Out] Timed out

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \sin(e + fx)) \sqrt{c + d \sin(e + fx)} \sin(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sin(f\*x+e)/(a+b\*sin(f\*x+e))/(c+d\*sin(f\*x+e))^(1/2),x)

[Out] Integral(1/((a + b\*sin(e + f\*x))\*sqrt(c + d\*sin(e + f\*x))\*sin(e + f\*x)), x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sin(f\*x+e)/(a+b\*sin(f\*x+e))/(c+d\*sin(f\*x+e))^(1/2),x, algorithm="giac")

[Out] integrate(1/((b\*sin(f\*x + e) + a)\*sqrt(d\*sin(f\*x + e) + c)\*sin(f\*x + e)), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin(e + f x) (a + b \sin(e + f x)) \sqrt{c + d \sin(e + f x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sin(e + f\*x)\*(a + b\*sin(e + f\*x))\*(c + d\*sin(e + f\*x))^(1/2)),x)

[Out] int(1/(sin(e + f\*x)\*(a + b\*sin(e + f\*x))\*(c + d\*sin(e + f\*x))^(1/2)), x)

$$3.42 \quad \int \frac{\sqrt{g \sin(e + fx)} \sqrt{a + b \sin(e + fx)}}{c + d \sin(e + fx)} dx$$

Optimal. Leaf size=254

$$\frac{2\sqrt{a+b} \sqrt{g} \sqrt{\frac{a(1-\csc(e+fx))}{a+b}} \sqrt{\frac{a(1+\csc(e+fx))}{a-b}} \Pi\left(\frac{a+b}{b}; \sin^{-1}\left(\frac{\sqrt{g} \sqrt{a+b \sin(e+fx)}}{\sqrt{a+b} \sqrt{g \sin(e+fx)}}\right)\right) \Big|_{-\frac{a+b}{a-b}}}{df}$$

[Out] 2\*EllipticPi(g^(1/2)\*(a+b\*sin(f\*x+e))^(1/2)/(a+b)^(1/2)/(g\*sin(f\*x+e))^(1/2), (a+b)/b, ((-a-b)/(a-b))^(1/2))\*(a+b)^(1/2)\*g^(1/2)\*(a\*(1-csc(f\*x+e))/(a+b))^(1/2)\*(a\*(1+csc(f\*x+e))/(a-b))^(1/2)\*tan(f\*x+e)/d/f-2\*(-a\*d+b\*c)\*EllipticPi(1/2\*(1-csc(f\*x+e))^(1/2)\*2^(1/2), 2\*c/(c+d), 2^(1/2)\*(a/(a+b))^(1/2))\*(-cot(f\*x+e)^2)^(1/2)\*((b+a\*csc(f\*x+e))/(a+b))^(1/2)\*(g\*sin(f\*x+e))^(1/2)\*tan(f\*x+e)/d/(c+d)/f/(a+b\*sin(f\*x+e))^(1/2)

Rubi [A]

time = 0.33, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 39,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {3008, 2888, 3016}

$$\frac{2\sqrt{g} \sqrt{a+b} \tan(e+fx) \sqrt{\frac{a(1-\csc(e+fx))}{a+b}} \sqrt{\frac{a(\csc(e+fx)+1)}{a-b}} \Pi\left(\frac{a+b}{b}; \text{ArcSin}\left(\frac{\sqrt{g} \sqrt{a+b \sin(e+fx)}}{\sqrt{a+b} \sqrt{g \sin(e+fx)}}\right)\right) \Big|_{-\frac{a+b}{a-b}} - \frac{2(bc-ad)\tan(e+fx) \sqrt{-\cot^2(e+fx)} \sqrt{g \sin(e+fx)} \sqrt{\frac{a \csc(e+fx)+b}{a+b}} \Pi\left(\frac{2c}{c+d}; \text{ArcSin}\left(\frac{\sqrt{1-\csc(e+fx)}}{\sqrt{2}}\right)\right) \Big|_{\frac{2a}{a+b}}}{df(c+d)\sqrt{a+b \sin(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[g\*Sin[e + f\*x]]\*Sqrt[a + b\*Sin[e + f\*x]])/(c + d\*Sin[e + f\*x]),x]

[Out] (2\*Sqrt[a + b]\*Sqrt[g]\*Sqrt[(a\*(1 - Csc[e + f\*x]))/(a + b)]\*Sqrt[(a\*(1 + Csc[e + f\*x]))/(a - b)]\*EllipticPi[(a + b)/b, ArcSin[(Sqrt[g]\*Sqrt[a + b\*Sin[e + f\*x]])/(Sqrt[a + b]\*Sqrt[g\*Sin[e + f\*x]])], -(a + b)/(a - b)]\*Tan[e + f\*x]/(d\*f) - (2\*(b\*c - a\*d)\*Sqrt[-Cot[e + f\*x]^2]\*Sqrt[(b + a\*Csc[e + f\*x])/(a + b)]\*EllipticPi[(2\*c)/(c + d), ArcSin[Sqrt[1 - Csc[e + f\*x]]/Sqrt[2]], (2\*a)/(a + b)]\*Sqrt[g\*Sin[e + f\*x]]\*Tan[e + f\*x]/(d\*(c + d)\*f\*Sqrt[a + b\*Sin[e + f\*x]])

Rule 2888

Int[Sqrt[(b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)]]/Sqrt[(c\_) + (d\_.)\*sin[(e\_.) + (f\_.)\*(x\_)]], x\_Symbol] :> Simp[2\*b\*(Tan[e + f\*x]/(d\*f))\*Rt[(c + d)/b, 2]\*Sqrt[c\*((1 + Csc[e + f\*x])/(c - d))\*Sqrt[c\*((1 - Csc[e + f\*x])/(c + d))]\*EllipticPi[(c + d)/d, ArcSin[Sqrt[c + d\*Sin[e + f\*x]]/Sqrt[b\*Sin[e + f\*x]]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/b]

Rule 3008

```
Int[(Sqrt[(g_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)])]/((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[b/d, In
t[Sqrt[g*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]], x], x] - Dist[(b*c - a*d)/
d, Int[Sqrt[g*Sin[e + f*x]]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x]))
, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
- b^2, 0] && NeQ[c^2 - d^2, 0]
```

### Rule 3016

```
Int[Sqrt[(g_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] := Simp[2*Sqrt[
-Cot[e + f*x]^2]*(Sqrt[g*Sin[e + f*x]]/(f*(c + d)*Cot[e + f*x]*Sqrt[a + b*S
in[e + f*x]))*Sqrt[(b + a*Csc[e + f*x])/(a + b)]*EllipticPi[2*(c/(c + d)),
ArcSin[Sqrt[1 - Csc[e + f*x]]/Sqrt[2]], 2*(a/(a + b))], x] /; FreeQ[{a, b,
c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0]
```

### Rubi steps

$$\int \frac{\sqrt{g \sin(e + fx)} \sqrt{a + b \sin(e + fx)}}{c + d \sin(e + fx)} dx = \frac{b \int \frac{\sqrt{g \sin(e + fx)}}{\sqrt{a + b \sin(e + fx)}} dx}{d} - \frac{(bc - ad) \int \frac{\sqrt{g \sin(e + fx)}}{\sqrt{a + b \sin(e + fx)}} dx}{d}$$

$$= \frac{2\sqrt{a+b} \sqrt{g} \sqrt{\frac{a(1 - \csc(e + fx))}{a+b}} \sqrt{\frac{a(1 + \csc(e + fx))}{a-b}}}{d}$$

**Mathematica** [C] Result contains complex when optimal does not.  
time = 39.21, size = 23019, normalized size = 90.63

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[g*Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]])/(c + d*Sin[e + f*
x]), x]
```

```
[Out] Result too large to show
```

**Maple** [C] Result contains complex when optimal does not.  
time = 2.96, size = 6211, normalized size = 24.45

method	result	size
--------	--------	------

default	Expression too large to display	6211
---------	---------------------------------	------

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sin(f*x+e))^(1/2)*(a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x,method=_
RETURNVERBOSE)
```

```
[Out] result too large to display
```

**Maxima [F]**

```
time = 0.00, size = 0, normalized size = 0.00
```

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sin(f*x+e))^(1/2)*(a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x, a
lgorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)*sqrt(g*sin(f*x + e))/(d*sin(f*x + e) + c
), x)
```

**Fricas [F(-1)]** Timed out

```
time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sin(f*x+e))^(1/2)*(a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

**Sympy [F]**

```
time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{\sqrt{g \sin(e + fx)} \sqrt{a + b \sin(e + fx)}}{c + d \sin(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sin(f*x+e))**(1/2)*(a+b*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e)),x)
```

```
[Out] Integral(sqrt(g*sin(e + f*x))*sqrt(a + b*sin(e + f*x))/(c + d*sin(e + f*x))
, x)
```

**Giac [F]**

```
time = 0.00, size = 0, normalized size = 0.00
```

could not integrate



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sin(f*x+e))^(1/2)*(a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)*sqrt(g*sin(f*x + e))/(d*sin(f*x + e) + c), x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{g \sin(e + f x)} \sqrt{a + b \sin(e + f x)}}{c + d \sin(e + f x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*sin(e + f*x))^(1/2)*(a + b*sin(e + f*x))^(1/2))/(c + d*sin(e + f*x)),x)
```

```
[Out] int(((g*sin(e + f*x))^(1/2)*(a + b*sin(e + f*x))^(1/2))/(c + d*sin(e + f*x)), x)
```

$$3.43 \quad \int \frac{\sqrt{a + b \sin(e + fx)}}{\sqrt{g \sin(e + fx)} (c + d \sin(e + fx))} dx$$

**Optimal.** Leaf size=250

$$\frac{2\sqrt{a+b} \sqrt{\frac{a(1-\csc(e+fx))}{a+b}} \sqrt{\frac{a(1+\csc(e+fx))}{a-b}} F\left(\sin^{-1}\left(\frac{\sqrt{g} \sqrt{a+b \sin(e+fx)}}{\sqrt{a+b} \sqrt{g \sin(e+fx)}}\right) \middle| -\frac{a+b}{a-b}\right) \tan(e)}{cf\sqrt{g}}$$

[Out] -2\*EllipticF(g^(1/2)\*(a+b\*sin(f\*x+e))^(1/2)/(a+b)^(1/2)/(g\*sin(f\*x+e))^(1/2), ((-a-b)/(a-b))^(1/2)\*(a+b)^(1/2)\*(a\*(1-csc(f\*x+e)))/(a+b)^(1/2)\*(a\*(1+csc(f\*x+e)))/(a-b))^(1/2)\*tan(f\*x+e)/c/f/g^(1/2)+2\*(-a\*d+b\*c)\*EllipticPi(1/2\*(1-csc(f\*x+e))^(1/2)\*2^(1/2), 2\*c/(c+d), 2^(1/2)\*(a/(a+b))^(1/2))\*(-cot(f\*x+e))^2)^(1/2)\*((b+a\*csc(f\*x+e))/(a+b)^(1/2)\*(g\*sin(f\*x+e))^(1/2)\*tan(f\*x+e)/c/(c+d)/f/g/(a+b\*sin(f\*x+e))^(1/2))

**Rubi [A]**

time = 0.34, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 39,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {3012, 2895, 3016}

$$\frac{2(bc-ad)\tan(e+fx)\sqrt{-\cot^2(e+fx)}\sqrt{g\sin(e+fx)}\sqrt{\frac{a\csc(e+fx)+b}{a+b}}\Pi\left(\frac{2c}{c+d}; \text{ArcSin}\left(\frac{\sqrt{1-\csc(e+fx)}}{\sqrt{2}}\right) \middle| \frac{2c}{a+b}\right)}{cf(g(c+d)\sqrt{a+b\sin(e+fx)})} - \frac{2\sqrt{a+b}\tan(e+fx)\sqrt{\frac{a(1-\csc(e+fx))}{a+b}}\sqrt{\frac{a(\csc(e+fx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{g}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{g\sin(e+fx)}}\right) \middle| -\frac{a+b}{a-b}\right)}{cf\sqrt{g}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b\*Sin[e + f\*x]]/(Sqrt[g\*Sin[e + f\*x]]\*(c + d\*Sin[e + f\*x])),x]

[Out] (-2\*Sqrt[a + b]\*Sqrt[(a\*(1 - Csc[e + f\*x]))/(a + b)]\*Sqrt[(a\*(1 + Csc[e + f\*x]))/(a - b)]\*EllipticF[ArcSin[(Sqrt[g]\*Sqrt[a + b\*Sin[e + f\*x]])/(Sqrt[a + b]\*Sqrt[g\*Sin[e + f\*x]])], -(a + b)/(a - b)]\*Tan[e + f\*x]/(c\*f\*Sqrt[g]) + (2\*(b\*c - a\*d)\*Sqrt[-Cot[e + f\*x]^2]\*Sqrt[(b + a\*Csc[e + f\*x])/(a + b)]\*EllipticPi[(2\*c)/(c + d), ArcSin[Sqrt[1 - Csc[e + f\*x]]/Sqrt[2]], (2\*a)/(a + b)]\*Sqrt[g\*Sin[e + f\*x]]\*Tan[e + f\*x])/(c\*(c + d)\*f\*g\*Sqrt[a + b\*Sin[e + f\*x]])

Rule 2895

Int[1/(Sqrt[(d\_)\*sin[(e\_) + (f\_)\*(x\_)]]\*Sqrt[(a\_) + (b\_)\*sin[(e\_) + (f\_)\*(x\_)]]), x\_Symbol] :> Simp[-2\*(Tan[e + f\*x]/(a\*f))\*Rt[(a + b)/d, 2]\*Sqrt[a\*((1 - Csc[e + f\*x])/(a + b))]\*Sqrt[a\*((1 + Csc[e + f\*x])/(a - b))]\*EllipticF[ArcSin[Sqrt[a + b\*Sin[e + f\*x]]/Sqrt[d\*Sin[e + f\*x]]/Rt[(a + b)/d, 2]], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && PosQ[(a + b)/d]

Rule 3012

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/(Sqrt[(g_)*sin[(e_) + (f_
)*(x_)]]*((c_) + (d_)*sin[(e_) + (f_)*(x_)])), x_Symbol] :> Dist[a/c, In
t[1/(Sqrt[g*Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]]), x], x] + Dist[(b*c - a
*d)/(c*g), Int[Sqrt[g*Sin[e + f*x]]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e
+ f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

### Rule 3016

```
Int[Sqrt[(g_)*sin[(e_) + (f_)*(x_)]]/(Sqrt[(a_) + (b_)*sin[(e_) + (f_
)*(x_)]]*((c_) + (d_)*sin[(e_) + (f_)*(x_)])), x_Symbol] :> Simp[2*Sqrt[
-Cot[e + f*x]^2]*(Sqrt[g*Sin[e + f*x]]/(f*(c + d)*Cot[e + f*x]*Sqrt[a + b*S
in[e + f*x]))*Sqrt[(b + a*Csc[e + f*x])/(a + b)]*EllipticPi[2*(c/(c + d)),
ArcSin[Sqrt[1 - Csc[e + f*x]]/Sqrt[2]], 2*(a/(a + b))], x] /; FreeQ[{a, b,
c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0]
```

### Rubi steps

$$\int \frac{\sqrt{a + b \sin(e + fx)}}{\sqrt{g \sin(e + fx)} (c + d \sin(e + fx))} dx = \frac{a \int \frac{1}{\sqrt{g \sin(e + fx)} \sqrt{a + b \sin(e + fx)}} dx}{c} + \frac{(bc - ad) \int \frac{1}{\sqrt{a + b \sin(e + fx)}} dx}{c}$$

$$= -\frac{2\sqrt{a+b} \sqrt{\frac{a(1 - \csc(e + fx))}{a+b}} \sqrt{\frac{a(1 + \csc(e + fx))}{a-b}} F\left(\sin^{-1}\left(\frac{\sqrt{a+b} \sqrt{1 - \csc(e + fx)}}{\sqrt{a-b}}\right)\right)}{cf\sqrt{g}}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 8202 vs. 2(250) = 500.

time = 48.67, size = 8202, normalized size = 32.81

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[g*Sin[e + f*x]]*(c + d*Sin[e + f*x
])),x]
```

```
[Out] Result too large to show
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 3297 vs. 2(231) = 462.

time = 0.29, size = 3298, normalized size = 13.19

method	result	size
default	Expression too large to display	3298

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))/(g*sin(f*x+e))^(1/2),x,method=_
RETURNVERBOSE)
```

```
[Out] 1/f*(-(cos(f*x+e)*a-(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)-a)/(b+(-a^2+b^
2)^(1/2))/sin(f*x+e))^(1/2)*((cos(f*x+e)*a+(-a^2+b^2)^(1/2)*sin(f*x+e)-b*si
n(f*x+e)-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1/2)*(a*(-1+cos(f*x+e)))/(b+(-a^2+
b^2)^(1/2))/sin(f*x+e))^(1/2)*(a*d-b*c)*(2*(-c^2+d^2)^(1/2)*(-a^2+b^2)^(1/2
)*EllipticPi((-cos(f*x+e)*a-(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)-a)/(b
+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2),-(b+(-a^2+b^2)^(1/2))*c/(a*(-c^2+d^2)^(
1/2)-c*(-a^2+b^2)^(1/2)+a*d-b*c),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b
^2)^(1/2))^(1/2)*b+2*(-c^2+d^2)^(1/2)*(-a^2+b^2)^(1/2)*EllipticPi((-cos(f
*x+e)*a-(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)-a)/(b+(-a^2+b^2)^(1/2))/si
n(f*x+e))^(1/2), (b+(-a^2+b^2)^(1/2))*c/(a*(-c^2+d^2)^(1/2)+c*(-a^2+b^2)^(1/
2)-a*d+b*c),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2)*b-4*
(-c^2+d^2)^(1/2)*(-a^2+b^2)^(1/2)*EllipticF((-cos(f*x+e)*a-(-a^2+b^2)^(1/2
)*sin(f*x+e)-b*sin(f*x+e)-a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2),1/2*2^(
1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2)*b-(-c^2+d^2)^(1/2)*Elli
pticPi((-cos(f*x+e)*a-(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)-a)/(b+(-a^2
+b^2)^(1/2))/sin(f*x+e))^(1/2),-(b+(-a^2+b^2)^(1/2))*c/(a*(-c^2+d^2)^(1/2)-
c*(-a^2+b^2)^(1/2)+a*d-b*c),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1
/2))^(1/2)*a^2+2*(-c^2+d^2)^(1/2)*EllipticPi((-cos(f*x+e)*a-(-a^2+b^2)^(1/
2)*sin(f*x+e)-b*sin(f*x+e)-a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2),-(b+(
-a^2+b^2)^(1/2))*c/(a*(-c^2+d^2)^(1/2)-c*(-a^2+b^2)^(1/2)+a*d-b*c),1/2*2^(1
/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2)*b^2-(-c^2+d^2)^(1/2)*Elli
pticPi((-cos(f*x+e)*a-(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)-a)/(b+(-a^
2+b^2)^(1/2))/sin(f*x+e))^(1/2), (b+(-a^2+b^2)^(1/2))*c/(a*(-c^2+d^2)^(1/2)+
c*(-a^2+b^2)^(1/2)-a*d+b*c),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1
/2))^(1/2)*a^2+2*(-c^2+d^2)^(1/2)*EllipticPi((-cos(f*x+e)*a-(-a^2+b^2)^(1/
2)*sin(f*x+e)-b*sin(f*x+e)-a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2), (b+(-
a^2+b^2)^(1/2))*c/(a*(-c^2+d^2)^(1/2)+c*(-a^2+b^2)^(1/2)-a*d+b*c),1/2*2^(1/
2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2)*b^2+2*(-c^2+d^2)^(1/2)*El
lipticF((-cos(f*x+e)*a-(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)-a)/(b+(-a^
2+b^2)^(1/2))/sin(f*x+e))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2
)^(1/2))^(1/2)*a^2-4*(-c^2+d^2)^(1/2)*EllipticF((-cos(f*x+e)*a-(-a^2+b^2)
^(1/2)*sin(f*x+e)-b*sin(f*x+e)-a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2),1/
2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2)*b^2-(-a^2+b^2)^(1/
2)*EllipticPi((-cos(f*x+e)*a-(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)-a)/(
b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2),-(b+(-a^2+b^2)^(1/2))*c/(a*(-c^2+d^2)
^(1/2)-c*(-a^2+b^2)^(1/2)+a*d-b*c),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+
```

$$\begin{aligned}
& b^2)^{(1/2)})^{(1/2)} * a * c + 2 * (-a^2 + b^2)^{(1/2)} * \text{EllipticPi}((- (\cos(f*x+e) * a - (-a^2 + \\
& b^2)^{(1/2)} * \sin(f*x+e) - b * \sin(f*x+e) - a) / (b + (-a^2 + b^2)^{(1/2)}) / \sin(f*x+e))^{(1/2)} \\
& ), -(b + (-a^2 + b^2)^{(1/2)}) * c / (a * (-c^2 + d^2)^{(1/2)} - c * (-a^2 + b^2)^{(1/2)} + a * d - b * c), 1 \\
& / 2 * 2^{(1/2)} * ((b + (-a^2 + b^2)^{(1/2)}) / (-a^2 + b^2)^{(1/2)})^{(1/2)} * b * d + (-a^2 + b^2)^{(1 \\
& / 2)} * \text{EllipticPi}((- (\cos(f*x+e) * a - (-a^2 + b^2)^{(1/2)} * \sin(f*x+e) - b * \sin(f*x+e) - a) / \\
& (b + (-a^2 + b^2)^{(1/2)}) / \sin(f*x+e))^{(1/2)}, (b + (-a^2 + b^2)^{(1/2)}) * c / (a * (-c^2 + d^2) \\
& ^{(1/2)} + c * (-a^2 + b^2)^{(1/2)} - a * d + b * c), 1/2 * 2^{(1/2)} * ((b + (-a^2 + b^2)^{(1/2)}) / (-a^2 + \\
& b^2)^{(1/2)})^{(1/2)} * a * c - 2 * (-a^2 + b^2)^{(1/2)} * \text{EllipticPi}((- (\cos(f*x+e) * a - (-a^2 + \\
& b^2)^{(1/2)} * \sin(f*x+e) - b * \sin(f*x+e) - a) / (b + (-a^2 + b^2)^{(1/2)}) / \sin(f*x+e))^{(1/2)} \\
& ), (b + (-a^2 + b^2)^{(1/2)}) * c / (a * (-c^2 + d^2)^{(1/2)} + c * (-a^2 + b^2)^{(1/2)} - a * d + b * c), 1/ \\
& 2 * 2^{(1/2)} * ((b + (-a^2 + b^2)^{(1/2)}) / (-a^2 + b^2)^{(1/2)})^{(1/2)} * b * d - \text{EllipticPi}((- ( \\
& \cos(f*x+e) * a - (-a^2 + b^2)^{(1/2)} * \sin(f*x+e) - b * \sin(f*x+e) - a) / (b + (-a^2 + b^2)^{(1/2)} \\
& )) / \sin(f*x+e))^{(1/2)}, -(b + (-a^2 + b^2)^{(1/2)}) * c / (a * (-c^2 + d^2)^{(1/2)} - c * (-a^2 + b^2) \\
& ^{(1/2)} + a * d - b * c), 1/2 * 2^{(1/2)} * ((b + (-a^2 + b^2)^{(1/2)}) / (-a^2 + b^2)^{(1/2)})^{(1/2)} \\
& ) * a^2 * d - \text{EllipticPi}((- (\cos(f*x+e) * a - (-a^2 + b^2)^{(1/2)} * \sin(f*x+e) - b * \sin(f*x+e) \\
& - a) / (b + (-a^2 + b^2)^{(1/2)}) / \sin(f*x+e))^{(1/2)}, -(b + (-a^2 + b^2)^{(1/2)}) * c / (a * (-c^2 \\
& + d^2)^{(1/2)} - c * (-a^2 + b^2)^{(1/2)} + a * d - b * c), 1/2 * 2^{(1/2)} * ((b + (-a^2 + b^2)^{(1/2)}) / ( \\
& -a^2 + b^2)^{(1/2)})^{(1/2)} * a * b * c + 2 * \text{EllipticPi}((- (\cos(f*x+e) * a - (-a^2 + b^2)^{(1/2)} \\
& * \sin(f*x+e) - b * \sin(f*x+e) - a) / (b + (-a^2 + b^2)^{(1/2)}) / \sin(f*x+e))^{(1/2)}, -(b + (-a^ \\
& 2 + b^2)^{(1/2)}) * c / (a * (-c^2 + d^2)^{(1/2)} - c * (-a^2 + b^2)^{(1/2)} + a * d - b * c), 1/2 * 2^{(1/2)} \\
& * ((b + (-a^2 + b^2)^{(1/2)}) / (-a^2 + b^2)^{(1/2)})^{(1/2)} * b^2 * d + \text{EllipticPi}((- (\cos(f*x \\
& + e) * a - (-a^2 + b^2)^{(1/2)} * \sin(f*x+e) - b * \sin(f*x+e) - a) / (b + (-a^2 + b^2)^{(1/2)}) / \sin( \\
& f*x+e))^{(1/2)}, (b + (-a^2 + b^2)^{(1/2)}) * c / (a * (-c^2 + d^2)^{(1/2)} + c * (-a^2 + b^2)^{(1/2)} \\
& - a * d + b * c), 1/2 * 2^{(1/2)} * ((b + (-a^2 + b^2)^{(1/2)}) / (-a^2 + b^2)^{(1/2)})^{(1/2)} * a^2 * d + \\
& \text{EllipticPi}((- (\cos(f*x+e) * a - (-a^2 + b^2)^{(1/2)} * \sin(f*x+e) - b * \sin(f*x+e) - a) / (b + ( \\
& -a^2 + b^2)^{(1/2)}) / \sin(f*x+e))^{(1/2)}, (b + (-a^2 + b^2)^{(1/2)}) * c / (a * (-c^2 + d^2)^{(1/2)} \\
& + c * (-a^2 + b^2)^{(1/2)} - a * d + b * c), 1/2 * 2^{(1/2)} * ((b + (-a^2 + b^2)^{(1/2)}) / (-a^2 + b^2) \\
& ^{(1/2)})^{(1/2)} * a * b * c - 2 * \text{EllipticPi}((- (\cos(f*x+e) * a - (-a^2 + b^2)^{(1/2)} * \sin(f*x+ \\
& e) - b * \sin(f*x+e) - a) / (b + (-a^2 + b^2)^{(1/2)}) / \sin(f*x+e))^{(1/2)}, (b + (-a^2 + b^2)^{(1/ \\
& 2)}) * c / (a * (-c^2 + d^2)^{(1/2)} + c * (-a^2 + b^2)^{(1/2)} - a * d + b * c), 1/2 * 2^{(1/2)} * ((b + (-a^2 \\
& + b^2)^{(1/2)}) / (-a^2 + b^2)^{(1/2)})^{(1/2)} * b^2 * d) * si...
\end{aligned}$$

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sin(f\*x+e))^(1/2)/(c+d\*sin(f\*x+e))/(g\*sin(f\*x+e))^(1/2),x, a  
algorithm="maxima")

[Out] integrate(sqrt(b\*sin(f\*x + e) + a)/((d\*sin(f\*x + e) + c)\*sqrt(g\*sin(f\*x + e  
))), x)

**Fricas [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sin(f\*x+e))^(1/2)/(c+d\*sin(f\*x+e))/(g\*sin(f\*x+e))^(1/2),x, algorithm="fricas")

[Out] integral(-sqrt(b\*sin(f\*x + e) + a)\*sqrt(g\*sin(f\*x + e))/(d\*g\*cos(f\*x + e)^2 - c\*g\*sin(f\*x + e) - d\*g), x)

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b \sin(e + fx)}}{\sqrt{g \sin(e + fx)} (c + d \sin(e + fx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sin(f\*x+e))\*\*(1/2)/(c+d\*sin(f\*x+e))/(g\*sin(f\*x+e))\*\*(1/2),x)

[Out] Integral(sqrt(a + b\*sin(e + f\*x))/(sqrt(g\*sin(e + f\*x))\*(c + d\*sin(e + f\*x))), x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sin(f\*x+e))^(1/2)/(c+d\*sin(f\*x+e))/(g\*sin(f\*x+e))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b\*sin(f\*x + e) + a)/((d\*sin(f\*x + e) + c)\*sqrt(g\*sin(f\*x + e))), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a + b \sin(e + fx)}}{\sqrt{g \sin(e + fx)} (c + d \sin(e + fx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*sin(e + f\*x))^(1/2)/((g\*sin(e + f\*x))^(1/2)\*(c + d\*sin(e + f\*x))),x)

[Out] int((a + b\*sin(e + f\*x))^(1/2)/((g\*sin(e + f\*x))^(1/2)\*(c + d\*sin(e + f\*x))), x)

$$3.44 \quad \int \frac{\sqrt{g \sin(e + fx)}}{\sqrt{a + b \sin(e + fx)} (c + d \sin(e + fx))} dx$$

**Optimal.** Leaf size=114

$$\frac{2\sqrt{-\cot^2(e + fx)} \sqrt{\frac{b + a \csc(e + fx)}{a + b}} \Pi\left(\frac{2c}{c+d}; \sin^{-1}\left(\frac{\sqrt{1 - \csc(e + fx)}}{\sqrt{2}}\right) \middle| \frac{2a}{a+b}\right) \sqrt{g \sin(e + fx)} \tan(e + fx)}{(c + d)f \sqrt{a + b \sin(e + fx)}}$$

[Out] 2\*EllipticPi(1/2\*(1-csc(f\*x+e))^(1/2)\*2^(1/2),2\*c/(c+d),2^(1/2)\*(a/(a+b))^(1/2))\*(-cot(f\*x+e)^2)^(1/2)\*((b+a\*csc(f\*x+e))/(a+b))^(1/2)\*(g\*sin(f\*x+e))^(1/2)\*tan(f\*x+e)/(c+d)/f/(a+b\*sin(f\*x+e))^(1/2)

**Rubi** [A]

time = 0.14, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 39,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$ , Rules used = {3016}

$$\frac{2 \tan(e + fx) \sqrt{-\cot^2(e + fx)} \sqrt{g \sin(e + fx)} \sqrt{\frac{a \csc(e + fx) + b}{a + b}} \Pi\left(\frac{2c}{c+d}; \text{ArcSin}\left(\frac{\sqrt{1 - \csc(e + fx)}}{\sqrt{2}}\right) \middle| \frac{2a}{a+b}\right)}{f(c + d) \sqrt{a + b \sin(e + fx)}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[g\*Sin[e + f\*x]]/(Sqrt[a + b\*Sin[e + f\*x]]\*(c + d\*Sin[e + f\*x])),x]

[Out] (2\*Sqrt[-Cot[e + f\*x]^2]\*Sqrt[(b + a\*Csc[e + f\*x])/(a + b)]\*EllipticPi[(2\*c)/(c + d), ArcSin[Sqrt[1 - Csc[e + f\*x]]/Sqrt[2]], (2\*a)/(a + b)]\*Sqrt[g\*Sin[e + f\*x]]\*Tan[e + f\*x])/((c + d)\*f\*Sqrt[a + b\*Sin[e + f\*x]])

Rule 3016

Int[Sqrt[(g\_.)\*sin[(e\_.) + (f\_.)\*(x\_)]]/(Sqrt[(a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])\*(x\_)])\*((c\_.) + (d\_.)\*sin[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] :> Simp[2\*Sqrt[-Cot[e + f\*x]^2]\*(Sqrt[g\*Sin[e + f\*x]]/(f\*(c + d)\*Cot[e + f\*x]\*Sqrt[a + b\*Sin[e + f\*x]]))\*Sqrt[(b + a\*Csc[e + f\*x])/(a + b)]\*EllipticPi[2\*(c/(c + d)), ArcSin[Sqrt[1 - Csc[e + f\*x]]/Sqrt[2]], 2\*(a/(a + b))], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rubi steps

$$\int \frac{\sqrt{g \sin(e + fx)}}{\sqrt{a + b \sin(e + fx)} (c + d \sin(e + fx))} dx = \frac{2\sqrt{-\cot^2(e + fx)} \sqrt{\frac{b + a \csc(e + fx)}{a + b}} \Pi\left(\frac{2c}{c+d}; \sin^{-1}\left(\frac{\sqrt{1 - \csc(e + fx)}}{\sqrt{2}}\right) \middle| \frac{2a}{a+b}\right) \sqrt{g \sin(e + fx)} \tan(e + fx)}{(c + d)f \sqrt{a + b \sin(e + fx)}}$$

**Mathematica [B]** Leaf count is larger than twice the leaf count of optimal. 3427 vs. 2(114) = 228.

time = 47.73, size = 3427, normalized size = 30.06

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[g*Sin[e + f*x]]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])),x]
```

```
[Out] (a*Sqrt[-a^2 + b^2]*((a*c + (b + Sqrt[-a^2 + b^2])*(-d + Sqrt[-c^2 + d^2]))*EllipticPi[(2*Sqrt[-a^2 + b^2]*c)/(b*c + Sqrt[-a^2 + b^2]*c - a*d + a*Sqrt[-c^2 + d^2]), ArcSin[Sqrt[(b + Sqrt[-a^2 + b^2] + a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(b + Sqrt[-a^2 + b^2])] + ((a*c) + (b + Sqrt[-a^2 + b^2])*(d + Sqrt[-c^2 + d^2]))*EllipticPi[(2*Sqrt[-a^2 + b^2]*c)/(b*c + Sqrt[-a^2 + b^2]*c - a*(d + Sqrt[-c^2 + d^2])), ArcSin[Sqrt[(b + Sqrt[-a^2 + b^2] + a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(b + Sqrt[-a^2 + b^2]))*Sqrt[Sin[e + f*x]]*Sqrt[g*Sin[e + f*x]]*Sqrt[(a*Sec[(e + f*x)/2]^2*(a + b*Sin[e + f*x]))/(a^2 - b^2)))/((b + Sqrt[-a^2 + b^2])^2*(b*c - a*d)*Sqrt[-c^2 + d^2]*f*(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])*Sqrt[-((a*Tan[(e + f*x)/2])/(b + Sqrt[-a^2 + b^2]))]*((a^2*Sqrt[-a^2 + b^2]*((a*c + (b + Sqrt[-a^2 + b^2])*(-d + Sqrt[-c^2 + d^2]))*EllipticPi[(2*Sqrt[-a^2 + b^2]*c)/(b*c + Sqrt[-a^2 + b^2]*c - a*d + a*Sqrt[-c^2 + d^2]), ArcSin[Sqrt[(b + Sqrt[-a^2 + b^2] + a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(b + Sqrt[-a^2 + b^2])] + ((a*c) + (b + Sqrt[-a^2 + b^2])*(d + Sqrt[-c^2 + d^2]))*EllipticPi[(2*Sqrt[-a^2 + b^2]*c)/(b*c + Sqrt[-a^2 + b^2]*c - a*(d + Sqrt[-c^2 + d^2])), ArcSin[Sqrt[(b + Sqrt[-a^2 + b^2] + a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(b + Sqrt[-a^2 + b^2]))*Sec[(e + f*x)/2]^2*Sqrt[Sin[e + f*x]]*Sqrt[(a*Sec[(e + f*x)/2]^2*(a + b*Sin[e + f*x]))/(a^2 - b^2)))/(4*(b + Sqrt[-a^2 + b^2])^3*(b*c - a*d)*Sqrt[-c^2 + d^2]*Sqrt[a + b*Sin[e + f*x]]*(-((a*Tan[(e + f*x)/2])/(b + Sqrt[-a^2 + b^2]))^(3/2)) - (a*b*Sqrt[-a^2 + b^2]*Cos[e + f*x]*((a*c + (b + Sqrt[-a^2 + b^2])*(-d + Sqrt[-c^2 + d^2]))*EllipticPi[(2*Sqrt[-a^2 + b^2]*c)/(b*c + Sqrt[-a^2 + b^2]*c - a*d + a*Sqrt[-c^2 + d^2]), ArcSin[Sqrt[(b + Sqrt[-a^2 + b^2] + a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(b + Sqrt[-a^2 + b^2])] + ((a*c) + (b + Sqrt[-a^2 + b^2])*(d + Sqrt[-c^2 + d^2]))*EllipticPi[(2*Sqrt[-a^2 + b^2]*c)/(b*c + Sqrt[-a^2 + b^2]*c - a*(d + Sqrt[-c^2 + d^2])), ArcSin[Sqrt[(b + Sqrt[-a^2 + b^2] + a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(b + Sqrt[-a^2 + b^2]))*Sqrt[Sin[e + f*x]]*Sqrt[(a*Sec[(e + f*x)/2]^2*(a + b*Sin[e + f*x]))/(a^2 - b^2)))/(2*(b + Sqrt[-a^2 + b^2])^2*(b*c - a*d)*Sqrt[-c^2 + d^2]*(a + b*Sin[e + f*x])^(3/2)*Sqrt[-((a*Tan[(e + f*x)/2])/(b + Sqrt[-a^2 + b^2]))]) + (a*Sqrt[-a^2 + b^2]*Cos[e + f*x]*((a*c + (b + Sqrt[-a^2 + b^2])*(-d + Sqrt[-c^2 + d^2]))*EllipticPi[(2*Sqrt[-a^2 + b^2]*c)/(b*c + Sqrt[-a^2 + b^2]*c - a*d + a*Sqrt[-c^2 + d^2]
```



2]), ArcSin[Sqrt[(b + Sqrt[-a^2 + b^2] + a\*Tan[(e + f\*x)/2])/Sqrt[-a^2 + b^2])/Sqrt[2]], (2\*Sqrt[-a^2 + b^2])/(b + Sqrt[-a^2 + b^2]) + (-a\*c) + (b + Sqrt[-a^2 + b^2])\*(d + Sqrt[-c^2 + d^2])\*EllipticPi[(2\*Sqrt[-a^2 + b^2]\*c)/(b\*c + Sqrt[-a^2 + b^2]\*c - a\*(d + Sqrt[-c^2 + d^2])), ArcSin[Sqrt[(b + Sqrt[-a^2 + b^2] + a\*Tan[(e + f\*x)/2])/Sqrt[-a^2 + b^2])/Sqrt[2]], (2\*Sqrt[-a^2 + b^2])/(b + Sqrt[-a^2 + b^2])\*Sqrt[(a\*Sec[(e + f\*x)/2]^2\*(a + b\*Sin[e + f\*x]))/(a^2 - b^2)]/(2\*(b + Sqrt[-a^2 + b^2])^2\*(b\*c - a\*d)\*Sqrt[-c^2 + d^2]\*Sqrt[Sin[e + f\*x]]\*Sqrt[a + b\*Sin[e + f\*x]]\*Sqrt[-((a\*Tan[(e + f\*x)/2])/(b + Sqrt[-a^2 + b^2]))]) + (a\*Sqrt[-a^2 + b^2]\*((a\*c + (b + Sqrt[-a^2 + b^2])\*(-d + Sqrt[-c^2 + d^2]))\*EllipticPi[(2\*Sqrt[-a^2 + b^2]\*c)/(b\*c + Sqrt[-a^2 + b^2]\*c - a\*d + a\*Sqrt[-c^2 + d^2])), ArcSin[Sqrt[(b + Sqrt[-a^2 + b^2] + a\*Tan[(e + f\*x)/2])/Sqrt[-a^2 + b^2])/Sqrt[2]], (2\*Sqrt[-a^2 + b^2])/(b + Sqrt[-a^2 + b^2]) + (-a\*c) + (b + Sqrt[-a^2 + b^2])\*(d + Sqrt[-c^2 + d^2])\*EllipticPi[(2\*Sqrt[-a^2 + b^2]\*c)/(b\*c + Sqrt[-a^2 + b^2]\*c - a\*(d + Sqrt[-c^2 + d^2])), ArcSin[Sqrt[(b + Sqrt[-a^2 + b^2] + a\*Tan[(e + f\*x)/2])/Sqrt[-a^2 + b^2])/Sqrt[2]], (2\*Sqrt[-a^2 + b^2])/(b + Sqrt[-a^2 + b^2])\*Sqrt[Sin[e + f\*x]]\*((a\*b\*Cos[e + f\*x]\*Sec[(e + f\*x)/2]^2)/(a^2 - b^2) + (a\*Sec[(e + f\*x)/2]^2\*(a + b\*Sin[e + f\*x])\*Tan[(e + f\*x)/2])/(a^2 - b^2)))/(2\*(b + Sqrt[-a^2 + b^2])^2\*(b\*c - a\*d)\*Sqrt[-c^2 + d^2]\*Sqrt[a + b\*Sin[e + f\*x]]\*Sqrt[(a\*Sec[(e + f\*x)/2]^2\*(a + b\*Sin[e + f\*x]))/(a^2 - b^2)]\*Sqrt[-((a\*Tan[(e + f\*x)/2])/(b + Sqrt[-a^2 + b^2]))]) + (a\*Sqrt[-a^2 + b^2]\*Sqrt[Sin[e + f\*x]]\*Sqrt[(a\*Sec[(e + f\*x)/2]^2\*(a + b\*Sin[e + f\*x]))/(a^2 - b^2)]\*((a\*(a\*c + (b + Sqrt[-a^2 + b^2])\*(-d + Sqrt[-c^2 + d^2]))\*Sec[(e + f\*x)/2]^2)/(4\*Sqrt[2]\*Sqrt[-a^2 + b^2]\*Sqrt[(b + Sqrt[-a^2 + b^2] + a\*Tan[(e + f\*x)/2])/Sqrt[-a^2 + b^2]]\*Sqrt[1 - (b + Sqrt[-a^2 + b^2] + a\*Tan[(e + f\*x)/2])/(2\*Sqrt[-a^2 + b^2])]\*Sqrt[1 - (b + Sqrt[-a^2 + b^2] + a\*Tan[(e + f\*x)/2])/(b + Sqrt[-a^2 + b^2])]\*(1 - (c\*(b + Sqrt[-a^2 + b^2] + a\*Tan[(e + f\*x)/2]))/(b\*c + Sqrt[-a^2 + b^2]\*c - a\*d + a\*Sqrt[-c^2 + d^2])))) + (a\*(-a\*c) + (b + Sqrt[-a^2 + b^2])\*(d + Sqrt[-c^2 + d^2]))\*Sec[(e + f\*x)/2]^2/(4\*Sqrt[2]\*Sqrt[-a^2 + b^2]\*Sqrt[(b + Sqrt[-a^2 + b^2] + a\*Tan[(e + f\*x)/2])/Sqrt[-a^2 + b^2]]\*Sqrt[1 - (b + Sqrt[-a^2 + b^2] + a\*Tan[(e + f\*x)/2])/(2\*Sqrt[-a^2 + b^2])]\*Sqrt[1 - (b + Sqrt[-a^2 + b^2] + ...

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 2932 vs.  $2(107) = 214$ .

time = 0.35, size = 2933, normalized size = 25.73

method	result	size
default	Expression too large to display	2933

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*sin(f\*x+e))^(1/2)/(c+d\*sin(f\*x+e))/(a+b\*sin(f\*x+e))^(1/2),x,method=\_RETURNVERBOSE)

[Out]  $-1/f*(2*(-c^2+d^2)^{(1/2)}*(-a^2+b^2)^{(1/2)}*EllipticPi((-(\cos(f*x+e)*a-(-a^2+b^2)^{(1/2)}*\sin(f*x+e)-b*\sin(f*x+e)-a)/(b+(-a^2+b^2)^{(1/2)})/\sin(f*x+e))^{(1/2)}$



$$2)^{(1/2)} * c / (a * (-c^2 + d^2)^{(1/2)} - c * (-a^2 + b^2)^{(1/2)} + a * d - b * c), 1/2 * 2^{(1/2)} * ((b + (-a^2 + b^2)^{(1/2)}) / (-a^2 + b^2)^{(1/2)})^{(1/2)} * a^2 * d - \text{EllipticPi}((-\cos(f * x + e) * a - (-a^2 + b^2)^{(1/2)} * \sin(f * x + e) - b * \sin(f * x + e) - a) / (b + (-a^2 + b^2)^{(1/2)}) / \sin(f * x + e))^{(1/2)}, -(b + (-a^2 + b^2)^{(1/2)}) * c / (a * (-c^2 + d^2)^{(1/2)} - c * (-a^2 + b^2)^{(1/2)} + a * d - b * c), 1/2 * 2^{(1/2)} * ((b + (-a^2 + b^2)^{(1/2)}) / (-a^2 + b^2)^{(1/2)})^{(1/2)} * a * b * c + 2 * \text{EllipticPi}((-\cos(f * x + e) * a - (-a^2 + b^2)^{(1/2)} * \sin(f * x + e) - b * \sin(f * x + e) - a) / (b + (-a^2 + b^2)^{(1/2)}) / \sin(f * x + e))^{(1/2)}, -(b + (-a^2 + b^2)^{(1/2)}) * c / (a * (-c^2 + d^2)^{(1/2)} - c * (-a^2 + b^2)^{(1/2)} + a * d - b * c), 1/2 * 2^{(1/2)} * ((b + (-a^2 + b^2)^{(1/2)}) / (-a^2 + b^2)^{(1/2)})^{(1/2)} * b^2 * d * 2^{(1/2)} * (g * \sin(f * x + e))^{(1/2)} * \sin(f * x + e) / (a + b * \sin(f * x + e))^{(1/2)} * (a * (-1 + \cos(f * x + e))) / (b + (-a^2 + b^2)^{(1/2)}) / \sin(f * x + e))^{(1/2)} * ((\cos(f * x + e) * a + (-a^2 + b^2)^{(1/2)} * \sin(f * x + e) - b * \sin(f * x + e) - a) / (-a^2 + b^2)^{(1/2)} / \sin(f * x + e))^{(1/2)} * (-\cos(f * x + e) * a - (-a^2 + b^2)^{(1/2)} * \sin(f * x + e) - b * \sin(f * x + e) - a) / (b + (-a^2 + b^2)^{(1/2)}) / \sin(f * x + e))^{(1/2)} / (-1 + \cos(f * x + e)) * c / (a * (-c^2 + d^2)^{(1/2)} - c * (-a^2 + b^2)^{(1/2)} + a * d - b * c) / (-c^2 + d^2)^{(1/2)} / (a * (-c^2 + d^2)^{(1/2)} + c * (-a^2 + b^2)^{(1/2)} - a * d + b * c)$$

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*sin(f\*x+e))^(1/2)/(c+d\*sin(f\*x+e))/(a+b\*sin(f\*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(g\*sin(f\*x + e))/(sqrt(b\*sin(f\*x + e) + a)\*(d\*sin(f\*x + e) + c)), x)

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*sin(f\*x+e))^(1/2)/(c+d\*sin(f\*x+e))/(a+b\*sin(f\*x+e))^(1/2),x, algorithm="fricas")

[Out] Timed out

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{g \sin(e + fx)}}{\sqrt{a + b \sin(e + fx)} (c + d \sin(e + fx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))/(a+b*sin(f*x+e))**(1/2),x)
[Out] Integral(sqrt(g*sin(e + f*x))/(sqrt(a + b*sin(e + f*x))*(c + d*sin(e + f*x))), x)
```

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))/(a+b*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(g*sin(f*x + e))/(sqrt(b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)), x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{g \sin(e + f x)}}{\sqrt{a + b \sin(e + f x)} (c + d \sin(e + f x))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sin(e + f*x))^(1/2)/((a + b*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))),x)
```

```
[Out] int((g*sin(e + f*x))^(1/2)/((a + b*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))), x)
```

$$3.45 \quad \int \frac{1}{\sqrt{g \sin(e + fx)} \sqrt{a + b \sin(e + fx)} (c + d \sin(e + fx))} dx$$

**Optimal.** Leaf size=246

$$\frac{2\sqrt{a+b} \sqrt{\frac{a(1-\csc(e+fx))}{a+b}} \sqrt{\frac{a(1+\csc(e+fx))}{a-b}} F\left(\sin^{-1}\left(\frac{\sqrt{g} \sqrt{a+b \sin(e+fx)}}{\sqrt{a+b} \sqrt{g \sin(e+fx)}}\right) \middle| -\frac{a+b}{a-b}\right) \tan\left(\frac{2d \tan(e+fx) \sqrt{-\cot^2(e+fx)} \sqrt{g \sin(e+fx)} \sqrt{\frac{a \csc(e+fx)+b}{a+b}} \Pi\left(\frac{2c}{c+d}; \operatorname{ArcSin}\left(\frac{\sqrt{1-\csc(e+fx)}}{\sqrt{2}}\right) \middle| \frac{2a}{a+b}\right) - 2\sqrt{a+b} \tan(e+fx) \sqrt{\frac{a(1-\csc(e+fx))}{a+b}} \sqrt{\frac{a(\csc(e+fx)+1)}{a-b}} F\left(\operatorname{ArcSin}\left(\frac{\sqrt{g} \sqrt{a+b \sin(e+fx)}}{\sqrt{a+b} \sqrt{g \sin(e+fx)}}\right) \middle| -\frac{a+b}{a-b}\right)}{acf \sqrt{g}}$$

[Out]  $-2 \operatorname{EllipticF}(g^{1/2} (a+b \sin(fx+e))^{1/2} / (a+b)^{1/2} / (g \sin(fx+e))^{1/2}, ((-a-b)/(a-b))^{1/2}) * (a+b)^{1/2} * (a(1-\csc(fx+e)) / (a+b))^{1/2} * (a(1+\csc(fx+e)) / (a-b))^{1/2} * \tan(fx+e) / a / c / f / g^{1/2} - 2 * d * \operatorname{EllipticPi}(1/2 * (1-\csc(fx+e))^{1/2} * 2^{1/2}, 2 * c / (c+d), 2^{1/2} * (a/(a+b))^{1/2}) * (-\cot(fx+e)^2)^{1/2} * ((b+a \csc(fx+e)) / (a+b))^{1/2} * (g \sin(fx+e))^{1/2} * \tan(fx+e) / c / (c+d) / f / g / (a+b \sin(fx+e))^{1/2}$

**Rubi [A]**

time = 0.37, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 39,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {3018, 2895, 3016}

$$\frac{2d \tan(e+fx) \sqrt{-\cot^2(e+fx)} \sqrt{g \sin(e+fx)} \sqrt{\frac{a \csc(e+fx)+b}{a+b}} \Pi\left(\frac{2c}{c+d}; \operatorname{ArcSin}\left(\frac{\sqrt{1-\csc(e+fx)}}{\sqrt{2}}\right) \middle| \frac{2a}{a+b}\right) - 2\sqrt{a+b} \tan(e+fx) \sqrt{\frac{a(1-\csc(e+fx))}{a+b}} \sqrt{\frac{a(\csc(e+fx)+1)}{a-b}} F\left(\operatorname{ArcSin}\left(\frac{\sqrt{g} \sqrt{a+b \sin(e+fx)}}{\sqrt{a+b} \sqrt{g \sin(e+fx)}}\right) \middle| -\frac{a+b}{a-b}\right)}{acf \sqrt{g}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[g\*Sin[e + f\*x]]\*Sqrt[a + b\*Sin[e + f\*x]]\*(c + d\*Sin[e + f\*x])), x]

[Out]  $(-2 * \operatorname{Sqrt}[a + b] * \operatorname{Sqrt}[(a * (1 - \operatorname{Csc}[e + f * x])) / (a + b)] * \operatorname{Sqrt}[(a * (1 + \operatorname{Csc}[e + f * x])) / (a - b)] * \operatorname{EllipticF}[\operatorname{ArcSin}[(\operatorname{Sqrt}[g] * \operatorname{Sqrt}[a + b * \operatorname{Sin}[e + f * x]]) / (\operatorname{Sqrt}[a + b] * \operatorname{Sqrt}[g * \operatorname{Sin}[e + f * x]])], -(a + b) / (a - b)] * \operatorname{Tan}[e + f * x] / (a * c * f * \operatorname{Sqrt}[g]) - (2 * d * \operatorname{Sqrt}[-\operatorname{Cot}[e + f * x]^2] * \operatorname{Sqrt}[(b + a * \operatorname{Csc}[e + f * x]) / (a + b)] * \operatorname{EllipticPi}[(2 * c) / (c + d), \operatorname{ArcSin}[\operatorname{Sqrt}[1 - \operatorname{Csc}[e + f * x]] / \operatorname{Sqrt}[2]], (2 * a) / (a + b)] * \operatorname{Sqrt}[g * \operatorname{Sin}[e + f * x]] * \operatorname{Tan}[e + f * x]) / (c * (c + d) * f * g * \operatorname{Sqrt}[a + b * \operatorname{Sin}[e + f * x]])$

Rule 2895

Int[1/(Sqrt[(d\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)]]\*Sqrt[(a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)]]), x\_Symbol] :> Simp[-2\*(Tan[e + f\*x]/(a\*f))\*Rt[(a + b)/d, 2]\*Sqrt[a\*((1 - Csc[e + f\*x])/(a + b))]\*Sqrt[a\*((1 + Csc[e + f\*x])/(a - b))]\*EllipticF[ArcSin[Sqrt[a + b\*Sin[e + f\*x]]/Sqrt[d\*Sin[e + f\*x]]/Rt[(a + b)/d, 2]], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && PosQ[(a + b)/d]

Rule 3016

```
Int[Sqrt[(g_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[2*Sqrt[
-Cot[e + f*x]^2]*(Sqrt[g*Sin[e + f*x]]/(f*(c + d)*Cot[e + f*x]*Sqrt[a + b*S
in[e + f*x]]))*Sqrt[(b + a*Csc[e + f*x])/(a + b)]*EllipticPi[2*(c/(c + d)),
ArcSin[Sqrt[1 - Csc[e + f*x]]/Sqrt[2]], 2*(a/(a + b))], x] /; FreeQ[{a, b,
c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0]
```

### Rule 3018

```
Int[1/(Sqrt[(g_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[1/c,
Int[1/(Sqrt[g*Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]]), x], x] - Dist[d/(c*g
), Int[Sqrt[g*Sin[e + f*x]]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x]))
, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
- b^2, 0] && NeQ[c^2 - d^2, 0]
```

### Rubi steps

$$\int \frac{1}{\sqrt{g \sin(e + fx)} \sqrt{a + b \sin(e + fx)} (c + d \sin(e + fx))} dx = \frac{\int \frac{1}{\sqrt{g \sin(e + fx)} \sqrt{a + b \sin(e + fx)}} dx}{c}$$

$$= -\frac{2\sqrt{a+b} \sqrt{\frac{a(1 - \csc(e + fx))}{a+b}} \sqrt{\frac{a(1 + \csc(e + fx))}{a+b}}}{c}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 4935 vs. 2(246) = 492.

time = 49.28, size = 4935, normalized size = 20.06

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[g*Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f
*x])), x]
```

```
[Out] (-4*Sqrt[-a^2 + b^2]*Cos[(e + f*x)/2]^4*(-2*(b + Sqrt[-a^2 + b^2])*(b*c - a
*d)*Sqrt[-c^2 + d^2]*EllipticF[ArcSin[Sqrt[(b + Sqrt[-a^2 + b^2]) + a*Tan[(e
+ f*x)/2]]/Sqrt[-a^2 + b^2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(b + Sqrt[-a^2
+ b^2])) - a*d*((a*c + (b + Sqrt[-a^2 + b^2])*(-d + Sqrt[-c^2 + d^2]))*Ell
ipticPi[(2*Sqrt[-a^2 + b^2]*c)/(b*c + Sqrt[-a^2 + b^2]*c - a*d + a*Sqrt[-c^
```

$$\begin{aligned}
& 2 + d^2]), \text{ArcSin}[\text{Sqrt}[(b + \text{Sqrt}[-a^2 + b^2] + a*\text{Tan}[(e + f*x)/2])/\text{Sqrt}[-a^2 + b^2]]/\text{Sqrt}[2]], (2*\text{Sqrt}[-a^2 + b^2])/(b + \text{Sqrt}[-a^2 + b^2]) + (-(a*c) \\
& + (b + \text{Sqrt}[-a^2 + b^2])*(d + \text{Sqrt}[-c^2 + d^2]))*\text{EllipticPi}[(2*\text{Sqrt}[-a^2 + b^2]*c)/(b*c + \text{Sqrt}[-a^2 + b^2]*c - a*(d + \text{Sqrt}[-c^2 + d^2])), \text{ArcSin}[\text{Sqrt}[(b + \text{Sqrt}[-a^2 + b^2] + a*\text{Tan}[(e + f*x)/2])/\text{Sqrt}[-a^2 + b^2]]/\text{Sqrt}[2]], (2* \\
& \text{Sqrt}[-a^2 + b^2])/(b + \text{Sqrt}[-a^2 + b^2]))*\text{Sqrt}[(a*\text{Sec}[(e + f*x)/2]^2*(a + b*\text{Sin}[e + f*x]))/(a^2 - b^2)]*(-((a*\text{Tan}[(e + f*x)/2])/(b + \text{Sqrt}[-a^2 + b^2] \\
& ]))^(3/2))/(a^2*c*(-(b*c) + a*d)*\text{Sqrt}[-c^2 + d^2]*f*\text{Sin}[e + f*x]^(3/2)*\text{Sqr} \\
& \text{t}[g*\text{Sin}[e + f*x]]*(a + b*\text{Sin}[e + f*x])*(c + d*\text{Sin}[e + f*x])*((3*\text{Sqrt}[-a^2 + b^2]*\text{Cos}[(e + f*x)/2]^2*(-2*(b + \text{Sqrt}[-a^2 + b^2])*(b*c - a*d)*\text{Sqrt}[-c^2 + d^2]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[(b + \text{Sqrt}[-a^2 + b^2] + a*\text{Tan}[(e + f*x)/2])/\text{Sqr} \\
& \text{t}[-a^2 + b^2]]/\text{Sqrt}[2]], (2*\text{Sqrt}[-a^2 + b^2])/(b + \text{Sqrt}[-a^2 + b^2])) - a*d \\
& *((a*c + (b + \text{Sqrt}[-a^2 + b^2])*(-d + \text{Sqrt}[-c^2 + d^2]))*\text{EllipticPi}[(2*\text{Sqrt}[-a^2 + b^2]*c)/(b*c + \text{Sqrt}[-a^2 + b^2]*c - a*d + a*\text{Sqrt}[-c^2 + d^2])), \text{ArcS} \\
& \text{in}[\text{Sqrt}[(b + \text{Sqrt}[-a^2 + b^2] + a*\text{Tan}[(e + f*x)/2])/\text{Sqrt}[-a^2 + b^2]]/\text{Sqrt}[2]], (2*\text{Sqrt}[-a^2 + b^2])/(b + \text{Sqrt}[-a^2 + b^2]) + (-(a*c) + (b + \text{Sqrt}[-a^2 + b^2])*(d + \text{Sqrt}[-c^2 + d^2]))*\text{EllipticPi}[(2*\text{Sqrt}[-a^2 + b^2]*c)/(b*c + \text{Sqrt}[-a^2 + b^2]*c - a*(d + \text{Sqrt}[-c^2 + d^2])), \text{ArcSin}[\text{Sqrt}[(b + \text{Sqrt}[-a^2 + b^2] + a*\text{Tan}[(e + f*x)/2])/\text{Sqrt}[-a^2 + b^2]]/\text{Sqrt}[2]], (2*\text{Sqrt}[-a^2 + b^2])/(b + \text{Sqrt}[-a^2 + b^2]))*\text{Sqrt}[(a*\text{Sec}[(e + f*x)/2]^2*(a + b*\text{Sin}[e + f*x] \\
& ))/(a^2 - b^2)]*\text{Sqrt}[-((a*\text{Tan}[(e + f*x)/2])/(b + \text{Sqrt}[-a^2 + b^2]))]/(a*(b + \text{Sqrt}[-a^2 + b^2])*c*(-(b*c) + a*d)*\text{Sqrt}[-c^2 + d^2]*\text{Sin}[e + f*x]^(3/2)*\text{S} \\
& \text{qrt}[a + b*\text{Sin}[e + f*x]]) + (2*b*\text{Sqrt}[-a^2 + b^2]*\text{Cos}[(e + f*x)/2]^4*\text{Cos}[e + f*x]*(-2*(b + \text{Sqrt}[-a^2 + b^2])*(b*c - a*d)*\text{Sqrt}[-c^2 + d^2]*\text{EllipticF}[\text{Arc} \\
& \text{Sin}[\text{Sqrt}[(b + \text{Sqrt}[-a^2 + b^2] + a*\text{Tan}[(e + f*x)/2])/\text{Sqrt}[-a^2 + b^2]]/\text{Sqrt}[2]], (2*\text{Sqrt}[-a^2 + b^2])/(b + \text{Sqrt}[-a^2 + b^2])) - a*d*((a*c + (b + \text{Sqrt}[-a^2 + b^2])*(-d + \text{Sqrt}[-c^2 + d^2]))*\text{EllipticPi}[(2*\text{Sqrt}[-a^2 + b^2]*c)/(b*c + \text{Sqrt}[-a^2 + b^2]*c - a*d + a*\text{Sqrt}[-c^2 + d^2])), \text{ArcSin}[\text{Sqrt}[(b + \text{Sqrt}[-a^2 + b^2] + a*\text{Tan}[(e + f*x)/2])/\text{Sqrt}[-a^2 + b^2]]/\text{Sqrt}[2]], (2*\text{Sqrt}[-a^2 + b^2])/(b + \text{Sqrt}[-a^2 + b^2])) + (-(a*c) + (b + \text{Sqrt}[-a^2 + b^2])*(d + \text{Sqrt}[-c^2 + d^2]))*\text{EllipticPi}[(2*\text{Sqrt}[-a^2 + b^2]*c)/(b*c + \text{Sqrt}[-a^2 + b^2]*c - a*(d + \text{Sqrt}[-c^2 + d^2])), \text{ArcSin}[\text{Sqrt}[(b + \text{Sqrt}[-a^2 + b^2] + a*\text{Tan}[(e + f*x)/2])/\text{Sqrt}[-a^2 + b^2]]/\text{Sqrt}[2]], (2*\text{Sqrt}[-a^2 + b^2])/(b + \text{Sqrt}[-a^2 + b^2]))*\text{Sqrt}[(a*\text{Sec}[(e + f*x)/2]^2*(a + b*\text{Sin}[e + f*x]))/(a^2 - b^2)]*(-( \\
& (a*\text{Tan}[(e + f*x)/2])/(b + \text{Sqrt}[-a^2 + b^2]))^(3/2))/(a^2*c*(-(b*c) + a*d)* \\
& \text{Sqrt}[-c^2 + d^2]*\text{Sin}[e + f*x]^(3/2)*(a + b*\text{Sin}[e + f*x])^(3/2) + (6*\text{Sqrt}[-a^2 + b^2]*\text{Cos}[(e + f*x)/2]^4*\text{Cos}[e + f*x]*(-2*(b + \text{Sqrt}[-a^2 + b^2])*(b*c - a*d)*\text{Sqrt}[-c^2 + d^2]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[(b + \text{Sqrt}[-a^2 + b^2] + a*\text{Tan}[(e + f*x)/2])/\text{Sqrt}[-a^2 + b^2]]/\text{Sqrt}[2]], (2*\text{Sqrt}[-a^2 + b^2])/(b + \text{Sqrt}[-a^2 + b^2])) - a*d*((a*c + (b + \text{Sqrt}[-a^2 + b^2])*(-d + \text{Sqrt}[-c^2 + d^2]))*\text{EllipticPi}[(2*\text{Sqrt}[-a^2 + b^2]*c)/(b*c + \text{Sqrt}[-a^2 + b^2]*c - a*d + a*\text{Sqrt}[-c^2 + d^2])), \text{ArcSin}[\text{Sqrt}[(b + \text{Sqrt}[-a^2 + b^2] + a*\text{Tan}[(e + f*x)/2])/\text{Sqrt}[-a^2 + b^2]]/\text{Sqrt}[2]], (2*\text{Sqrt}[-a^2 + b^2])/(b + \text{Sqrt}[-a^2 + b^2])) + (-(a*c) + (b + \text{Sqrt}[-a^2 + b^2])*(d + \text{Sqrt}[-c^2 + d^2]))*\text{EllipticPi}[(2*\text{Sqrt}[-a^2 + b^2]*c)/(b*c + \text{Sqrt}[-a^2 + b^2]*c - a*(d + \text{Sqrt}[-c^2 + d^2])), \text{ArcSin}[\text{Sqr}
\end{aligned}$$

```

rt[(b + Sqrt[-a^2 + b^2] + a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt[2]],
(2*Sqrt[-a^2 + b^2])/(b + Sqrt[-a^2 + b^2]))*Sqrt[(a*Sec[(e + f*x)/2]^2*(
a + b*Sin[e + f*x]))/(a^2 - b^2)]*(-((a*Tan[(e + f*x)/2])/(b + Sqrt[-a^2 +
b^2]))^(3/2))/(a^2*c*(-(b*c) + a*d)*Sqrt[-c^2 + d^2]*Sin[e + f*x]^(5/2)*Sq
rt[a + b*Sin[e + f*x]]) + (8*Sqrt[-a^2 + b^2]*Cos[(e + f*x)/2]^3*(-2*(b + S
qrt[-a^2 + b^2])*(b*c - a*d)*Sqrt[-c^2 + d^2]*EllipticF[ArcSin[Sqrt[(b + Sq
rt[-a^2 + b^2] + a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt[2]], (2*Sqrt[-a
^2 + b^2])/(b + Sqrt[-a^2 + b^2]) - a*d*((a*c + (b + Sqrt[-a^2 + b^2])*(-d
+ Sqrt[-c^2 + d^2]))*EllipticPi[(2*Sqrt[-a^2 + b^2]*c)/(b*c + Sqrt[-a^2 +
b^2]*c - a*d + a*Sqrt[-c^2 + d^2]), ArcSin[Sqrt[(b + Sqrt[-a^2 + b^2] + a*T
an[(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(b + Sqrt
[-a^2 + b^2])) + (-a*c) + (b + Sqrt[-a^2 + b^2])*(d + Sqrt[-c^2 + d^2]))*E
llipticPi[(2*Sqrt[-a^2 + b^2]*c)/(b*c + Sqrt[-a^2 + b^2]*c - a*(d + Sqrt[-c
^2 + d^2])), ArcSin[Sqrt[(b + Sqrt[-a^2 + b^2] + a*Tan[(e + f*x)/2])/Sqrt[-
a^2 + b^2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(b + Sqrt[-a^2 + b^2])))*Sin[(e
+ f*x)/2]*Sqrt[(a*Sec[(e + f*x)/2]^2*(a + b*Sin[e + f*x]))/(a^2 - b^2)]*(-
((a*Tan[(e + f*x)/2])/(b + Sqrt[-a^2 + b^2]))^...

```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 3697 vs.  $2(227) = 454$ .

time = 0.29, size = 3698, normalized size = 15.03

method	result	size
default	Expression too large to display	3698

Verification of antiderivative is not currently implemented for this CAS.

```

[In] int(1/(c+d*sin(f*x+e))/(g*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(1/2),x,method
=_RETURNVERBOSE)

```

```

[Out] -1/f/(a+b*sin(f*x+e))^(1/2)*(4*EllipticF((-cos(f*x+e)*a-(-a^2+b^2)^(1/2))*s
in(f*x+e)-b*sin(f*x+e)-a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2),1/2*2^(1/2
))*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*a*b*d*(-a^2+b^2)^(1/2)*(-c
^2+d^2)^(1/2)-4*EllipticF((-cos(f*x+e)*a-(-a^2+b^2)^(1/2))*sin(f*x+e)-b*sin
(f*x+e)-a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2),1/2*2^(1/2))*((b+(-a^2+b^2
)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*b^2*c*(-a^2+b^2)^(1/2)*(-c^2+d^2)^(1/2)-2
*EllipticF((-cos(f*x+e)*a-(-a^2+b^2)^(1/2))*sin(f*x+e)-b*sin(f*x+e)-a)/(b+(
-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2),1/2*2^(1/2))*((b+(-a^2+b^2)^(1/2))/(-a^2+
b^2)^(1/2))^(1/2))*a^3*d*(-c^2+d^2)^(1/2)+2*EllipticF((-cos(f*x+e)*a-(-a^2
+b^2)^(1/2))*sin(f*x+e)-b*sin(f*x+e)-a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/
2),1/2*2^(1/2))*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*a^2*b*c*(-c^2
+d^2)^(1/2)+4*EllipticF((-cos(f*x+e)*a-(-a^2+b^2)^(1/2))*sin(f*x+e)-b*sin(f
*x+e)-a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2),1/2*2^(1/2))*((b+(-a^2+b^2)
^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*a*b^2*d*(-c^2+d^2)^(1/2)-4*EllipticF((-cos
(f*x+e)*a-(-a^2+b^2)^(1/2))*sin(f*x+e)-b*sin(f*x+e)-a)/(b+(-a^2+b^2)^(1/2))/
sin(f*x+e))^(1/2),1/2*2^(1/2))*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2)

```





```
a*d-b*c),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*a*b*d^2
*(-a^2+b^2)^(1/2)+EllipticPi((-cos(f*x+e)*a-(-a^2+b^2)^(1/2)*sin(f*x+e)-b*
sin(f*x+e)-a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2),-(b+(-a^2+b^2)^(1/2))*
c/(a*(-c^2+d^2)^(1/2)-c*(-a^2+b^2)^(1/2)+a*d-b*c),1/2*2^(1/2)*((b+(-a^2+b^2
)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*a^3*d^2+EllipticPi((-cos(f*x+e)*a-(-a^2+
b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)-a)/(b+(-a^2+...
```

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*sin(f*x+e))/(g*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(1/2),x,
algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)*sqrt(g*sin(f*x +
e))), x)
```

**Fricas [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*sin(f*x+e))/(g*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(1/2),x,
algorithm="fricas")
```

```
[Out] integral(-sqrt(b*sin(f*x + e) + a)*sqrt(g*sin(f*x + e))/((b*c + a*d)*g*cos(
f*x + e)^2 - (b*c + a*d)*g + (b*d*g*cos(f*x + e)^2 - (a*c + b*d)*g)*sin(f*x
+ e)), x)
```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{g \sin(e + fx)} \sqrt{a + b \sin(e + fx)} (c + d \sin(e + fx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*sin(f*x+e))/(g*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(1/2),
x)
```

```
[Out] Integral(1/(sqrt(g*sin(e + f*x))*sqrt(a + b*sin(e + f*x))*(c + d*sin(e + f*
x))), x)
```

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*sin(f*x+e))/(g*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(1/2),x,
algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)*sqrt(g*sin(f*x +
e))), x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{g \sin(e + f x)} \sqrt{a + b \sin(e + f x)} (c + d \sin(e + f x))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((g*sin(e + f*x))^(1/2)*(a + b*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x
))),x)
```

```
[Out] int(1/((g*sin(e + f*x))^(1/2)*(a + b*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x
))), x)
```

$$3.46 \quad \int \frac{\sqrt{g \sin(e + fx)} \sqrt{c + d \sin(e + fx)}}{a + b \sin(e + fx)} dx$$

Optimal. Leaf size=254

$$\frac{2\sqrt{c+d} \sqrt{g} \sqrt{\frac{c(1 - \csc(e + fx))}{c+d}} \sqrt{\frac{c(1 + \csc(e + fx))}{c-d}} \Pi\left(\frac{c+d}{d}; \sin^{-1}\left(\frac{\sqrt{g} \sqrt{c+d \sin(e + fx)}}{\sqrt{c+d} \sqrt{g \sin(e + fx)}}\right) \mid -\frac{c+d}{c-d}\right)}{bf}$$

[Out] 2\*EllipticPi(g^(1/2)\*(c+d\*sin(f\*x+e))^(1/2)/(c+d)^(1/2)/(g\*sin(f\*x+e))^(1/2), (c+d)/d, ((-c-d)/(c-d))^(1/2)\*(c+d)^(1/2)\*g^(1/2)\*(c\*(1-csc(f\*x+e))/(c+d))^(1/2)\*(c\*(1+csc(f\*x+e))/(c-d))^(1/2)\*tan(f\*x+e)/b/f+2\*(-a\*d+b\*c)\*EllipticPi(1/2\*(1-csc(f\*x+e))^(1/2)\*2^(1/2), 2\*a/(a+b), 2^(1/2)\*(c/(c+d))^(1/2))\*(-cot(f\*x+e)^2)^(1/2)\*((d+c\*csc(f\*x+e))/(c+d))^(1/2)\*(g\*sin(f\*x+e))^(1/2)\*tan(f\*x+e)/b/(a+b)/f/(c+d\*sin(f\*x+e))^(1/2)

Rubi [A]

time = 0.34, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 39,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {3008, 2888, 3016}

$$\frac{2(bc-ad)\tan(e+fx)\sqrt{-\cot^2(e+fx)}\sqrt{g\sin(e+fx)}\sqrt{\frac{c\csc(e+fx)+d}{c+d}}\Pi\left(\frac{2c}{c+d}; \text{ArcSin}\left(\frac{\sqrt{1-\csc(e+fx)}}{\sqrt{2}}\right) \mid \frac{2c}{c+d}\right)}{bf(a+b)\sqrt{c+d\sin(e+fx)}} + \frac{2\sqrt{g}\sqrt{c+d}\tan(e+fx)\sqrt{\frac{c(1-\csc(e+fx))}{c+d}}\sqrt{\frac{c(\csc(e+fx)+1)}{c-d}}\Pi\left(\frac{c+d}{d}; \text{ArcSin}\left(\frac{\sqrt{g}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{g\sin(e+fx)}}\right) \mid -\frac{c+d}{c-d}\right)}{bf}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[g\*Sin[e + f\*x]]\*Sqrt[c + d\*Sin[e + f\*x]])/(a + b\*Sin[e + f\*x]),x]

[Out] (2\*Sqrt[c + d]\*Sqrt[g]\*Sqrt[(c\*(1 - Csc[e + f\*x]))/(c + d)]\*Sqrt[(c\*(1 + Csc[e + f\*x]))/(c - d)]\*EllipticPi[(c + d)/d, ArcSin[(Sqrt[g]\*Sqrt[c + d\*Sin[e + f\*x]])/(Sqrt[c + d]\*Sqrt[g\*Sin[e + f\*x]])], -(c + d)/(c - d)]\*Tan[e + f\*x]/(b\*f) + (2\*(b\*c - a\*d)\*Sqrt[-Cot[e + f\*x]^2]\*Sqrt[(d + c\*Csc[e + f\*x])/(c + d)]\*EllipticPi[(2\*a)/(a + b), ArcSin[Sqrt[1 - Csc[e + f\*x]]/Sqrt[2]], (2\*c)/(c + d)]\*Sqrt[g\*Sin[e + f\*x]]\*Tan[e + f\*x]/(b\*(a + b)\*f\*Sqrt[c + d\*Sin[e + f\*x]])

Rule 2888

Int[Sqrt[(b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)]]/Sqrt[(c\_) + (d\_.)\*sin[(e\_.) + (f\_.)\*(x\_)]], x\_Symbol] :> Simp[2\*b\*(Tan[e + f\*x]/(d\*f))\*Rt[(c + d)/b, 2]\*Sqrt[c\*((1 + Csc[e + f\*x])/(c - d))]\*Sqrt[c\*((1 - Csc[e + f\*x])/(c + d))]\*EllipticPi[(c + d)/d, ArcSin[Sqrt[c + d\*Sin[e + f\*x]]/Sqrt[b\*Sin[e + f\*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/b]

Rule 3008

```
Int[(Sqrt[(g_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)])]/((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[b/d, In
t[Sqrt[g*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]], x], x] - Dist[(b*c - a*d)/
d, Int[Sqrt[g*Sin[e + f*x]]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x]))
, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
- b^2, 0] && NeQ[c^2 - d^2, 0]
```

### Rule 3016

```
Int[Sqrt[(g_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] := Simp[2*Sqrt[
-Cot[e + f*x]^2]*(Sqrt[g*Sin[e + f*x]]/(f*(c + d)*Cot[e + f*x]*Sqrt[a + b*S
in[e + f*x]))*Sqrt[(b + a*Csc[e + f*x])/(a + b)]*EllipticPi[2*(c/(c + d)),
ArcSin[Sqrt[1 - Csc[e + f*x]]/Sqrt[2]], 2*(a/(a + b))], x] /; FreeQ[{a, b,
c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0]
```

### Rubi steps

$$\int \frac{\sqrt{g \sin(e + fx)} \sqrt{c + d \sin(e + fx)}}{a + b \sin(e + fx)} dx = \frac{d \int \frac{\sqrt{g \sin(e + fx)}}{\sqrt{c + d \sin(e + fx)}} dx}{b} - \frac{(-bc + ad) \int \frac{\sqrt{g \sin(e + fx)}}{(a + b \sin(e + fx)) \sqrt{c + d \sin(e + fx)}} dx}{b}$$

$$= \frac{2\sqrt{c+d} \sqrt{g} \sqrt{\frac{c(1 - \csc(e + fx))}{c + d}} \sqrt{\frac{c(1 + \csc(e + fx))}{c - d}} \Pi}{b}$$

**Mathematica** [C] Result contains complex when optimal does not.  
time = 39.50, size = 23019, normalized size = 90.63

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[g*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])/(a + b*Sin[e + f*
x]), x]
```

```
[Out] Result too large to show
```

**Maple** [C] Result contains complex when optimal does not.  
time = 0.72, size = 6200, normalized size = 24.41

method	result	size
--------	--------	------

default	Expression too large to display	6200
---------	---------------------------------	------

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(1/2)*(g*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x,method=_
RETURNVERBOSE)
```

```
[Out] result too large to display
```

**Maxima [F]**

```
time = 0.00, size = 0, normalized size = 0.00
```

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)*(g*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, a
lgorithm="maxima")
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)*sqrt(g*sin(f*x + e))/(b*sin(f*x + e) + a
), x)
```

**Fricas [F(-1)]** Timed out

```
time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)*(g*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

**Sympy [F]**

```
time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{\sqrt{g \sin(e + fx)} \sqrt{c + d \sin(e + fx)}}{a + b \sin(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(1/2)*(g*sin(f*x+e))**(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Integral(sqrt(g*sin(e + f*x))*sqrt(c + d*sin(e + f*x))/(a + b*sin(e + f*x))
, x)
```

**Giac [F]**

```
time = 0.00, size = 0, normalized size = 0.00
```

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)*(g*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)*sqrt(g*sin(f*x + e))/(b*sin(f*x + e) + a), x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{g \sin(e + f x)} \sqrt{c + d \sin(e + f x)}}{a + b \sin(e + f x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^(1/2))/(a + b*sin(e + f*x)),x)
```

```
[Out] int(((g*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^(1/2))/(a + b*sin(e + f*x)), x)
```

$$3.47 \quad \int \frac{\sqrt{g \sin(e + fx)}}{(a + b \sin(e + fx)) \sqrt{c + d \sin(e + fx)}} dx$$

**Optimal.** Leaf size=114

$$\frac{2\sqrt{-\cot^2(e + fx)} \sqrt{\frac{d + c \csc(e + fx)}{c + d}} \Pi\left(\frac{2a}{a+b}; \sin^{-1}\left(\frac{\sqrt{1 - \csc(e + fx)}}{\sqrt{2}}\right) \middle| \frac{2c}{c+d}\right) \sqrt{g \sin(e + fx)} \tan(e + fx)}{(a + b)f \sqrt{c + d \sin(e + fx)}}$$

[Out] 2\*EllipticPi(1/2\*(1-csc(f\*x+e))^(1/2)\*2^(1/2), 2\*a/(a+b), 2^(1/2)\*(c/(c+d))^(1/2))\*(-cot(f\*x+e)^2)^(1/2)\*((d+c\*csc(f\*x+e))/(c+d))^(1/2)\*(g\*sin(f\*x+e))^(1/2)\*tan(f\*x+e)/(a+b)/f/(c+d\*sin(f\*x+e))^(1/2)

**Rubi [A]**

time = 0.14, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 39,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$ , Rules used = {3016}

$$\frac{2 \tan(e + fx) \sqrt{-\cot^2(e + fx)} \sqrt{g \sin(e + fx)} \sqrt{\frac{c \csc(e + fx) + d}{c + d}} \Pi\left(\frac{2a}{a+b}; \text{ArcSin}\left(\frac{\sqrt{1 - \csc(e + fx)}}{\sqrt{2}}\right) \middle| \frac{2c}{c+d}\right)}{f(a + b) \sqrt{c + d \sin(e + fx)}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[g\*Sin[e + f\*x]]/((a + b\*Sin[e + f\*x])\*Sqrt[c + d\*Sin[e + f\*x]]),x]

[Out] (2\*Sqrt[-Cot[e + f\*x]^2]\*Sqrt[(d + c\*Csc[e + f\*x))/(c + d]]\*EllipticPi[(2\*a)/(a + b), ArcSin[Sqrt[1 - Csc[e + f\*x]]/Sqrt[2]], (2\*c)/(c + d)]\*Sqrt[g\*Sin[e + f\*x]]\*Tan[e + f\*x])/((a + b)\*f\*Sqrt[c + d\*Sin[e + f\*x]])

Rule 3016

Int[Sqrt[(g\_.)\*sin[(e\_.) + (f\_.)\*(x\_)]]/(Sqrt[(a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])], x\_Symbol] :> Simp[2\*Sqrt[-Cot[e + f\*x]^2]\*(Sqrt[g\*Sin[e + f\*x]]/(f\*(c + d)\*Cot[e + f\*x]\*Sqrt[a + b\*Sin[e + f\*x]]))\*Sqrt[(b + a\*Csc[e + f\*x])/(a + b)]\*EllipticPi[2\*(c/(c + d)), ArcSin[Sqrt[1 - Csc[e + f\*x]]/Sqrt[2]], 2\*(a/(a + b))], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rubi steps

$$\int \frac{\sqrt{g \sin(e + fx)}}{(a + b \sin(e + fx)) \sqrt{c + d \sin(e + fx)}} dx = \frac{2\sqrt{-\cot^2(e + fx)} \sqrt{\frac{d + c \csc(e + fx)}{c + d}} \Pi\left(\frac{2a}{a+b}; \sin^{-1}\left(\frac{\sqrt{1 - \csc(e + fx)}}{\sqrt{2}}\right) \middle| \frac{2c}{c+d}\right) \sqrt{g \sin(e + fx)} \tan(e + fx)}{(a + b)f \sqrt{c + d \sin(e + fx)}}$$



**Mathematica [B]** Leaf count is larger than twice the leaf count of optimal. 3429 vs. 2(114) = 228.

time = 47.48, size = 3429, normalized size = 30.08

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[g*Sin[e + f*x]]/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x
]]) ,x]
```

```
[Out] -((c*Sqrt[-c^2 + d^2]*((-a*c) + (b + Sqrt[-a^2 + b^2])*(d + Sqrt[-c^2 + d^
2]))*EllipticPi[(2*a*Sqrt[-c^2 + d^2])/(-b*c) - Sqrt[-a^2 + b^2]*c + a*(d
+ Sqrt[-c^2 + d^2])], ArcSin[Sqrt[(d + Sqrt[-c^2 + d^2] + c*Tan[(e + f*x)/2
])/Sqrt[-c^2 + d^2]]/Sqrt[2]], (2*Sqrt[-c^2 + d^2])/(d + Sqrt[-c^2 + d^2]))
+ (a*c + (-b + Sqrt[-a^2 + b^2])*(d + Sqrt[-c^2 + d^2]))*EllipticPi[(2*a*S
qrt[-c^2 + d^2])/(-b*c) + Sqrt[-a^2 + b^2]*c + a*(d + Sqrt[-c^2 + d^2])],
ArcSin[Sqrt[(d + Sqrt[-c^2 + d^2] + c*Tan[(e + f*x)/2])/Sqrt[-c^2 + d^2]]/S
qrt[2]], (2*Sqrt[-c^2 + d^2])/(d + Sqrt[-c^2 + d^2]))*Sqrt[Sin[e + f*x]]*S
qrt[g*Sin[e + f*x]]*Sqrt[(c*Sec[(e + f*x)/2]^2*(c + d*Sin[e + f*x]))/(c^2 -
d^2)]/(Sqrt[-a^2 + b^2]*(b*c - a*d)*(d + Sqrt[-c^2 + d^2])^2*f*(a + b*Sin
[e + f*x])*(c + d*Sin[e + f*x])*Sqrt[-((c*Tan[(e + f*x)/2])/(d + Sqrt[-c^2
+ d^2]))]*(-1/4*(c^2*Sqrt[-c^2 + d^2]*((-a*c) + (b + Sqrt[-a^2 + b^2])*(d
+ Sqrt[-c^2 + d^2]))*EllipticPi[(2*a*Sqrt[-c^2 + d^2])/(-b*c) - Sqrt[-a^2
+ b^2]*c + a*(d + Sqrt[-c^2 + d^2])], ArcSin[Sqrt[(d + Sqrt[-c^2 + d^2] + c
*Tan[(e + f*x)/2])/Sqrt[-c^2 + d^2]]/Sqrt[2]], (2*Sqrt[-c^2 + d^2])/(d + Sq
rt[-c^2 + d^2])) + (a*c + (-b + Sqrt[-a^2 + b^2])*(d + Sqrt[-c^2 + d^2]))*E
llipticPi[(2*a*Sqrt[-c^2 + d^2])/(-b*c) + Sqrt[-a^2 + b^2]*c + a*(d + Sqrt
[-c^2 + d^2])], ArcSin[Sqrt[(d + Sqrt[-c^2 + d^2] + c*Tan[(e + f*x)/2])/Sqr
t[-c^2 + d^2]]/Sqrt[2]], (2*Sqrt[-c^2 + d^2])/(d + Sqrt[-c^2 + d^2]))*Sec[
(e + f*x)/2]^2*Sqrt[Sin[e + f*x]]*Sqrt[(c*Sec[(e + f*x)/2]^2*(c + d*Sin[e +
f*x]))/(c^2 - d^2)]/(Sqrt[-a^2 + b^2]*(b*c - a*d)*(d + Sqrt[-c^2 + d^2])^
3*Sqrt[c + d*Sin[e + f*x]]*(-((c*Tan[(e + f*x)/2])/(d + Sqrt[-c^2 + d^2]))
^(3/2)) + (c*d*Sqrt[-c^2 + d^2]*Cos[e + f*x]*((-a*c) + (b + Sqrt[-a^2 + b^
2])*(d + Sqrt[-c^2 + d^2]))*EllipticPi[(2*a*Sqrt[-c^2 + d^2])/(-b*c) - Sqr
t[-a^2 + b^2]*c + a*(d + Sqrt[-c^2 + d^2])], ArcSin[Sqrt[(d + Sqrt[-c^2 + d
^2] + c*Tan[(e + f*x)/2])/Sqrt[-c^2 + d^2]]/Sqrt[2]], (2*Sqrt[-c^2 + d^2])/
(d + Sqrt[-c^2 + d^2])) + (a*c + (-b + Sqrt[-a^2 + b^2])*(d + Sqrt[-c^2 + d
^2]))*EllipticPi[(2*a*Sqrt[-c^2 + d^2])/(-b*c) + Sqrt[-a^2 + b^2]*c + a*(d
+ Sqrt[-c^2 + d^2])], ArcSin[Sqrt[(d + Sqrt[-c^2 + d^2] + c*Tan[(e + f*x)/
2])/Sqrt[-c^2 + d^2]]/Sqrt[2]], (2*Sqrt[-c^2 + d^2])/(d + Sqrt[-c^2 + d^2]
)]*Sqrt[Sin[e + f*x]]*Sqrt[(c*Sec[(e + f*x)/2]^2*(c + d*Sin[e + f*x]))/(c^2
- d^2)]/(2*Sqrt[-a^2 + b^2]*(b*c - a*d)*(d + Sqrt[-c^2 + d^2])^2*(c + d*S
in[e + f*x])^(3/2)*Sqrt[-((c*Tan[(e + f*x)/2])/(d + Sqrt[-c^2 + d^2]))] -
(c*Sqrt[-c^2 + d^2]*Cos[e + f*x]*((-a*c) + (b + Sqrt[-a^2 + b^2])*(d + Sqr
t[-c^2 + d^2]))*EllipticPi[(2*a*Sqrt[-c^2 + d^2])/(-b*c) - Sqrt[-a^2 + b^2]
```

$$] * c + a * (d + \sqrt{-c^2 + d^2})) , \text{ArcSin}[\sqrt{(d + \sqrt{-c^2 + d^2} + c * \tan((e + f * x) / 2)) / \sqrt{-c^2 + d^2}}] / \sqrt{2}] , (2 * \sqrt{-c^2 + d^2}) / (d + \sqrt{-c^2 + d^2})] + (a * c + (-b + \sqrt{-a^2 + b^2})) * (d + \sqrt{-c^2 + d^2})) * \text{EllipticPi}[(2 * a * \sqrt{-c^2 + d^2}) / (- (b * c) + \sqrt{-a^2 + b^2} * c + a * (d + \sqrt{-c^2 + d^2}))] , \text{ArcSin}[\sqrt{(d + \sqrt{-c^2 + d^2} + c * \tan((e + f * x) / 2)) / \sqrt{-c^2 + d^2}}] / \sqrt{2}] , (2 * \sqrt{-c^2 + d^2}) / (d + \sqrt{-c^2 + d^2})] * \sqrt{(c * \text{Sec}[(e + f * x) / 2]^2 * (c + d * \sin[e + f * x])) / (c^2 - d^2))} / (2 * \sqrt{-a^2 + b^2} * (b * c - a * d)) * (d + \sqrt{-c^2 + d^2})^2 * \sqrt{\sin[e + f * x]} * \sqrt{c + d * \sin[e + f * x]} * \sqrt{-((c * \tan[(e + f * x) / 2]) / (d + \sqrt{-c^2 + d^2}))} - (c * \sqrt{-c^2 + d^2}) * ((- (a * c) + (b + \sqrt{-a^2 + b^2})) * (d + \sqrt{-c^2 + d^2})) * \text{EllipticPi}[(2 * a * \sqrt{-c^2 + d^2}) / (- (b * c) - \sqrt{-a^2 + b^2} * c + a * (d + \sqrt{-c^2 + d^2}))] , \text{ArcSin}[\sqrt{(d + \sqrt{-c^2 + d^2} + c * \tan((e + f * x) / 2)) / \sqrt{-c^2 + d^2}}] / \sqrt{2}] , (2 * \sqrt{-c^2 + d^2}) / (d + \sqrt{-c^2 + d^2})] + (a * c + (-b + \sqrt{-a^2 + b^2})) * (d + \sqrt{-c^2 + d^2})) * \text{EllipticPi}[(2 * a * \sqrt{-c^2 + d^2}) / (- (b * c) + \sqrt{-a^2 + b^2} * c + a * (d + \sqrt{-c^2 + d^2}))] , \text{ArcSin}[\sqrt{(d + \sqrt{-c^2 + d^2} + c * \tan((e + f * x) / 2)) / \sqrt{-c^2 + d^2}}] / \sqrt{2}] , (2 * \sqrt{-c^2 + d^2}) / (d + \sqrt{-c^2 + d^2})] * \sqrt{\sin[e + f * x]} * ((c * d * \cos[e + f * x]) * \text{Sec}[(e + f * x) / 2]^2) / (c^2 - d^2) + (c * \text{Sec}[(e + f * x) / 2]^2 * (c + d * \sin[e + f * x])) * \tan[(e + f * x) / 2] / (c^2 - d^2)) / (2 * \sqrt{-a^2 + b^2} * (b * c - a * d)) * (d + \sqrt{-c^2 + d^2})^2 * \sqrt{c + d * \sin[e + f * x]} * \sqrt{(c * \text{Sec}[(e + f * x) / 2]^2 * (c + d * \sin[e + f * x])) / (c^2 - d^2)} * \sqrt{-((c * \tan[(e + f * x) / 2]) / (d + \sqrt{-c^2 + d^2}))} - (c * \sqrt{-c^2 + d^2}) * \sqrt{\sin[e + f * x]} * \sqrt{(c * \text{Sec}[(e + f * x) / 2]^2 * (c + d * \sin[e + f * x])) / (c^2 - d^2)} * ((c * (- (a * c) + (b + \sqrt{-a^2 + b^2})) * (d + \sqrt{-c^2 + d^2})) * \text{Sec}[(e + f * x) / 2]^2) / (4 * \sqrt{2} * \sqrt{-c^2 + d^2}) * \sqrt{(d + \sqrt{-c^2 + d^2} + c * \tan((e + f * x) / 2)) / \sqrt{-c^2 + d^2}}] * \sqrt{1 - (d + \sqrt{-c^2 + d^2} + c * \tan((e + f * x) / 2)) / (2 * \sqrt{-c^2 + d^2})}] * \sqrt{1 - (d + \sqrt{-c^2 + d^2} + c * \tan((e + f * x) / 2)) / (d + \sqrt{-c^2 + d^2})}] * (1 - (a * (d + \sqrt{-c^2 + d^2} + c * \tan((e + f * x) / 2))) / (- (b * c) - \sqrt{-a^2 + b^2} * c + a * (d + \sqrt{-c^2 + d^2}))) + (c * (a * c + (-b + \sqrt{-a^2 + b^2})) * (d + \sqrt{-c^2 + d^2})) * \text{Sec}[(e + f * x) / 2]^2) / (4 * \sqrt{2} * \sqrt{-c^2 + d^2}) * \sqrt{(d + \sqrt{-c^2 + d^2} + c * \tan((e + f * x) / 2)) / \sqrt{-c^2 + d^2}}] * \sqrt{1 - (d + \sqrt{-c^2 + d^2} + c * \tan((e + f * x) / 2)) / (2 * \sqrt{-c^2 + d^2})} \dots$$

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 2924 vs.  $2(107) = 214$ .

time = 0.39, size = 2925, normalized size = 25.66

method	result	size
default	Expression too large to display	2925

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)`

[Out]  $-1/f * (\text{EllipticPi}(-(\cos(f * x + e) * c - (-c^2 + d^2)^{1/2} * \sin(f * x + e) - d * \sin(f * x + e) - c) / ((-c^2 + d^2)^{1/2} + d) / \sin(f * x + e))^{1/2}, ((-c^2 + d^2)^{1/2} + d) * a / (a * (-c^2 + d^2$



/2)\*(((−c<sup>2</sup>+d<sup>2</sup>)<sup>(1/2)</sup>+d)/(−c<sup>2</sup>+d<sup>2</sup>)<sup>(1/2)</sup>)<sup>(1/2)</sup>)\*c<sup>2</sup>\*(−a<sup>2</sup>+b<sup>2</sup>)<sup>(1/2)</sup>+2\*EllipticPi((−(cos(f\*x+e)\*c−(−c<sup>2</sup>+d<sup>2</sup>)<sup>(1/2)</sup>\*sin(f\*x+e)−d\*sin(f\*x+e)−c)/((−c<sup>2</sup>+d<sup>2</sup>)<sup>(1/2)</sup>+d)/sin(f\*x+e))<sup>(1/2)</sup>,((−c<sup>2</sup>+d<sup>2</sup>)<sup>(1/2)</sup>+d)\*a/(a\*(−c<sup>2</sup>+d<sup>2</sup>)<sup>(1/2)</sup>−c\*(−a<sup>2</sup>+b<sup>2</sup>)<sup>(1/2)</sup>+a\*d−b\*c),1/2\*2<sup>(1/2)</sup>\*(((−c<sup>2</sup>+d<sup>2</sup>)<sup>(1/2)</sup>+d)/(−c<sup>2</sup>+d<sup>2</sup>)<sup>(1/2)</sup>)<sup>(1/2)</sup>)\*d<sup>2</sup>\*(−a<sup>2</sup>+b<sup>2</sup>)<sup>(1/2)</sup>+2\*EllipticPi((−(cos(f\*x+e)\*c−(−c<sup>2</sup>+d<sup>2</sup>)<sup>(1/2)</sup>\*sin(f\*x+e)−d\*sin(f\*x+e)−c)/((−c<sup>2</sup>+d<sup>2</sup>)<sup>(1/2)</sup>+d)/sin(f\*x+e))<sup>(1/2)</sup>,((−c<sup>2</sup>+d<sup>2</sup>)<sup>(1/2)</sup>+d)\*a/(a\*(−c<sup>2</sup>+d<sup>2</sup>)<sup>(1/2)</sup>−c\*(−a<sup>2</sup>+b<sup>2</sup>)<sup>(1/2)</sup>+a\*d−b\*c),1/2\*2<sup>(1/2)</sup>\*(((−c<sup>2</sup>+d<sup>2</sup>)<sup>(1/2)</sup>+d)/(−c<sup>2</sup>+d<sup>2</sup>)<sup>(1/2)</sup>)<sup>(1/2)</sup>)\*d\*(−c<sup>2</sup>+d<sup>2</sup>)<sup>(1/2)</sup>\*(−a<sup>2</sup>+b<sup>2</sup>)<sup>(1/2)</sup>)\*2<sup>(1/2)</sup>\*(g\*sin(f\*x+e))<sup>(1/2)</sup>\*sin(f\*x+e)/(c+d\*sin(f\*x+e))<sup>(1/2)</sup>\*(c\*(−1+cos(f\*x+e)))/((−c<sup>2</sup>+d<sup>2</sup>)<sup>(1/2)</sup>+d)/sin(f\*x+e))<sup>(1/2)</sup>\*((cos(f\*x+e)\*c+(−c<sup>2</sup>+d<sup>2</sup>)<sup>(1/2)</sup>\*sin(f\*x+e)−d\*sin(f\*x+e)−c)/(−c<sup>2</sup>+d<sup>2</sup>)<sup>(1/2)</sup>/sin(f\*x+e))<sup>(1/2)</sup>\*((−cos(f\*x+e)\*c−(−c<sup>2</sup>+d<sup>2</sup>)<sup>(1/2)</sup>\*sin(f\*x+e)−d\*sin(f\*x+e)−c)/((−c<sup>2</sup>+d<sup>2</sup>)<sup>(1/2)</sup>+d)/sin(f\*x+e))<sup>(1/2)</sup>/((−1+cos(f\*x+e))\*a/(−a\*(−c<sup>2</sup>+d<sup>2</sup>)<sup>(1/2)</sup>+c\*(−a<sup>2</sup>+b<sup>2</sup>)<sup>(1/2)</sup>−a\*d+b\*c)/(a\*(−c<sup>2</sup>+d<sup>2</sup>)<sup>(1/2)</sup>+c\*(−a<sup>2</sup>+b<sup>2</sup>)<sup>(1/2)</sup>+a\*d−b\*c)/(−a<sup>2</sup>+b<sup>2</sup>)<sup>(1/2)</sup>

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*sin(f\*x+e))<sup>(1/2)</sup>/(a+b\*sin(f\*x+e))/(c+d\*sin(f\*x+e))<sup>(1/2)</sup>,x, algorithm="maxima")

[Out] integrate(sqrt(g\*sin(f\*x + e))/((b\*sin(f\*x + e) + a)\*sqrt(d\*sin(f\*x + e) + c)), x)

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*sin(f\*x+e))<sup>(1/2)</sup>/(a+b\*sin(f\*x+e))/(c+d\*sin(f\*x+e))<sup>(1/2)</sup>,x, algorithm="fricas")

[Out] Timed out

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{g \sin(e + fx)}}{(a + b \sin(e + fx)) \sqrt{c + d \sin(e + fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sin(f*x+e))**(1/2)/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))**(1/2),x)
[Out] Integral(sqrt(g*sin(e + f*x))/((a + b*sin(e + f*x))*sqrt(c + d*sin(e + f*x))), x)
```

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(g*sin(f*x + e))/((b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)), x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{g \sin(e + f x)}}{(a + b \sin(e + f x)) \sqrt{c + d \sin(e + f x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sin(e + f*x))^(1/2)/((a + b*sin(e + f*x))*(c + d*sin(e + f*x))^(1/2)),x)
```

```
[Out] int((g*sin(e + f*x))^(1/2)/((a + b*sin(e + f*x))*(c + d*sin(e + f*x))^(1/2)), x)
```

### 3.48 $\int \csc(e+fx) \sqrt{a+b \sin(e+fx)} \sqrt{c+d \sin(e+fx)}$

**Optimal.** Leaf size=391

$$\frac{2\sqrt{c+d} \Pi\left(\frac{a(c+d)}{(a+b)c}; \sin^{-1}\left(\frac{\sqrt{a+b} \sqrt{c+d \sin(e+fx)}}{\sqrt{c+d} \sqrt{a+b \sin(e+fx)}}\right) \middle| \frac{(a-b)(c+d)}{(a+b)(c-d)}\right) \sec(e+fx) \sqrt{-\frac{(bc-ad)(1-\sin(e+fx))}{(c+d)(a+b \sin(e+fx))}}}{\sqrt{a+b} f}$$

[Out]  $-2*\text{EllipticPi}((a+b)^{(1/2)}*(c+d*\sin(f*x+e))^{(1/2)}/(c+d)^{(1/2)}/(a+b*\sin(f*x+e))^{(1/2)}, a*(c+d)/(a+b)/c, ((a-b)*(c+d)/(a+b)/(c-d))^{(1/2)})*\sec(f*x+e)*(a+b*\sin(f*x+e))*(c+d)^{(1/2)}*(-(-a*d+b*c)*(1-\sin(f*x+e))/(c+d)/(a+b*\sin(f*x+e)))^{(1/2)}*((-a*d+b*c)*(1+\sin(f*x+e))/(c-d)/(a+b*\sin(f*x+e)))^{(1/2)}/f/(a+b)^{(1/2)}+2*\text{EllipticPi}((a+b)^{(1/2)}*(c+d*\sin(f*x+e))^{(1/2)}/(c+d)^{(1/2)}/(a+b*\sin(f*x+e))^{(1/2)}, b*(c+d)/(a+b)/d, ((a-b)*(c+d)/(a+b)/(c-d))^{(1/2)})*\sec(f*x+e)*(a+b*\sin(f*x+e))*(c+d)^{(1/2)}*(-(-a*d+b*c)*(1-\sin(f*x+e))/(c+d)/(a+b*\sin(f*x+e)))^{(1/2)}*((-a*d+b*c)*(1+\sin(f*x+e))/(c-d)/(a+b*\sin(f*x+e)))^{(1/2)}/f/(a+b)^{(1/2)}$

**Rubi [A]**

time = 0.37, antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$ , Rules used = {3028, 2890, 3024}

$$\frac{2\sqrt{c+d} \sec(e+fx)(a+b \sin(e+fx)) \sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(c+d)(a+b \sin(e+fx))}} \Pi\left(\frac{bc-ad}{(a+b)c}, \text{ArcSin}\left(\frac{\sqrt{a+b} \sqrt{c+d \sin(e+fx)}}{\sqrt{c+d} \sqrt{a+b \sin(e+fx)}}\right) \middle| \frac{(a-b)(c+d)}{(a+b)(c-d)}\right) + 2\sqrt{c+d} \sec(e+fx)(a+b \sin(e+fx)) \sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(c+d)(a+b \sin(e+fx))}} \Pi\left(\frac{bc-ad}{(a+b)d}, \text{ArcSin}\left(\frac{\sqrt{a+b} \sqrt{c+d \sin(e+fx)}}{\sqrt{c+d} \sqrt{a+b \sin(e+fx)}}\right) \middle| \frac{(a-b)(c+d)}{(a+b)(c-d)}\right)}{f\sqrt{a+b}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Csc}[e+f*x]*\text{Sqrt}[a+b*\text{Sin}[e+f*x]]*\text{Sqrt}[c+d*\text{Sin}[e+f*x]],x]$

[Out]  $(-2*\text{Sqrt}[c+d]*\text{EllipticPi}[(a*(c+d))/((a+b)*c), \text{ArcSin}[(\text{Sqrt}[a+b]*\text{Sqrt}[c+d*\text{Sin}[e+f*x]])/(\text{Sqrt}[c+d]*\text{Sqrt}[a+b*\text{Sin}[e+f*x]])], ((a-b)*(c+d))/((a+b)*(c-d))]*\text{Sec}[e+f*x]*\text{Sqrt}[-(((b*c-a*d)*(1-\text{Sin}[e+f*x])))/((c+d)*(a+b*\text{Sin}[e+f*x]))]*\text{Sqrt}[(b*c-a*d)*(1+\text{Sin}[e+f*x])]/((c-d)*(a+b*\text{Sin}[e+f*x]))]*(a+b*\text{Sin}[e+f*x])]/(\text{Sqrt}[a+b]*f) + (2*\text{Sqrt}[c+d]*\text{EllipticPi}[(b*(c+d))/((a+b)*d), \text{ArcSin}[(\text{Sqrt}[a+b]*\text{Sqrt}[c+d*\text{Sin}[e+f*x]])/(\text{Sqrt}[c+d]*\text{Sqrt}[a+b*\text{Sin}[e+f*x]])], ((a-b)*(c+d))/((a+b)*(c-d))]*\text{Sec}[e+f*x]*\text{Sqrt}[-(((b*c-a*d)*(1-\text{Sin}[e+f*x])))/((c+d)*(a+b*\text{Sin}[e+f*x]))]*\text{Sqrt}[(b*c-a*d)*(1+\text{Sin}[e+f*x])]/((c-d)*(a+b*\text{Sin}[e+f*x]))]*(a+b*\text{Sin}[e+f*x])]/(\text{Sqrt}[a+b]*f)$

Rule 2890

$\text{Int}[\text{Sqrt}[(a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]]/\text{Sqrt}[(c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_.)]]], x\_Symbol] \rightarrow \text{Simp}[2*((a + b*\text{Sin}[e + f*x])/(d*f*\text{Rt}[(a + b)/(c + d), 2]*\text{Cos}[e + f*x]))*\text{Sqrt}[(b*c - a*d)*((1 + \text{Sin}[e + f*x])/((c - d)*(a + b*\text{Sin}[e + f*x])))]*\text{Sqrt}[(-b*c - a*d)*((1 - \text{Sin}[e + f*x])/((c + d)*(a + b*\text{Sin}[e + f*x])))]], x\_Symbol]$

```
b*Sin[e + f*x])))*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d))), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

### Rule 3024

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/(sin[(e_) + (f_)*(x_)]*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Simp[-2*((a + b*Sin[e + f*x])/(c*f*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]))*Sqrt[(-b*c - a*d)*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e + f*x])))]*EllipticPi[a*((c + d)/(c*(a + b))), ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d))), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

### Rule 3028

```
Int[(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]])/sin[(e_) + (f_)*(x_)], x_Symbol] := Dist[d, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[c, Int[Sqrt[a + b*Sin[e + f*x]]/(Sin[e + f*x]*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && (NeQ[a^2 - b^2, 0] || NeQ[c^2 - d^2, 0])
```

### Rubi steps

$$\int \csc(e + fx) \sqrt{a + b \sin(e + fx)} \sqrt{c + d \sin(e + fx)} dx = c \int \frac{\csc(e + fx) \sqrt{a + b \sin(e + fx)}}{\sqrt{c + d \sin(e + fx)}} dx + d \int \frac{\csc(e + fx) \sqrt{c + d \sin(e + fx)}}{\sqrt{a + b \sin(e + fx)}} dx$$

$$= -\frac{2\sqrt{c+d} \Pi\left(\frac{a(c+d)}{(a+b)c}; \sin^{-1}\left(\frac{\sqrt{a+b} \sqrt{c+d \sin(e+fx)}}{\sqrt{c+d} \sqrt{a+b \sin(e+fx)}}\right)\right)}{\sqrt{a+bf}}$$

### Mathematica [A]

time = 0.19, size = 274, normalized size = 0.70

$$\frac{2\sqrt{c+d} \left( \Pi\left(\frac{a(c+d)}{(a+b)c}; \sin^{-1}\left(\frac{\sqrt{a+b} \sqrt{c+d \sin(e+fx)}}{\sqrt{c+d} \sqrt{a+b \sin(e+fx)}}\right)\right) \frac{(a-b)(c+d)}{(a+b)(c-d)} - \Pi\left(\frac{b(c+d)}{(a+b)d}; \sin^{-1}\left(\frac{\sqrt{a+b} \sqrt{c+d \sin(e+fx)}}{\sqrt{c+d} \sqrt{a+b \sin(e+fx)}}\right)\right) \frac{(a-b)(c+d)}{(a+b)(c-d)} \right) \sec(e+fx) \sqrt{\frac{(bc-ad)(-1+\sin(e+fx))}{(c+d)(a+b \sin(e+fx))}} \sqrt{\frac{(bc-ad)(1+\sin(e+fx))}{(c-d)(a+b \sin(e+fx))}}}{\sqrt{a+bf}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]], x]
```

```
[Out] (-2*Sqrt[c + d]*(EllipticPi[(a*(c + d))/((a + b)*c), ArcSin[(Sqrt[a + b]*Sqrt[c + d]*Sin[e + f*x])]/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x])]), ((a - b)*(c + d))/((a + b)*(c - d))] - EllipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*Sqrt[c + d]*Sin[e + f*x])]/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x])]), ((a - b)*(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(-1 + Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x]))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x])]/(Sqrt[a + b]*f)
```

**Maple [C]** Result contains complex when optimal does not.

time = 4.44, size = 268466, normalized size = 686.61

method	result	size
default	Expression too large to display	268466

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(1/2)/sin(f*x+e),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(1/2)/sin(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/sin(f*x + e), x)
```

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(1/2)/sin(f*x+e),x, algorithm="fricas")
```

```
[Out] Timed out
```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b \sin(e + fx)} \sqrt{c + d \sin(e + fx)}}{\sin(e + fx)} dx$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**(1/2)*(c+d*sin(f*x+e))**(1/2)/sin(f*x+e),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x))*sqrt(c + d*sin(e + f*x))/sin(e + f*x), x)
```

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(1/2)/sin(f*x+e),x, algor  
ithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/sin(f*x + e), x  
)
```

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a + b \sin(e + f x)} \sqrt{c + d \sin(e + f x)}}{\sin(e + f x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^(1/2))/sin(e + f*x),x)
```

```
[Out] int(((a + b*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^(1/2))/sin(e + f*x), x  
)
```

$$3.49 \quad \int \frac{\csc(e+fx) \sqrt{a+b \sin(e+fx)}}{\sqrt{c+d \sin(e+fx)}} dx$$

Optimal. Leaf size=198

$$\frac{2\sqrt{c+d} \Pi\left(\frac{a(c+d)}{(a+b)c}; \sin^{-1}\left(\frac{\sqrt{a+b} \sqrt{c+d \sin(e+fx)}}{\sqrt{c+d} \sqrt{a+b \sin(e+fx)}}\right) \middle| \frac{(a-b)(c+d)}{(a+b)(c-d)}\right) \sec(e+fx) \sqrt{-\frac{(bc-ad)(1-\sin(e+fx))}{(c+d)(a+b \sin(e+fx))}}}{\sqrt{a+b} cf}$$

[Out]  $-2*\text{EllipticPi}((a+b)^{(1/2)}*(c+d*\sin(f*x+e))^{(1/2)}/(c+d)^{(1/2)}/(a+b*\sin(f*x+e))^{(1/2)}, a*(c+d)/(a+b)/c, ((a-b)*(c+d)/(a+b)/(c-d))^{(1/2)})*\sec(f*x+e)*(a+b*\sin(f*x+e))*(c+d)^{(1/2)}*(-(-a*d+b*c)*(1-\sin(f*x+e))/(c+d)/(a+b*\sin(f*x+e)))^{(1/2)}*((-a*d+b*c)*(1+\sin(f*x+e))/(c-d)/(a+b*\sin(f*x+e)))^{(1/2)}/c/f/(a+b)^{(1/2)}$

Rubi [A]

time = 0.13, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$ , Rules used = {3024}

$$\frac{2\sqrt{c+d} \sec(e+fx)(a+b \sin(e+fx)) \sqrt{-\frac{(bc-ad)(1-\sin(e+fx))}{(c+d)(a+b \sin(e+fx))}} \sqrt{\frac{(bc-ad)(\sin(e+fx)+1)}{(c-d)(a+b \sin(e+fx))}} \Pi\left(\frac{a(c+d)}{(a+b)c}, \text{ArcSin}\left(\frac{\sqrt{a+b} \sqrt{c+d \sin(e+fx)}}{\sqrt{c+d} \sqrt{a+b \sin(e+fx)}}\right) \middle| \frac{(a-b)(c+d)}{(a+b)(c-d)}\right)}{cf\sqrt{a+b}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(\text{Csc}[e+f*x]*\text{Sqrt}[a+b*\text{Sin}[e+f*x]])/\text{Sqrt}[c+d*\text{Sin}[e+f*x]],x]$

[Out]  $(-2*\text{Sqrt}[c+d]*\text{EllipticPi}[(a*(c+d))/((a+b)*c), \text{ArcSin}[(\text{Sqrt}[a+b]*\text{Sqrt}[c+d*\text{Sin}[e+f*x]])/(\text{Sqrt}[c+d]*\text{Sqrt}[a+b*\text{Sin}[e+f*x]])], ((a-b)*(c+d))/((a+b)*(c-d))]*\text{Sec}[e+f*x]*\text{Sqrt}[-(((b*c-a*d)*(1-\text{Sin}[e+f*x])))/((c+d)*(a+b*\text{Sin}[e+f*x]))]*\text{Sqrt}[(b*c-a*d)*(1+\text{Sin}[e+f*x])]/((c-d)*(a+b*\text{Sin}[e+f*x]))]*(a+b*\text{Sin}[e+f*x]))/(\text{Sqrt}[a+b]*c*f)$

Rule 3024

$\text{Int}[\text{Sqrt}[(a_)+(b_)*\sin[(e_)+(f_)*(x_)]]/(\sin[(e_)+(f_)*(x_)]*\text{Sqrt}[(c_)+(d_)*\sin[(e_)+(f_)*(x_)]]), x\_Symbol] \rightarrow \text{Simp}[-2*((a+b*\text{Sin}[e+f*x])/c*f*\text{Rt}[(a+b)/(c+d), 2]*\text{Cos}[e+f*x])*\text{Sqrt}[-(b*c-a*d)*((1-\text{Sin}[e+f*x])/((c+d)*(a+b*\text{Sin}[e+f*x])))]*\text{Sqrt}[(b*c-a*d)*((1+\text{Sin}[e+f*x])/((c-d)*(a+b*\text{Sin}[e+f*x])))]*\text{EllipticPi}[a*((c+d)/(c*(a+b))), \text{ArcSin}[\text{Rt}[(a+b)/(c+d), 2]*(\text{Sqrt}[c+d*\text{Sin}[e+f*x]])/\text{Sqrt}[a+b*\text{Sin}[e+f*x]]], (a-b)*((c+d)/((a+b)*(c-d)))]], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c-a*d, 0] \&\& \text{NeQ}[a^2-b^2, 0] \&\& \text{NeQ}[c^2-d^2, 0]$

### Rubi steps

$$\int \frac{\csc(e + fx) \sqrt{a + b \sin(e + fx)}}{\sqrt{c + d \sin(e + fx)}} dx = -\frac{2\sqrt{c + d} \Pi\left(\frac{a(c+d)}{(a+b)c}; \sin^{-1}\left(\frac{\sqrt{a+b} \sqrt{c + d \sin(e + fx)}}{\sqrt{c+d} \sqrt{a + b \sin(e + fx)}}\right)\right) \Big|_{\frac{(a-b)(c+d)}{(a+b)(c-d)}}}{\sqrt{a+b} cf}$$

### Mathematica [A]

time = 0.11, size = 197, normalized size = 0.99

$$\frac{2\sqrt{c+d} \Pi\left(\frac{a(c+d)}{(a+b)c}; \sin^{-1}\left(\frac{\sqrt{a+b} \sqrt{c+d \sin(e+fx)}}{\sqrt{c+d} \sqrt{a+b \sin(e+fx)}}\right)\right) \Big|_{\frac{(a-b)(c+d)}{(a+b)(c-d)}} \sec(e+fx) \sqrt{\frac{(-bc+ad)(1-\sin(e+fx))}{(c+d)(a+b \sin(e+fx))}} \sqrt{\frac{(bc-ad)(1+\sin(e+fx))}{(c-d)(a+b \sin(e+fx))}} (a+b \sin(e+fx))}{\sqrt{a+b} cf}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[e + f*x]*Sqrt[a + b*Sin[e + f*x]])/Sqrt[c + d*Sin[e + f*x]], x]
```

```
[Out] (-2*Sqrt[c + d]*EllipticPi[(a*(c + d))/((a + b)*c), ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[((-b*c) + a*d)*(1 - Sin[e + f*x]])/((c + d)*(a + b*Sin[e + f*x]))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x])/(Sqrt[a + b]*c*f)
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 1923 vs. 2(183) = 366.

time = 0.59, size = 1924, normalized size = 9.72

method	result	size
default	Expression too large to display	1924

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^(1/2)/sin(f*x+e)/(c+d*sin(f*x+e))^(1/2), x, method=_RETURNVERBOSE)
```

```
[Out] 4/f*((cos(f*x+e)*(-a^2+b^2)^(1/2)-b*cos(f*x+e)-a*sin(f*x+e)+(-a^2+b^2)^(1/2)-b)*(-a*(-c^2+d^2)^(1/2)+c*(-a^2+b^2)^(1/2)-a*d+b*c)/(cos(f*x+e)*(-a^2+b^2)^(1/2)+b*cos(f*x+e)+a*sin(f*x+e)+(-a^2+b^2)^(1/2)+b)/(a*(-c^2+d^2)^(1/2)+c*(-a^2+b^2)^(1/2)+a*d-b*c)^(1/2)*((c*sin(f*x+e)+(-c^2+d^2)^(1/2)*cos(f*x+e)+d*cos(f*x+e)+(-c^2+d^2)^(1/2)+d)/(cos(f*x+e)*(-a^2+b^2)^(1/2)+b*cos(f*x+e)+a*sin(f*x+e)+(-a^2+b^2)^(1/2)+b)*(-a^2+b^2)^(1/2)*a/(a*(-c^2+d^2)^(1/2)+c*(-a^2+b^2)^(1/2)+a*d-b*c)^(1/2)*(-((-c^2+d^2)^(1/2)*cos(f*x+e)-d*cos(f*x+e)-c*sin(f*x+e)+(-c^2+d^2)^(1/2)-d)/(cos(f*x+e)*(-a^2+b^2)^(1/2)+b*cos(f*x+e)+a*sin(f*x+e)+(-a^2+b^2)^(1/2)+b))
```

$$\begin{aligned}
& e) + a \sin(f*x+e) + (-a^2+b^2)^{(1/2)+b} * (-a^2+b^2)^{(1/2)} * a / (-a * (-c^2+d^2)^{(1/2)} \\
& + c * (-a^2+b^2)^{(1/2)+a*d-b*c})^{(1/2)} * (a+b*\sin(f*x+e))^{(1/2)} * (c+d*\sin(f*x+e)) \\
& ^{(1/2)} * (1+\cos(f*x+e))^2 * (-1+\cos(f*x+e))^2 * ((-a^2+b^2)^{(1/2)} * (-c^2+d^2)^{(1/2)} \\
& ) * \cos(f*x+e) * a^2 - (-a^2+b^2)^{(1/2)} * (-c^2+d^2)^{(1/2)} * \cos(f*x+e) * b^2 - (-a^2+b^2 \\
& )^{(1/2)} * (-c^2+d^2)^{(1/2)} * \sin(f*x+e) * a * b + (-a^2+b^2)^{(1/2)} * \cos(f*x+e) * a^2 * d - ( \\
& -a^2+b^2)^{(1/2)} * \cos(f*x+e) * b^2 * d + (-a^2+b^2)^{(1/2)} * \sin(f*x+e) * a^2 * c - (-a^2+b^ \\
& 2)^{(1/2)} * \sin(f*x+e) * a * b * d + (-c^2+d^2)^{(1/2)} * \cos(f*x+e) * a^2 * b - (-c^2+d^2)^{(1/2)} \\
& ) * \cos(f*x+e) * b^3 + (-c^2+d^2)^{(1/2)} * \sin(f*x+e) * a^3 - (-c^2+d^2)^{(1/2)} * \sin(f*x+e \\
& ) * a * b^2 - \cos(f*x+e) * a^3 * c + \cos(f*x+e) * a^2 * b * d + \cos(f*x+e) * a * b^2 * c - \cos(f*x+e) * b \\
& ^3 * d + \sin(f*x+e) * a^3 * d - \sin(f*x+e) * a * b^2 * d - (-a^2+b^2)^{(1/2)} * (-c^2+d^2)^{(1/2)} * \\
& b^2 + (-a^2+b^2)^{(1/2)} * a * b * c - (-a^2+b^2)^{(1/2)} * b^2 * d + (-c^2+d^2)^{(1/2)} * a^2 * b - (- \\
& c^2+d^2)^{(1/2)} * b^3 + a^2 * b * d - b^3 * d * (\text{EllipticPi}(((\cos(f*x+e)) * (-a^2+b^2)^{(1/2)} \\
& - b * \cos(f*x+e) - a * \sin(f*x+e) + (-a^2+b^2)^{(1/2)} - b) * (-a * (-c^2+d^2)^{(1/2)} + c * (-a^2 \\
& + b^2)^{(1/2)} - a * d + b * c) / (\cos(f*x+e) * (-a^2+b^2)^{(1/2)} + b * \cos(f*x+e) + a * \sin(f*x+e) \\
& + (-a^2+b^2)^{(1/2)+b} / (a * (-c^2+d^2)^{(1/2)} + c * (-a^2+b^2)^{(1/2)} + a * d - b * c))^{(1/2)} \\
& , (a * (-c^2+d^2)^{(1/2)} + c * (-a^2+b^2)^{(1/2)} + a * d - b * c) / (a * (-c^2+d^2)^{(1/2)} - c * (-a^ \\
& 2+b^2)^{(1/2)} + a * d - b * c) , ((a * (-c^2+d^2)^{(1/2)} + c * (-a^2+b^2)^{(1/2)} - a * d + b * c) * (a * ( \\
& -c^2+d^2)^{(1/2)} + c * (-a^2+b^2)^{(1/2)} + a * d - b * c) / (-a * (-c^2+d^2)^{(1/2)} + c * (-a^2+b^ \\
& 2)^{(1/2)} + a * d - b * c) / (-a * (-c^2+d^2)^{(1/2)} + c * (-a^2+b^2)^{(1/2)} - a * d + b * c))^{(1/2)} - \\
& \text{EllipticPi}(((\cos(f*x+e)) * (-a^2+b^2)^{(1/2)} - b * \cos(f*x+e) - a * \sin(f*x+e) + (-a^2+b^ \\
& 2)^{(1/2)} - b) * (-a * (-c^2+d^2)^{(1/2)} + c * (-a^2+b^2)^{(1/2)} - a * d + b * c) / (\cos(f*x+e) * (- \\
& a^2+b^2)^{(1/2)} + b * \cos(f*x+e) + a * \sin(f*x+e) + (-a^2+b^2)^{(1/2)+b} / (a * (-c^2+d^2)^{(1/2)} \\
& + c * (-a^2+b^2)^{(1/2)} + a * d - b * c))^{(1/2)} , (b + (-a^2+b^2)^{(1/2)}) * (a * (-c^2+d^2) \\
& ^{(1/2)} + c * (-a^2+b^2)^{(1/2)} + a * d - b * c) / (-b + (-a^2+b^2)^{(1/2)}) / (-a * (-c^2+d^2)^{(1/ \\
& 2)} + c * (-a^2+b^2)^{(1/2)} - a * d + b * c) , ((a * (-c^2+d^2)^{(1/2)} + c * (-a^2+b^2)^{(1/2)} - a * d + \\
& b * c) * (a * (-c^2+d^2)^{(1/2)} + c * (-a^2+b^2)^{(1/2)} + a * d - b * c) / (-a * (-c^2+d^2)^{(1/2)} + c \\
& * (-a^2+b^2)^{(1/2)} + a * d - b * c) / (-a * (-c^2+d^2)^{(1/2)} + c * (-a^2+b^2)^{(1/2)} - a * d + b * c) \\
& )^{(1/2)})) / \sin(f*x+e)^4 / (b * \cos(f*x+e)^2 * d - \sin(f*x+e) * a * d - \sin(f*x+e) * b * c - a * c - \\
& b * d) * a^2 / (-a * (-c^2+d^2)^{(1/2)} + c * (-a^2+b^2)^{(1/2)} - a * d + b * c) / (-a^2+b^2)^{(1/2)} / \\
& (b + (-a^2+b^2)^{(1/2)}) / (-b + (-a^2+b^2)^{(1/2)})
\end{aligned}$$

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sin(f\*x+e))^(1/2)/sin(f\*x+e)/(c+d\*sin(f\*x+e))^(1/2),x, algorith="maxima")

[Out] integrate(sqrt(b\*sin(f\*x + e) + a)/(sqrt(d\*sin(f\*x + e) + c)\*sin(f\*x + e)), x)

**Fricas [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sin(f\*x+e))^(1/2)/sin(f\*x+e)/(c+d\*sin(f\*x+e))^(1/2),x, algorithm="fricas")

[Out] integral(-sqrt(b\*sin(f\*x + e) + a)\*sqrt(d\*sin(f\*x + e) + c)/(d\*cos(f\*x + e)^2 - c\*sin(f\*x + e) - d), x)

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b \sin(e + f x)}}{\sqrt{c + d \sin(e + f x)} \sin(e + f x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sin(f\*x+e))\*\*(1/2)/sin(f\*x+e)/(c+d\*sin(f\*x+e))\*\*(1/2),x)

[Out] Integral(sqrt(a + b\*sin(e + f\*x))/(sqrt(c + d\*sin(e + f\*x))\*sin(e + f\*x)), x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sin(f\*x+e))^(1/2)/sin(f\*x+e)/(c+d\*sin(f\*x+e))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b\*sin(f\*x + e) + a)/(sqrt(d\*sin(f\*x + e) + c)\*sin(f\*x + e)), x)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a + b \sin(e + f x)}}{\sin(e + f x) \sqrt{c + d \sin(e + f x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*sin(e + f\*x))^(1/2)/(sin(e + f\*x)\*(c + d\*sin(e + f\*x))^(1/2)),x)

[Out] int((a + b\*sin(e + f\*x))^(1/2)/(sin(e + f\*x)\*(c + d\*sin(e + f\*x))^(1/2)), x)

$$3.50 \quad \int \frac{\csc(e+fx)}{\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}} dx$$

**Optimal.** Leaf size=398

$$\frac{2\sqrt{c+d} \Pi\left(\frac{a(c+d)}{(a+b)c}; \sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right) \mid \frac{(a-b)(c+d)}{(a+b)(c-d)}\right) \sec(e+fx) \sqrt{-\frac{(bc-ad)(1-\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}}{a\sqrt{a+b}cf}$$

```
[Out] -2*EllipticPi((a+b)^(1/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(1/2), a*(c+d)/(a+b)/c, ((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*sin(f*x+e))*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)/a/c/f/(a+b)^(1/2)-2*b*EllipticF((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2), ((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*(a+b)^(1/2)*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/a/(-a*d+b*c)/f/(c+d)^(1/2)
```

**Rubi [A]**

time = 0.39, antiderivative size = 398, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$ , Rules used = {3026, 2897, 3024}

$$\frac{2\sqrt{a+b} \sec(e+fx) \sqrt{c+d\sin(e+fx)} \sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}} \sqrt{\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}} F\left(\text{ArcSin}\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right) \mid \frac{(a-b)(c+d)}{(a+b)(c-d)}\right) \sec(e+fx) \sqrt{-\frac{(bc-ad)(1-\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}}{af\sqrt{a+b}(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[Csc[e + f\*x]/(Sqrt[a + b\*Sin[e + f\*x]]\*Sqrt[c + d\*Sin[e + f\*x]]), x]

```
[Out] (-2*Sqrt[c + d]*EllipticPi[(a*(c + d))/((a + b)*c), ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x])))/((c + d)*(a + b*Sin[e + f*x]))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x])]/(a*Sqrt[a + b]*c*f) - (2*b*Sqrt[a + b]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))] *Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x])]/(a*Sqrt[c + d]*(b*c - a*d)*f)
```

Rule 2897

```
Int[1/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)])], x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
```

```
)/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d))), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

### Rule 3024

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/(sin[(e_) + (f_)*(x_)]*Sqr
t[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Simp[-2*((a + b*Sin[
e + f*x])/(c*f*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]))*Sqrt[(-(b*c - a*d))*((
1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[(b*c - a*d)*((1 + S
in[e + f*x])/((c - d)*(a + b*Sin[e + f*x])))]*EllipticPi[a*((c + d)/(c*(a +
b))), ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*S
in[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d))), x] /; FreeQ[{a, b, c,
d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
]
```

### Rule 3026

```
Int[1/(sin[(e_) + (f_)*(x_)]*Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*S
qrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[-b/a, Int[1/(
Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] + Dist[1/a, Int[
Sqrt[a + b*Sin[e + f*x]]/(Sin[e + f*x]*Sqrt[c + d*Sin[e + f*x]]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && (NeQ[a^2 - b^2, 0] ||
NeQ[c^2 - d^2, 0])
```

### Rubi steps

$$\int \frac{\csc(e + fx)}{\sqrt{a + b \sin(e + fx)} \sqrt{c + d \sin(e + fx)}} dx = \frac{\int \frac{\csc(e + fx) \sqrt{a + b \sin(e + fx)}}{\sqrt{c + d \sin(e + fx)}} dx}{a} - b \int \frac{1}{\sqrt{a + b \sin(e + fx)}} dx$$

$$= \frac{2\sqrt{c + d} \Pi\left(\frac{a(c + d)}{(a + b)c}; \sin^{-1}\left(\frac{\sqrt{a + b} \sqrt{c + d \sin(e + fx)}}{\sqrt{c + d} \sqrt{a + b \sin(e + fx)}}\right)\right)}{a}$$

### Mathematica [A]

time = 1.65, size = 374, normalized size = 0.94

$$2 \operatorname{sec}(e + fx) \left( \frac{({}^{(c+d)} \operatorname{asin}^{-1} \left( \frac{\sqrt{a+b} \sqrt{c+d \sin(e+fx)}}{\sqrt{c+d} \sqrt{a+b \sin(e+fx)}} \right)) \left( \frac{(bc-ad)(-1+\sin(e+fx))}{(c+d)(a+b \sin(e+fx))} \right) \sqrt{\frac{(bc-ad)(1+\sin(e+fx))}{(c-d)(a+b \sin(e+fx))}}}{({}^{(a+b \sin(e+fx))} \operatorname{asin}^{-1} \left( \frac{\sqrt{c+d} \sqrt{a+b \sin(e+fx)}}{\sqrt{a+b} \sqrt{c+d \sin(e+fx)}} \right)) \left( \frac{(bc-ad)(-1+\sin(e+fx))}{(a+b)(c+d \sin(e+fx))} \right) \sqrt{\frac{(bc-ad)(1+\sin(e+fx))}{(a-b)(c+d \sin(e+fx))}})}{a \sqrt{a+b} \sqrt{c+d} f} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
x]
```

```
[Out] (2*Sec[e + f*x]*(-(((c + d)*EllipticPi[(a*(c + d))/((a + b)*c), ArcSin[(Sqr
t[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])],
((a - b)*(c + d))/((a + b)*(c - d))]*Sqrt[((b*c - a*d)*(-1 + Sin[e + f*x])
)/((c + d)*(a + b*Sin[e + f*x]))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c
- d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x])/c) - (b*(a + b)*Elliptic
F[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin
[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sqrt[((-(b*c) + a*d)*(-1
+ Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[((-(b*c) + a*d)*(1 +
Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x]))]*(c + d*Sin[e + f*x]))/(b*c
- a*d)))/(a*Sqrt[a + b]*Sqrt[c + d]*f)
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 26839 vs.  $2(368) = 736$ .

time = 0.42, size = 26840, normalized size = 67.44

method	result	size
default	Expression too large to display	26840

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(f*x+e)/(a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] result too large to display
```

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(f*x+e)/(a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sin(f*x + e)
), x)
```

**Fricas [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate(1/sin(f\*x+e)/(a+b\*sin(f\*x+e))^(1/2)/(c+d\*sin(f\*x+e))^(1/2),x, algorithm="fricas")

[Out] Timed out

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b \sin(e + fx)} \sqrt{c + d \sin(e + fx)} \sin(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sin(f\*x+e)/(a+b\*sin(f\*x+e))\*\*(1/2)/(c+d\*sin(f\*x+e))\*\*(1/2),x)

[Out] Integral(1/(sqrt(a + b\*sin(e + f\*x))\*sqrt(c + d\*sin(e + f\*x))\*sin(e + f\*x)), x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sin(f\*x+e)/(a+b\*sin(f\*x+e))^(1/2)/(c+d\*sin(f\*x+e))^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(b\*sin(f\*x + e) + a)\*sqrt(d\*sin(f\*x + e) + c)\*sin(f\*x + e)), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sin(e + fx) \sqrt{a + b \sin(e + fx)} \sqrt{c + d \sin(e + fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sin(e + f\*x)\*(a + b\*sin(e + f\*x))^(1/2)\*(c + d\*sin(e + f\*x))^(1/2)), x)

[Out] int(1/(sin(e + f\*x)\*(a + b\*sin(e + f\*x))^(1/2)\*(c + d\*sin(e + f\*x))^(1/2)), x)

### 3.51 $\int (a + a \sin(e + fx))^m (A + B \sin(e + fx))^p (c - c \sin(e + fx))^n dx$

**Optimal.** Leaf size=157

$$\frac{2^{\frac{1}{2}+n} F_1\left(\frac{1}{2} + m; \frac{1}{2} - n, -p; \frac{3}{2} + m; \frac{1}{2}(1 + \sin(e + fx)), -\frac{B(1 + \sin(e + fx))}{A - B}\right) \sec(e + fx) (1 - \sin(e + fx))^{\frac{1}{2}-n} (a + af(1 + 2m))}{af(1 + 2m)}$$

[Out]  $2^{(1/2+n)} \text{AppellF1}(1/2+m, -p, 1/2-n, 3/2+m, -B*(1+\sin(f*x+e))/(A-B), 1/2+1/2*\sin(f*x+e)) * \sec(f*x+e) * (1-\sin(f*x+e))^{(1/2-n)} * (a+a*\sin(f*x+e))^{(1+m)} * (A+B*\sin(f*x+e))^p * (c-c*\sin(f*x+e))^n / a / f / (1+2*m) / (((A+B*\sin(f*x+e))/(A-B))^p)$

**Rubi [A]**

time = 0.20, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {3087, 145, 144, 143}

$$\frac{2^{n+\frac{1}{2}} \sec(e + fx) (1 - \sin(e + fx))^{\frac{1}{2}-n} (a \sin(e + fx) + a)^{m+1} (c - c \sin(e + fx))^n (A + B \sin(e + fx))^p \left(\frac{A+B \sin(e+fx)}{A-B}\right)^{-p} F_1\left(m + \frac{1}{2}; \frac{1}{2} - n, -p; m + \frac{3}{2}; \frac{1}{2}(\sin(e + fx) + 1), -\frac{B(\sin(e+fx)+1)}{A-B}\right)}{af(2m+1)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + a*\text{Sin}[e + f*x])^m * (A + B*\text{Sin}[e + f*x])^p * (c - c*\text{Sin}[e + f*x])^n, x]$

[Out]  $(2^{(1/2 + n)} \text{AppellF1}[1/2 + m, 1/2 - n, -p, 3/2 + m, (1 + \text{Sin}[e + f*x])/2, -(B*(1 + \text{Sin}[e + f*x]))/(A - B)]) * \text{Sec}[e + f*x] * (1 - \text{Sin}[e + f*x])^{(1/2 - n)} * (a + a*\text{Sin}[e + f*x])^{(1 + m)} * (A + B*\text{Sin}[e + f*x])^p * (c - c*\text{Sin}[e + f*x])^n / (a*f*(1 + 2*m) * ((A + B*\text{Sin}[e + f*x])/(A - B))^p)$

Rule 143

$\text{Int}[(a + b*x)^m * (c + d*x)^n * (e + f*x)^p, x\_Symbol] \rightarrow \text{Simp}[(a + b*x)^{m+1} / (b*(m+1) * (b/(b*c - a*d))^{n+1} * (b/(b*e - a*f))^p) * \text{AppellF1}[m+1, -n, -p, m+2, (-d)*((a + b*x)/(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /;$  FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b\*c - a\*d), 0] && GtQ[b/(b\*e - a\*f), 0] && !(GtQ[d/(d\*a - c\*b), 0] && GtQ[d/(d\*e - c\*f), 0] && SimplerQ[c + d\*x, a + b\*x]) && !(GtQ[f/(f\*a - e\*b), 0] && GtQ[f/(f\*c - e\*d), 0] && SimplerQ[e + f\*x, a + b\*x])

Rule 144

$\text{Int}[(a + b*x)^m * (c + d*x)^n * (e + f*x)^p, x\_Symbol] \rightarrow \text{Dist}[(e + f*x)^{\text{FracPart}[p]} / ((b/(b*e - a*f))^{\text{IntPart}[p]} * (b*((e + f*x)/(b*e - a*f)))^{\text{FracPart}[p]}), \text{Int}[(a + b*x)^m * (c + d*x)^n * (b/(b*e - a*f) + b*f*x/(b*e - a*f))^p, x], x] /;$  FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b

\*c - a\*d), 0] && !GtQ[b/(b\*e - a\*f), 0]

### Rule 145

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))
^(p_), x_Symbol] := Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && !GtQ[b/
(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x] && !SimplerQ[e + f*x, a +
b*x]
```

### Rule 3087

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])^(p_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol]
:= Dist[Sqrt[a + b*Sin[e + f*x]]*(Sqrt[c + d*Sin[e + f*x]]/(f*Cos[e + f*x])
), Subst[Int[(a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2)*(A + B*x)^p, x], x, Si
n[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n, p}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 - b^2, 0]
```

### Rubi steps

$$\begin{aligned} \int (a + a \sin(e + fx))^m (A + B \sin(e + fx))^p (c - c \sin(e + fx))^n dx &= \frac{(\sec(e + fx) \sqrt{a + a \sin(e + fx)}) \sqrt{a + a \sin(e + fx)}}{\sec(e + fx) \sqrt{a + a \sin(e + fx)}} (A + B \sin(e + fx))^p (c - c \sin(e + fx))^n \\ &= \frac{(\sec(e + fx) \sqrt{a + a \sin(e + fx)}) (A + B \sin(e + fx))^p (c - c \sin(e + fx))^n}{\sec(e + fx) \sqrt{a + a \sin(e + fx)}} \\ &= \frac{(2^{-\frac{1}{2}+n} \sec(e + fx) \sqrt{a + a \sin(e + fx)}) (A + B \sin(e + fx))^p (c - c \sin(e + fx))^n}{\sec(e + fx) \sqrt{a + a \sin(e + fx)}} \\ &= \frac{2^{\frac{1}{2}+n} F_1\left(\frac{1}{2} + m; \frac{1}{2} - n, -p; \frac{3}{2} + m; \frac{1}{2}\right)}{\sec(e + fx) \sqrt{a + a \sin(e + fx)}} \end{aligned}$$

### Mathematica [A]

time = 0.80, size = 168, normalized size = 1.07

$$\frac{2F_1\left(\frac{1}{2} + n; \frac{1}{2} - n, -p; \frac{3}{2} + n; \cos^2\left(\frac{1}{4}(2e + \pi + 2fx)\right), \frac{2B \sin^2\left(\frac{1}{4}(2e - \pi + 2fx)\right)}{A+B}\right) \cot\left(\frac{1}{4}(2e + \pi + 2fx)\right) (a(1 + \sin(e + fx)))^m (A + B \sin(e + fx))^p \left(\frac{A+B \sin(e + fx)}{A+B}\right)^{-p} (c - c \sin(e + fx))^n \sin^2\left(\frac{1}{4}(2e + \pi + 2fx)\right)^{\frac{1}{2}-m}}{f + 2fn}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a\*Sin[e + f\*x])^m\*(A + B\*Sin[e + f\*x])^p\*(c - c\*Sin[e + f\*x])^n,x]

[Out] (-2\*AppellF1[1/2 + n, 1/2 - m, -p, 3/2 + n, Cos[(2\*e + Pi + 2\*f\*x)/4]^2, (2\*B\*Sin[(2\*e - Pi + 2\*f\*x)/4]^2)/(A + B)]\*Cot[(2\*e + Pi + 2\*f\*x)/4]\*(a\*(1 + Sin[e + f\*x]))^m\*(A + B\*Sin[e + f\*x])^p\*(c - c\*Sin[e + f\*x])^n\*(Sin[(2\*e + Pi + 2\*f\*x)/4]^2)^(1/2 - m)/((f + 2\*f\*n)\*((A + B\*Sin[e + f\*x])/(A + B))^p)

**Maple [F]**

time = 0.20, size = 0, normalized size = 0.00

$$\int (a + a \sin(fx + e))^m (A + B \sin(fx + e))^p (c - c \sin(fx + e))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a\*sin(f\*x+e))^m\*(A+B\*sin(f\*x+e))^p\*(c-c\*sin(f\*x+e))^n,x)

[Out] int((a+a\*sin(f\*x+e))^m\*(A+B\*sin(f\*x+e))^p\*(c-c\*sin(f\*x+e))^n,x)

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*sin(f\*x+e))^m\*(A+B\*sin(f\*x+e))^p\*(c-c\*sin(f\*x+e))^n,x, algorithm="maxima")

[Out] integrate((B\*sin(f\*x + e) + A)^p\*(a\*sin(f\*x + e) + a)^m\*(-c\*sin(f\*x + e) + c)^n, x)

**Fricas [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*sin(f\*x+e))^m\*(A+B\*sin(f\*x+e))^p\*(c-c\*sin(f\*x+e))^n,x, algorithm="fricas")

[Out] integral((B\*sin(f\*x + e) + A)^p\*(a\*sin(f\*x + e) + a)^m\*(-c\*sin(f\*x + e) + c)^n, x)

**Sympy [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*sin(f\*x+e))\*m\*(A+B\*sin(f\*x+e))\*p\*(c-c\*sin(f\*x+e))\*n,x)

[Out] Timed out

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*sin(f\*x+e))^m\*(A+B\*sin(f\*x+e))^p\*(c-c\*sin(f\*x+e))^n,x, algorithm="giac")

[Out] integrate((B\*sin(f\*x + e) + A)^p\*(a\*sin(f\*x + e) + a)^m\*(-c\*sin(f\*x + e) + c)^n, x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int (A + B \sin(e + f x))^p (a + a \sin(e + f x))^m (c - c \sin(e + f x))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*sin(e + f\*x))^p\*(a + a\*sin(e + f\*x))^m\*(c - c\*sin(e + f\*x))^n,x)

[Out] int((A + B\*sin(e + f\*x))^p\*(a + a\*sin(e + f\*x))^m\*(c - c\*sin(e + f\*x))^n, x)



# Chapter 4

## Appendix

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## 4.1 Download section

The following zip files contain the raw integrals used in this test.

**Mathematica format** Mathematica\_syntax.zip

**Maple and Mupad format** Maple\_syntax.zip

**Sympy format** SYMPY\_syntax.zip

**Sage math format** SAGE\_syntax.zip

## 4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

### 4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7, 2022. add second output which gives reason for the grade *)
(*           Small rewrite of logic in main function to make it*)
(*           match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*           GradeAntiderivative[result,optimal] returns*)
(* "F" if the result fails to integrate an expression that*)
(*           is integrable*)
(* "C" if result involves higher level functions than necessary*)
(* "B" if result is more than twice the size of the optimal*)
(*           antiderivative*)
(* "A" if result can be considered optimal*)
```



```

GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
  expnResult = ExpnType[result];
  expnOptimal = ExpnType[optimal];
  leafCountResult = LeafCount[result];
  leafCountOptimal = LeafCount[optimal];

  (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
  If[expnResult<=expnOptimal,
    If[Not[FreeQ[result,Complex]], (*result contains complex*)
      If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
        If[leafCountResult<=2*leafCountOptimal,
          finalresult={"A","none"}
          ,(*ELSE*)
          finalresult={"B","Both result and optimal contain complex but leaf count
        ]
        ,(*ELSE*)
        finalresult={"C","Result contains complex when optimal does not."}
      ]
      ,(*ELSE*)(*result does not contains complex*)
      If[leafCountResult<=2*leafCountOptimal,
        finalresult={"A","none"}
        ,(*ELSE*)
        finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
      ]
    ]
    ,(*ELSE*)(*expnResult>expnOptimal*)
    If[FreeQ[result,Integrate] && FreeQ[result,Int],
      finalresult={"C","Result contains higher order function than in optimal. Order "<
    ,
    finalresult={"F","Contains unresolved integral."}
  ]
];

finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)

```



```

ExpIntegralE, ExpIntegralEi, LogIntegral,
SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
Gamma, LogGamma, PolyGamma,
Zeta, PolyLog, ProductLog,
EllipticF, EllipticE, EllipticPi
},func]

HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]

AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]

```

## 4.2.2 Maple grading function

```

# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017 Use Maple leaf count instead since buildin
#Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017 corrected the check for complex result
#Nasser 10/27/2017 check for leafsize and do not call ExpnType()
#
# if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
#
# see problem 156, file Apostol_Problems
#Nasser 4/07/2022 add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
      leaf_count_optimal,
      ExpnType_result,
      ExpnType_optimal,
      debug:=false;

  leaf_count_result:=leafcount(result);
  #do NOT call ExpnType() if leaf size is too large. Recursion problem
  if leaf_count_result > 500000 then
    return "B","result has leaf size over 500,000. Avoiding possible recursion issues";
  fi;

  leaf_count_optimal := leafcount(optimal);
  ExpnType_result := ExpnType(result);
  ExpnType_optimal := ExpnType(optimal);

```

```

    if debug then
        print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
    fi;

# If result and optimal are mathematical expressions,
# GradeAntiderivative[result,optimal] returns
# "F" if the result fails to integrate an expression that
#   is integrable
# "C" if result involves higher level functions than necessary
# "B" if result is more than twice the size of the optimal
#   antiderivative
# "A" if result can be considered optimal

#This check below actually is not needed, since I only
#call this grading only for passed integrals. i.e. I check
#for "F" before calling this. But no harm of keeping it here.
#just in case.

if not type(result,freeof('int')) then
    return "F","Result contains unresolved integral";
fi;

if ExpnType_result<=ExpnType_optimal then
    if debug then
        print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A","";
            else
                return "B",cat("Both result and optimal contain complex but leaf count of r
                    convert(leaf_count_result,string)," vs. $2 (" ,
                    convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co

        end if
    else #result contains complex but optimal is not
        if debug then
            print("result contains complex but optimal is not");
        fi;
        return "C","Result contains complex when optimal does not.";
    fi;
else # result do not contain complex

```

```

    # this assumes optimal do not as well. No check is needed here.
    if debug then
        print("result do not contain complex, this assumes optimal do not as well")
    fi;
    if leaf_count_result<=2*leaf_count_optimal then
        if debug then
            print("leaf_count_result<=2*leaf_count_optimal");
        fi;
        return "A","";
    else
        if debug then
            print("leaf_count_result>2*leaf_count_optimal");
        fi;
        return "B",cat("Leaf count of result is larger than twice the leaf count of o
                        convert(leaf_count_result,string)," $ vs. $2(",
                        convert(leaf_count_optimal,string),")=",convert(2*leaf_cou

    fi;
    fi;
else #ExpnType(result) > ExpnType(optimal)
    if debug then
        print("ExpnType(result) > ExpnType(optimal)");
    fi;
    return "C",cat("Result contains higher order function than in optimal. Order ",
                  convert(ExpnType_result,string)," vs. order ",
                  convert(ExpnType_optimal,string),".");
fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
    return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function

```

```

# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'^^') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
      max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'+`) or type(expn,'*`) then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
    9
  end if
end proc:

ElementaryFunctionQ := proc(func)
  member(func,[
    exp,log,ln,
    sin,cos,tan,cot,sec,csc,

```

```

    arcsin,arccos,arctan,arccot,arcsec,arccsc,
    sinh,cosh,tanh,coth,sech,csch,
    arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func, [
    erf,erfc,erfi,
    FresnelS,FresnelC,
    Ei,Ei,Li,Si,Ci,Shi,Chi,
    GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
    EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func, [Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func, [AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u)
  else
    apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
  MmaTranslator[Mma][LeafCount](u);
end proc:

```

### 4.2.3 Sympy grading function

```

#Dec 24, 2019. Nasser M. Abbasi:
#      Port of original Maple grading function by
#      Albert Rich to use with Sympy/Python
#Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
#      added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
        asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
        asinh,acosh,atanh,acoth,asech,acsch
    ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
        fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
        gamma,loggamma,digamma,zeta,polylog,LambertW,
        elliptic_f,elliptic_e,elliptic_pi,exp_polar
    ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

```



```

except AttributeError as error:
    return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow): #type(expn,'^')
        if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
            return expnType(expn.args[0]) #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'+' or type(expn,'*')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:

```

```

    m1 = max(map(expnType, list(expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation = ""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation = ""
                    else:
                        grade = "B"
                        grade_annotation = "Both result and optimal contain complex but leaf count of result is larger"
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation = "Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation = ""
                else:
                    grade = "B"
                    grade_annotation = "Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_count_result)
            else:
                grade = "C"
                grade_annotation = "Result contains higher order function than in optimal. Order "+str(ExpnType_result)

```

```

# print("Before returning. grade=", grade, " grade_annotation=", grade_annotation)

return grade, grade_annotation

```

#### 4.2.4 SageMath grading function

```

# Dec 24, 2019. Nasser: Ported original Maple grading function by
#       Albert Rich to use with Sagemath. This is used to
#       grade Fracas, Giac and Maxima results.
# Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#       'arctan2', 'floor', 'abs', 'log_integral'
# June 4, 2022 Made default grade_annotation "none" instead of "" due
#       issue later when reading the file.
# July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    # print("Enter tree_size, expr is ", expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow: # isinstance(expr, Pow):
        if expr.operands()[1] == 1/2: # expr.args[1] == Rational(1,2):
            if debug: print("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

```

```

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
        'sin','cos','tan','cot','sec','csc',
        'arcsin','arccos','arctan','arccot','arcsec','arccsc',
        'sinh','cosh','tanh','coth','sech','csch',
        'arcsinh','arccosh','arctanh','arcoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
    ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")

    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral',
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
        'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func , " is special_function")
        else:
            print ("func ", func , " is NOT special_function")

    return m

def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric'] #[appellf1] can't find this in sagemath

```

```

def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-try:
    if expn.parent() is SR:
        return expn.operator() is None
    if expn.parent() in (ZZ, QQ, AA, QQbar):
        return expn in expn.parent() # Should always return True
    if hasattr(expn.parent(), "base_ring") and hasattr(expn.parent(), "gens"):
        return expn in expn.parent().base_ring() or expn in expn.parent().gens()

    return False

except AttributeError as error:
    print("Exception,AttributeError in is_atom")
    print ("caught exception" , type(error).__name__ )
    return False

def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list: #isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow: #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer: #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0]) #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational: #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)

```

```

    return 1
  else:
    return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.args[0]))
  else:
    return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()): #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(4,m1) #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(5,m1) #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(6,m1) #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
    #is checked before calling the grading function that is passed.
    #but kept it here.
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):

    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

```

```

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation = "none"
            else:
                grade = "B"
                grade_annotation = "Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation = "Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation = "none"
        else:
            grade = "B"
            grade_annotation = "Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation = "Result contains higher order function than in optimal. Order "+str(expnType_resu

print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation

```